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ON S-DISCONNECTED SPACES

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Abstract. The structure of the class of S-disconnected spaces is studied. Two types of S-disconnectedness of topological spaces are introduced. Properties of these spaces in the context of connectedness of spaces are investigated.

1. Introduction

A certain class of non-Hausdorff spaces, called irreducible spaces, was introduced by MacDonald [18]. Pipitone and Russo [27] have defined S-connected spaces. In [34] Thompson proved that these two notions are equivalent. It should be also noticed that Levine has defined the so-called D-spaces [16], which are irreducible spaces, in fact. On the other hand, the notion of hyperconnected spaces, due to Steen and Seebach [32] is equivalent to the notion of D-spaces (Sharma [29]). Some properties of hyperconnected spaces were investigated by Noiri [22].

2. Preliminaries

Throughout the present paper (X, τ) and (Y, σ) denote topological spaces on which no separation axioms are assumed. The closure (resp. interior) in (X, τ) of a subset S of (X, τ) will be denoted by $\mathrm{cl}\,(S)$ (resp. $\mathrm{int}\,(S)$). The set S is said to be regular open (resp. regular closed) in (X, τ), if $S = \mathrm{int}\,(\mathrm{cl}\,(S))$ (resp. $S = \mathrm{cl}\,(\mathrm{int}\,(S))$. A subset S of X is said to be semi-open [15] (resp. α-open [21]) if $S \subset \mathrm{cl}\,(\mathrm{int}\,(S))$ (resp. $S \subset \mathrm{int}\,(\mathrm{cl}\,(\mathrm{int}\,(S))))$. Levine defined [15] S as semi-open if there exists an open subset G of (X, τ) such that $G \subset S \subset \mathrm{cl}\,(G)$. The complement of a semi-open set is said to be semi-closed [4]. The semi-closure of a subset S of (X, τ) [4], denoted by $\mathrm{scl}\,(S)$, is defined as an intersection of all semi-closed sets of (X, τ) containing S. The set $\mathrm{scl}\,(S)$ is semi-closed. The semi-interior of S in (X, τ) [4], denoted by $\mathrm{sint}\,(S)$, is defined as a union of all semi-open subsets A of (X, τ) such that $A \subset S$. It is well known that

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$X \setminus \operatorname{sint}(A) = \operatorname{scl}(X \setminus A)$ and $X \setminus \operatorname{scl}(A) = \operatorname{sint}(X \setminus A)$ [4, Theorem 1.6]. The family of all semi-open (resp. semi-closed; α-open; closed; regular open) subsets of (X,τ) we denote by $\mathrm{SO}(X,\tau)$ (resp. $\mathrm{SC}(X,\tau)$; τ^α; $\mathrm{c}(\tau)$; $\mathrm{RO}(X,\tau)$). The family τ^α forms a topology on X, different from τ, in general. The following inclusions hold in each (X,τ): $\tau \subset \tau^\alpha \subset \mathrm{SO}(X,\tau)$. The inclusion $\tau \subset \mathrm{SO}(X,\tau)$ implies $\mathrm{c}(\tau) \subset \mathrm{SC}(X,\tau)$. The reverses of these inclusions are not necessarily true, in general. A topological space (X,τ) is said to be semi-connected (briefly: S-connected), if X is not the union of two disjoint nonempty semi-open subsets of (X,τ). In the opposite case (X,τ) is called semi-disconnected (briefly: S-disconnected). Pipitone and Russo [27, Esempio 3.3, 11, p. 30] showed that connectedness does not imply S-connectedness, in general. A topological space (X,τ) is said to be extremally disconnected (briefly: e.d.), if $\mathrm{cl}(G) \in \tau$ for each $G \in \tau$.

