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Book reviews

Mark Elin, Simeon Reich and David Shoikhet, Numerical range of holomorphic mappings and applications, Cham: Birkhäuser xiv + 229 p. 2019.
ISBN 978-3-030-05019-1/hbk; 978-3-030-05020-7/ebook.[1]

For a closed linear operator A defined on a dense subspace $\mathcal{D}_A$ of a Hilbert space $\mathcal{H}$ the numerical range is defined by

$$V(A) = \{\langle Ax, x\rangle : x \in \mathcal{D}_A,\ \|x\| = 1\}.$$

This definition was extended by Lumer (1961) to a complex Banach space X:

$$V(A) = \{\langle Ax, x^*\rangle : x \in \mathcal{D}_A,\ \|x\| = 1,\ x^* \in J(x)\},$$

where $J(x) = \{x^* \in X^* : \langle x, x^*\rangle = \|x\|^2 = \|x^*\|^2\}$ and X^* is the dual of X.

In both cases the numerical radius of the operator A is defined by

$$|V(A)| = \sup\{|\lambda| : \lambda \in V(A)\}.$$

The operator A is called Hermitian if $V(A) \subset \mathbb{R}$ and dissipative if

$$\{\operatorname{Re}\lambda : \lambda \in V(A)\} \subset (-\infty, 0],$$

or, equivalently, $\|(tI - A)x\| \geq t\|x\|$ for all $x \in \mathcal{D}_A$ and all $t > 0$.

Numerical ranges turned out to be an essential tool in the study of semigroups of linear operators on Banach spaces, mainly due to the famous Lumer-Phillips theorem (1961): if A is dissipative and for some $\lambda_0 > 0$ (hence, for all $\lambda > 0$) $(\lambda_0 I - A)\mathcal{D}_A = X$, then A is the infinitesimal generator of a C_0-semigroup of linear contractions on X.

The theory of linear semigroups of operators on a Banach space is presented in the first chapter of the book, including the Hille-Yosida and Lumer-Phillips theorems, the analytic extension of semigroups of linear operators, as well as an overview of ergodic theory with emphasis on some classical and recent results on Cesàro and Abel averages.

Harris (1971) defined the numerical range for holomorphic functions in the following way. For a convex domain $\mathcal{D}$ in a Banach space X and $x \in \partial\mathcal{D}$ put

$$Q(x) = \{\ell \in X^* : \ell(x) = 1, \operatorname{Re}\ell(y) \leq 1,\ \forall y \in \mathcal{D}\}.$$

[1]Our colleague, Prof. Gabriela Kohr, enthusiastically embarked on writing this review. Unfortunately, her untimely death, a sad loss for all of us, prevented her from completing this task. This review is dedicated to her fond memory (S.C.).

For a holomorphic function $h : \mathcal{D} \to X$ admitting a continuous extension to $\overline{\mathcal{D}}$ define the numerical range of h on $\mathcal{D}$ by

$$V_{\mathcal{D}}(h) = \{\ell(h(x)) : x \in \partial\mathcal{D},\ \ell \in Q(x)\}\,,$$

and let $|V_{\mathcal{D}}(h)| = \sup\{|\lambda| : \lambda \in V_{\mathcal{D}}(h)\}$ be the numerical radius of the function h on $\mathcal{D}$.

In the second chapter, *Numerical range*, the authors generalize Harris' theory of the numerical range of holomorphic mappings. The main properties of the so-called quasi-dissipative mappings and their growth estimates are studied, including a nonlinear analog of the Lumer-Phillips theorem in the study of nonlinear semigroups and their applications to evolution equations and to geometric properties of holomorphic mappings in finite- and infinite-dimensional Banach spaces.

Another area of applications is that of fixed points, treated in the third chapter, 3. *Fixed points of holomorphic mappings.* The classical result in this field is the Grand Fixed Point Theorem (as it is called by the authors) due to Denjoy and Wolff (1926), but some extensions of the Earle-Hamilton and Bohl-Poincaré-Krasnoselskii Theorems, including their connections with Schwarz-Pick systems of pseudometrics and pseudo-contractive mappings, are presented as well. As the authors mention, another goal is to prove existence and uniqueness of fixed points of holomorphic mappings (not necessarily bounded) acting on the open unit ball of a Banach space.