3. p. S-disconnectedness and s.p. S-disconnectedness

In 1983 Janković proved the following characterization of e.d. spaces: *an (X,τ) is e.d. if and only if* $\mathrm{SO}(X,\tau) = \tau^\alpha$ [13, Theorem 2.9(f)]. Later (in 1984), Reilly and Vamanamurthy showed that (X,τ) is disconnected if and only if (X,τ^α) is disconnected [28, Theorem 2]. These two theorems give a motivation to investigate S-disconnectedness of not e.d. spaces from the connectedness point of view. For e.d. spaces we have what follows: *an (X,τ) is disconnected if and only if it is S-disconnected* [12, Theorem 3.2(2)].

Definition 3.1. A not e.d. topological space (X,τ) is called to be ***properly S-disconnected*** (briefly: ***p. S-disc.***), if there exist $A, U \subset X$ such that $A \in \mathrm{SO}(X,\tau) \setminus \tau^\alpha$, $U \in \tau^\alpha$, $U \cup A = X$, and $U \cap A = \emptyset$.

Theorem 3.2. *Let (X,τ) be a topological space. The following are equivalent:*

1. *(X,τ) is p. S-disc.*
2. *There exist $A, U \subset X$ such that $A \in \mathrm{SO}(X,\tau) \setminus \tau^\alpha$, $U \in \mathrm{RO}(X,\tau)$, $U \cup A = X$, and $U \cap A = \emptyset$.*
3. *There exist $A, U \subset X$ such that $A \in \mathrm{SO}(X,\tau) \setminus \tau$, $U \in \mathrm{RO}(X,\tau)$, $U \cup A = X$, and $U \cap A = \emptyset$.*
4. *There exist $A, U \subset X$ such that $A \in \mathrm{SO}(X,\tau) \setminus \tau^\alpha$, $U \in \tau$, $U \cup A = X$, and $U \cap A = \emptyset$.*
5. *There exist $A, U \subset X$ such that $A \in \mathrm{SO}(X,\tau) \setminus \mathrm{RO}(X,\tau)$, $U \in \mathrm{RO}(X,\tau)$, $U \cup A = X$, and $U \cap A = \emptyset$.*
6. *There exist $A, U \subset X$ such that $A \in \mathrm{SO}(X,\tau) \setminus \tau$, $U \in \tau$, $U \cup A = X$, and $U \cap A = \emptyset$.*

Proof. Implications: (2)⇒(3), (4)⇒(1), (3)⇒(5), (3)⇒(6) are obvious.
(1)⇒(2). By hypothesis there exist sets $A \in \mathrm{SO}(X,\tau) \setminus \tau^\alpha$, $U \in \tau^\alpha$ such that $U \cup A = X$ and $U \cap A = \emptyset$. (2) follows from [9, Lemma 2.2], because $U \in \tau^\alpha \cap \mathrm{SC}(X,\tau)$.
(3)⇒(4). Let $A, U \subset X$ be such that $A \in \mathrm{SO}(X,\tau) \setminus \tau$, $U \in \mathrm{RO}(X,\tau)$, $U \cup A = X$, and $U \cap A = \emptyset$. Suppose $A \in \tau^\alpha \setminus \tau$. Hence $A \subset \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))$ and A is regular closed. Therefore $A \in \tau$. A contradiction.
(5)⇒(3). Suppose $A \in \tau \setminus \mathrm{RO}(X,\tau)$. Then $A = \mathrm{int}(A) = \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))$, because A is regular closed. Hence A is regular open. A contradiction.
(6)⇒(3). Use [9, Lemma 2.2(2)]. □

Let us remark that in Definition 3.1 and in conditions (2)–(6) of Theorem 3.2 we have $\emptyset \neq U \neq X$.

Example 3.3. (a). Consider $X = \{a,b,c\}$ with the topology

$$\tau = \big\{\emptyset, X, \{a\}, \{b\}, \{a,b\}\big\}.$$

Since $\mathrm{SO}(X,\tau) = \big\{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}\big\}$ and $\tau^\alpha = \tau$, then the equality $X = \{a\} \cup \{b,c\}$ implies p. S-disconnectedness of (X,τ).
(b). Take the space of reals $\mathbb{R}$ with the usual topology. Then $\mathbb{R}$ is p. S-disc., since $\mathbb{R} = (-\infty, a] \cup (a, +\infty)$.

Definition 3.4. A not e.d. topological space (X,τ) is called to be ***super-properly S-disconnected*** (briefly: ***s.p. S-disc.***), if there exist $A, B \subset X$ such that $A, B \in \mathrm{SO}(X,\tau) \setminus \tau^\alpha$, $A \cup B = X$, and $A \cap B = \emptyset$.

Example 3.5. Let $X = \{a,b,c,d\}$ and $\tau = \big\{\emptyset, X, \{a\}, \{d\}, \{a,d\}\big\}$. Since $\{a,b\}$, $\{c,d\} \in \mathrm{SO}(X,\tau) \setminus \tau^\alpha$ and $X = \{a,b\} \cup \{c,d\}$, then (X,τ) is s.p. S-disc.

It should be noticed that the space from Example 3.3 is not s.p. S-disc.

The following remark is obvious.

Remark 3.6. A topological space (X,τ) is S-disconnected if and only if (X,τ) is s.p. S-disc. or p. S-disc. or disconnected.

If (X,τ) is p. S-disc. or s.p. S-disc., then there exists $A \in \mathrm{SO}(X,\tau) \setminus \tau$. The reverse implication is not true, in general, as the following example shows.

Example 3.7. Let $X = \{a,b,c\}$, $\tau = \big\{\emptyset, X, \{a\}\big\}$. For this space we have $\mathrm{SO}(X,\tau) \setminus \tau = \big\{\{a,b\}, \{a,c\}\big\}$.

Observe that the spaces in Examples 3.3 and 3.5 are connected.

Remark 3.8. Example 3.7 shows that there exists a connected space, which is not p. S-disc.

Example 3.9. Let $X = \mathbb{R}^2 \setminus D$, where $D = \{(x, y) : x = 0\}$. In X consider the subset topology τ of the Euclidean topology of the plane. If $U = \{(x, y) \in X : x < 0\}$ and $V = \{(x, y) \in X : x > 0\}$, then it is clear that (X, τ) is not connected. Let now

$$A = \{(x, y) \in X : y < 0\} \cup \{(x, y) \in X : y = 0, x \in \mathbb{Q}\},$$
$$B = \{(x, y) \in X : y > 0\} \cup \{(x, y) \in X : y = 0, x \in \mathbb{R} \setminus \mathbb{Q}\},$$

where $\mathbb{Q}$ stands for the set of rationals. One easily checks that $A, B \in \mathrm{SO}(X, \tau) \setminus \tau^\alpha$. This shows that (X, τ) is s.p. S-disc. Note that if $a < b$ and $ab \neq 0$, then we can put also

$$A = \{(x, y) \in X : y < 0\} \cup \{(x, y) \in X : y = 0,\ x = a \text{ or } x > b\},$$
$$B = \{(x, y) \in X : y > 0\} \cup \{(x, y) \in X : y = 0,\ x < a \text{ or } a < x \leq b\}.$$

Example 3.10. Let $X = \{a, b, c, d\}$ and

$$\tau = \big\{\emptyset, X, \{a\}, \{b, c, d\}, \{b\}, \{d\}, \{b, d\}, \{a, b\}, \{a, d\}, \{a, b, d\}\big\}.$$

For this space we have $\tau = \tau^\alpha$ and $\mathrm{SO}(X, \tau) = \tau \cup \big\{\{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}\big\}$. Partitions $X = \{a\} \cup \{b, c, d\} = \{a, d\} \cup \{b, c\}$ show respectively that (X, τ) is disconnected and p. S-disc. One observes that this space is not s.p. S-disc.

Example 3.11. Let $X = \{a, b, c\}$, $\tau = \big\{\emptyset, X, \{a\}, \{b, c\}\big\}$. The space (X, τ) is disconnected and not p. S-disc.