A good companion in reading this chapter is the book by

S. Reich and D. Shoikhet, *Nonlinear semigroups, fixed points, and geometry of domains in Banach spaces*, Imperial College Press, London, 2005,

where, in the fifth chapter, Denjoy-Wolff type results are presented at length.

Ch. 4, *Semigroups of holomorphic mappings*, is concerned with certain autonomous dynamical systems acting on the open unit ball of a complex Banach space. This study is motivated by the fact that if such a system is differentiable with respect to time, then its derivative is a holomorphically dissipative mapping.

In Ch. 5, *The ergodic theory for holomorphic mappings*, the ergodic properties of a holomorphic mapping around its fixed points are studied. Special attention is paid to the so-called power bounded, dissipative and pseudo-contractive mappings.

The last chapter of the book, Ch. 6. *Applications*, is devoted to applications of the numerical range to diverse geometric and analytic problems – radii of starlikeness and spirallikeness, semigroups of composition operators on H^p-spaces, etc.

Combining methods from various areas of mathematical analysis (understood in a wide sense – functional analysis, operator theory, operator equations, holomorphic vector functions) and presenting both classical results and new developments, many of the latter due to the authors, this fine book reflects the authors' encyclopedic knowledge of mathematics as well as their ability to present the results in an accessible and clear manner. The book sheds a new light on the numerical range of holomorphic mappings and its applications and invites people, especially young researchers, to push further research in these areas.

S. Cobzaş

Dorin Andrica and Ovidiu Bagdasar, Recurrent sequences. Key results, applications, and problems, Problem Books in Mathematics. Cham: Springer 2020, xiv+402 p.
ISBN 978-3-030-51501-0/hbk; 978-3-030-51502-7/ebook.

The recurrence is a central theme in many fields of mathematics, primarily in the study of dynamical systems, but also in the theory of algorithms, numerical analysis, etc. It has deep applications in biology, physics, computer science, signal processing, economics.

The present book is devoted to sequences defined by recurrence relations, both in the real and complex field of numbers. The first two chapters, 1. *Introduction to recurrence relations* and 2. *Basic recurrent sequences*, are concerned with some fundamental results on recurrence sequences (including existence and uniqueness) and the basic recurrent sequences and polynomials – Fibonacci, Lucas, Pell, or Lucas-Pell. Homographic recurrences defined by linear fractional transformations in the complex plane are also discussed in the second chapter. The reading of these chapters, as well as of Chpter 5, requires only college algebra, complex numbers, analysis, and basic combinatorics, while for Chapters 3, 4, and 6, some basic results in number theory, linear algebra, and complex analysis are needed.

Chapter 3, *Arithmetic and trigonometric properties of some classical recurrent sequences*, is concerned with further properties and formulae for some classical recurrent sequences, while Chapter 4, *Generating functions*, treats the important topic of generating functions, both ordinary and exponential, for classical recurrent sequences. This chapter also contains a new version of Cauchy's integral formula, obtained by the authors, with applications to exact integral formulae for the coefficients of some classical polynomials as well as for some classical sequences.

Chapter 5, *More on second-order linear recurrent sequences*, is mainly concerned with the important class of Horadam sequences, including graphical representations in the complex plane of the orbits of these sequences. Here many original results of the authors are included.

Chapter 6, *Higher order linear recurrent sequences*, presents the dynamics of complex linear recurrent sequences of higher order and investigates the periodicity, geometric structure, and enumeration of the periodic patterns.

Chapter 7, *Recurrences in Olympiad training,* contains 123 olympiad training problems involving recurrent sequences, solved in detail in Chapter 8. *Solutions to proposed problems.*

Written in a clear and alive style, the book contains many results concerning recurrent sequences, reflecting the current research in the field and including authors' contributions. The theoretical results are illustrated by numerous examples and diagrams and practical applications to algebra, number theory, geometry of the complex plane, discrete mathematics, or combinatorics, are given.

The book is of interest to researchers working in this area and in related domains, but college or university students and their instructors will also find a lot of useful material in it.

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