Theorem 3.12. *A topological space (X, τ) is s.p. S-disc. if and only if there exists a set $A \in \mathrm{SO}(X, \tau) \setminus \tau^\alpha$ with $\mathrm{scl}(A) \in \big(\mathrm{SO}(X, \tau) \setminus \tau^\alpha\big) \cap \big(\mathrm{SC}(X, \tau) \setminus \mathrm{c}(\tau^\alpha)\big)$.*

Proof. Necessity. Let (X, τ) be s.p. S-disc., i.e., for certain $A, B \in \mathrm{SO}(X, \tau) \setminus \tau^\alpha$ we have $X = A \cup B$ and $A \cap B = \emptyset$. Clearly $A, B \in \mathrm{SC}(X, \tau) \setminus \mathrm{c}(\tau^\alpha)$. Thus for A we obtain $\mathrm{scl}(A) = A \in \big(\mathrm{SO}(X, \tau) \setminus \tau^\alpha\big) \cap \big(\mathrm{SC}(X, \tau) \setminus \mathrm{c}(\tau^\alpha)\big)$ (analogously for B).
Sufficiency. Let (X, τ) be such a space that for a certain $U \in \mathrm{SO}(X, \tau) \setminus \tau^\alpha$ we have $\mathrm{scl}(U) \in \big(\mathrm{SO}(X, \tau) \setminus \tau^\alpha\big) \cap \big(\mathrm{SC}(X, \tau) \setminus \mathrm{c}(\tau^\alpha)\big)$. Put $A = \mathrm{scl}(U)$. So, for $B = X \setminus \mathrm{scl}(U)$ we infer without difficulties that $B \in \mathrm{SO}(X, \tau) \setminus \tau^\alpha$. Therefore (X, τ) is s.p. S-disc. and the proof is complete. □

Lemma 3.13. *Assume that for a (X, τ) the two conditions below hold.*

- ($\star$) *There exist disjoint subsets $A \in \mathrm{SO}(X, \tau) \setminus \tau^\alpha$, $B \in \mathrm{SO}(X, \tau) \setminus \{\emptyset\}$ with $X = A \cup B$.*
- ($\star\star$) *There exists a point $x \in \big(A \setminus \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))\big) \setminus (\mathrm{cl}(B) \setminus B)$, where $\mathrm{cl}(B) \neq X$.*

Then (X, τ) is disconnected.

Proof. Suppose (X,τ) is connected. We have

$$X = \operatorname{int}\big(\operatorname{cl}(\operatorname{int}(A)) \cup \operatorname{cl}(\operatorname{int}(B))\big) \subset \operatorname{int}(\operatorname{cl}(\operatorname{int}(A))) \cup \operatorname{cl}(\operatorname{int}(B)) \subset X$$

(see [1, Lemma 1.1]) and $\operatorname{int}(A) \neq \emptyset \neq \operatorname{int}(B)$. Thus, $X = \operatorname{int}(\operatorname{cl}(\operatorname{int}(A))) \cup \operatorname{cl}(\operatorname{int}(B))$. One easily checks that

$$\operatorname{int}(\operatorname{cl}(\operatorname{int}(A))) \cap \operatorname{cl}(\operatorname{int}(B)) = \emptyset \tag{3.1}$$

and similarly

$$\operatorname{int}(\operatorname{cl}(\operatorname{int}(B))) \cap \operatorname{cl}(\operatorname{int}(A)) = \emptyset. \tag{3.2}$$

Since $\operatorname{int}(\operatorname{cl}(\operatorname{int}(A))) \cap \operatorname{int}(\operatorname{cl}(\operatorname{int}(B))) = \emptyset$, $\operatorname{int}(\operatorname{cl}(\operatorname{int}(A))) \neq \emptyset \neq \operatorname{int}(\operatorname{cl}(\operatorname{int}(B)))$, we infer from the supposition that $X \setminus \big(\operatorname{int}(\operatorname{cl}(\operatorname{int}(A))) \cup \operatorname{int}(\operatorname{cl}(\operatorname{int}(B)))\big) \neq \emptyset$. So, we obtain $X = \operatorname{int}(\operatorname{cl}(\operatorname{int}(A))) \cup \operatorname{int}(\operatorname{cl}(\operatorname{int}(B))) \cup (\operatorname{cl}(B) \cap \operatorname{cl}(A))$, because $\operatorname{cl}(\operatorname{int}(\operatorname{cl}(S))) = \operatorname{cl}(S)$ for any semi-open subset of every topological space. Let $\operatorname{cl}(A) \neq X$ (the case $\operatorname{cl}(A) = X$ we leave to the reader). It is easy to see that we have $\operatorname{cl}(A) \setminus A = X \setminus (A \cup \operatorname{int}(B))$, $\operatorname{cl}(B) \setminus B = X \setminus (B \cup \operatorname{int}(A))$, and consequently $(\operatorname{cl}(A) \setminus A) \cap (\operatorname{cl}(B) \setminus B) = \emptyset$. So, we get what follows: $X = \operatorname{int}(\operatorname{cl}(\operatorname{int}(A))) \cup \operatorname{int}(\operatorname{cl}(\operatorname{int}(B))) \cup \big((A \cup (\operatorname{cl}(A) \setminus A)) \cap (B \cup (\operatorname{cl}(B) \setminus B))\big) = \operatorname{int}(\operatorname{cl}(\operatorname{int}(A))) \cup \operatorname{int}(\operatorname{cl}(\operatorname{int}(B))) \cup \big(A \cap (\operatorname{cl}(B) \setminus B)\big) \cup \big(B \cap (\operatorname{cl}(A) \setminus A)\big)$. Let x be a point fulfilling the condition $(\star\star)$. We shall show that $x \notin \operatorname{int}(\operatorname{cl}(\operatorname{int}(B)))$. Suppose not. By (3.2) we get $\operatorname{int}(\operatorname{cl}(\operatorname{int}(B))) \cap \operatorname{int}(A) = \emptyset$; hence $x \notin \operatorname{cl}(\operatorname{int}(A))$ what contradicts $x \in A \in \operatorname{SO}(X,\tau)$. Therefore $x \in A \cap (\operatorname{cl}(B) \setminus B)$. But, $x \notin \operatorname{cl}(B) \setminus B$ by $(\star\star)$. This shows that (X,τ) is disconnected. □

Theorem 3.14. *Each s.p. S-disc. space fulfilling the condition $(\star\star)$ is disconnected.*

Proof. It follows directly from Definition 3.4 and Lemma 3.13. □

Here, from the connectedness and e.d. points of view, the following is worth noticing.

Example 3.15. A space (X,τ) may be disconnected and not e.d. Consider $X = \{a,b,c,d\}$ and $\tau = \big\{\emptyset, X, \{a\}, \{b\}, \{d\}, \{a,b\}, \{a,d\}, \{b,d\}, \{b,c,d\}, \{a,b,d\}\big\}$. We have $X = \{a\} \cup \{b,c,d\}$ and $\operatorname{cl}(\{a,b\}) = \{a,b,c\} \notin \tau$.

Example 3.3(b) guarantees the existence of a not e.d. space which is connected.

Example 3.16. (a). Let $X = \{a,b,c\}$ and $\tau = \big\{\emptyset, X, \{a\}, \{a,b\}\big\}$. This space is e.d. and connected. See also Example 3.7.
(b). The space from Example 3.11 is e.d. and disconnected.

4. **Some properties**

Lemma 4.1. *Let* (X,τ) *be any space. If* $S \in \mathrm{SO}\,(X,\tau) \setminus \tau^{\alpha}$ *then* $\mathrm{cl}\,(\mathrm{int}\,(S)) \in \mathrm{SO}\,(X,\tau) \setminus \tau^{\alpha}$.

Proof. It is clear that $\mathrm{cl}\,(\mathrm{int}\,(S)) \in \mathrm{SO}\,(X,\tau)$. If we suppose $\mathrm{cl}\,(\mathrm{int}\,(S)) \in \tau^{\alpha}$, then $S \subset \mathrm{cl}\,(\mathrm{int}\,(S)) \subset \mathrm{int}\,(\mathrm{cl}\,(\mathrm{int}\,(\mathrm{cl}\,(\mathrm{int}\,(S))))) = \mathrm{int}\,(\mathrm{cl}\,(\mathrm{int}\,(S)))$. A contradiction. □

Theorem 4.2. *If* (X,τ) *is s.p. S-disc., then* (X,τ) *is p. S-disc.*

Proof. Assume that (X,τ) is s.p. S-disc. Then, for certain $A, B \in \mathrm{SO}\,(X,\tau) \setminus \tau^{\alpha} \subset \mathrm{SO}\,(X,\tau) \setminus \tau$ we have $X = A \cup B$ and $A \cap B = \emptyset$. Clearly $A \cup \mathrm{cl}\,(\mathrm{int}\,(B)) = X$ and hence $\mathrm{int}\,(A) \cup \mathrm{cl}\,(\mathrm{int}\,(B)) \subset X$. But, with [1, Lemma 1.1(b)] we obtain $X = \mathrm{int}\,\big(A \cup \mathrm{cl}\,(\mathrm{int}\,(B))\big) \subset \mathrm{int}\,(A) \cup \mathrm{cl}\,(\mathrm{int}\,(B))$. So, consequently

$$X = \mathrm{int}\,(A) \cup \mathrm{cl}\,(\mathrm{int}\,(B)).$$

It is easy to check that $\mathrm{int}\,(A) \cap \mathrm{cl}\,(\mathrm{int}\,(B)) = \emptyset$. Observe that $\mathrm{int}\,(A) \neq \emptyset$ and $\mathrm{cl}\,(\mathrm{int}\,(B))$ is a nonempty semi-open subset of (X,τ), which is not open (by Lemma 4.1). Thus, by Theorem 3.2(6), (X,τ) is p. S-disc. □

Theorem 4.2 implies the following obvious corollary.

Corollary 4.3. *If* (X,τ) *is s.p. S-disc., then there exists an* $A \subset X$ *such that* $A \in \mathrm{c}\,(\tau) \cap \big(\mathrm{SO}\,(X,\tau) \setminus \tau\big)$.

Theorem 4.4. *A connected topological space* (X,τ) *is p. S-disc. if and only if there exists* $A \in \mathrm{SO}\,(X,\tau) \setminus \tau$ *with* $\mathrm{cl}\,(A) \notin \tau$.

Proof. We apply Theorem 3.2(6). Necessity is obvious. For a strong sufficiency, i.e., with *any* (X,τ), suppose that $A \in \mathrm{SO}\,(X,\tau) \setminus \tau$ and $\mathrm{cl}\,(A) \notin \tau$. Then, since $\mathrm{cl}\,(A) \in \mathrm{SO}\,(X,\tau)$, from $X = (X \setminus \mathrm{cl}\,(A)) \cup \mathrm{cl}\,(A)$ it follows that (X,τ) is p. S-disc. □

Remark 4.5. If a space (X,τ) is not e.d. then there exists an $A \in \mathrm{SO}\,(X,\tau) \setminus \tau^{\alpha}$ with $\mathrm{scl}\,(A) \notin \tau$.

Proof. Suppose for each $A \in \mathrm{SO}\,(X,\tau) \setminus \tau^{\alpha}$, $\mathrm{scl}\,(A) \in \tau$. Since (X,τ) is not e.d., there is an $A' \in \tau^{\alpha}$ such that $\mathrm{scl}\,(A') \notin \tau$ [31, Theorem 2.1(iii)]. But with [14, Proposition 2.7(a)] we have $\mathrm{scl}\,(A') = \mathrm{int}\,(\mathrm{cl}\,(A'))$. A contradiction. □

Corollary 4.6. *If a space* (X,τ) *is connected and not p. S-disc., then for each* $A \in \mathrm{SO}\,(X,\tau) \setminus \tau$ *we have* $\mathrm{cl}\,(A) = X$.

Proof. By Theorem 4.4 we get that either $X = \mathrm{cl}\,(A)$ or $X \neq \mathrm{cl}\,(A) \in \tau$, but obviously the second relation is not possible. □

Theorem 4.7. *Let* (X,τ) *be a connected topological space. Then, the following are equivalent:*

(a) (X,τ) *is s.p. S-disc. or p. S-disc.*

(b) *There exists an* $A \in \mathrm{SO}(X,\tau) \setminus \tau$ *with* $\mathrm{scl}(A) \neq X$.

Proof. Strong (a)⇒(b). Let (X,τ) be p. S-disc. Then $X = U \cup A$ for such sets $U \in \tau \setminus \{X, \emptyset\}$, $A \in \mathrm{SO}(X,\tau) \setminus \tau$ that $U \cap A = \emptyset$. Consider the set $\mathrm{scl}(A)$. Since A is closed, then $\mathrm{scl}(A) = A \neq X$.

(b)⇒(a). Assume that for a certain $A' \in \mathrm{SO}(X,\tau) \setminus \tau$ we have $\mathrm{scl}(A') \neq X$. Put $A = \mathrm{scl}(A')$. Hence $\emptyset \neq B = X \setminus A \neq X$ and by [33, Corollary 2.2] we have $A, B \in \mathrm{SO}(X,\tau)$. The sets A and B cannot be both α-open in (X,τ), since (X,τ) is connected by hypothesis. Thus our space is s.p. S-disc. or p. S-disc. □

Lemma 4.8. *If a connected space* (X,τ) *is p. S-disc., then there exist sets* $U, V \in \mathrm{RO}(X,\tau) \setminus \{\emptyset\}$ *such that* $X = \mathrm{cl}(U) \cup V$, $\mathrm{cl}(U) \cap V = \emptyset$ *and* $\mathrm{cl}(U) \cap \mathrm{cl}(V) \neq \emptyset$.

Proof. Let (X,τ) be p. S-disc. and connected. By Theorem 3.2(5) there exist sets $A \in \mathrm{SO}(X,\tau) \setminus \mathrm{RO}(X,\tau)$, $V \in \mathrm{RO}(X,\tau)$ such that $X = A \cup V$ and $A \cap V = \emptyset$ (obviously $V \neq \emptyset$). Then $A \in \big(\mathrm{SO}(X,\tau) \cap \mathrm{SC}(X,\tau)\big) \setminus \{\emptyset, X\}$ and by [6, Proposition 2.1(c)] there exists a set $U \in \mathrm{RO}(X,\tau) \setminus \{\emptyset\}$ such that $U \subset A \subset \mathrm{cl}(U)$. Hence $A = \mathrm{cl}(A) = \mathrm{cl}(U)$ and $\mathrm{cl}(U) \cap V = \emptyset$. Observe that if $\mathrm{cl}(U) \cap \mathrm{cl}(V) = \emptyset$, then (X,τ) is disconnected and this contradicts connectedness of (X,τ). Therefore, $\mathrm{cl}(U) \cap \mathrm{cl}(V) \neq \emptyset$. □

By the proof of Lemma 4.8 it can be easily deduced what follows.

Theorem 4.9. *If a connected space* (X,τ) *is p. S-disc., then there exists an open but not regular open, disconnected subset of* (X,τ).

Proof. Our consideration relies on the proof of Lemma 4.8 (including the notation). We shall show only that the set $U \cup V$ is not regular open. Suppose that $U \cup V \in \mathrm{RO}(X,\tau)$. Hence $\mathrm{int}\big(\mathrm{cl}(U) \cup \mathrm{cl}(V)\big) = U \cup V \subsetneq X$. But, $\mathrm{int}\big(\mathrm{cl}(U) \cup \mathrm{cl}(V)\big) = X$, a contradiction. □

Corollary 4.10. *If* (X,τ) *is S-disconnected and connected, then there exists an open disconnected subset of* (X,τ).

Proof. See Remark 3.6 and Theorem 4.9. □

Lemma 4.11. *If a space* (X,τ) *is connected and if there exist sets* $U, V \in \mathrm{RO}(X,\tau) \setminus \{\emptyset\}$ *such that* $X = \mathrm{cl}(U) \cup V$ *and* $\mathrm{cl}(U) \cap V = \emptyset$, *then* (X,τ) *is p. S-disc.*

Proof. The set $\mathrm{cl}(U) \in \mathrm{SO}(X,\tau) \setminus \tau$, because (X,τ) is connected. So, by Theorem 3.2(3), (X,τ) is p. S-disc. □

Theorem 4.12. *Let a space* (X,τ) *be connected. Then the following are equivalent:*

1. (X,τ) *is p. S-disc.*
2. *There exist* $U, V \in \mathrm{RO}(X,\tau) \setminus \{\emptyset\}$ *such that* $X = \mathrm{cl}(U) \cup V$, $\mathrm{cl}(U) \cap V = \emptyset$.
3. *There exist* $U, V \in \tau \setminus \{\emptyset\}$ *such that* $X = \mathrm{cl}(U) \cup V$, $\mathrm{cl}(U) \cap V = \emptyset$.

4. *There exist* $U, V \in \tau^\alpha \setminus \{\emptyset\}$ *such that* $X = \mathrm{cl}\,(U) \cup V$, $\mathrm{cl}\,(U) \cap V = \emptyset$.
5. *There exist* $U, V \in \mathrm{RO}\,(X, \tau^\alpha) \setminus \{\emptyset\}$ *such that* $X = \alpha\text{-}\mathrm{cl}\,(U) \cup V$, $\alpha\text{-}\mathrm{cl}\,(U) \cap V = \emptyset$, *where* $\alpha\text{-}\mathrm{cl}\,(.)$ *denotes the closure operator with respect to* τ^α*-topology on* X.
6. *There exist* $U, V \in \tau^\alpha \setminus \{\emptyset\}$ *such that* $X = \alpha\text{-}\mathrm{cl}\,(U) \cup V$, $\alpha\text{-}\mathrm{cl}\,(U) \cap V = \emptyset$.

Proof. (1)⇔(2). Follows by Lemmas 4.8 and 4.11.
(2)⇒(3). Obvious.
(2)⇐(3). It can be easily seen that (3)⇒(1): by Theorem 3.2(6) and connectedness of (X, τ).
(3)⇒(4). Obvious.
(3)⇐(4). We shall show only that (4)⇒(1). By hypothesis we have $U \subset \mathrm{int}\,(\mathrm{cl}\,(\mathrm{int}\,(U)))$ and $U \neq \emptyset$. Hence $\mathrm{cl}\,(U) \in \mathrm{SO}\,(X, \tau)$ and $\mathrm{cl}\,(U) \neq \emptyset$. Also, $\mathrm{cl}\,(U) \notin \tau^\alpha$ up to connectedness of (X, τ) [28, Theorem 2]. Therefore (X, τ) is p. S-disc.
(5)⇔(2) and (6)⇔(4) follow by the proof of [14, Corollary 2.3] and [14, Proposition 2.2]. □

Remark 4.13. In Theorem 3.2, the class $\mathrm{SO}\,(X, \tau)$ can be replaced also by $\mathrm{SO}\,(X, \tau^\alpha)$ [21, Proposition 3] and the class $\mathrm{RO}\,(X, \tau)$ by $\mathrm{RO}\,(X, \tau^\alpha)$.

Theorem 4.14. *Let* (X, τ) *be a connected space. The following are equivalent:*

1. (X, τ) *is p.* S*-disc.*
2. *There exists a set* $B \in \mathrm{SC}\,(X, \tau)$ *such that* $B \neq X$ *and* $\mathrm{int}\,(B) \neq \emptyset$.
3. *There exists a set* $B \in \mathrm{SC}\,(X, \tau)$ *such that* $B \neq X$ *and* $\mathrm{sint}\,(B) \neq \emptyset$.

Proof. (1)⇒(2). Let (X, τ) be p. S-disc. By hypothesis the space (X, τ) is connected. On the other hand, from Theorem 3.2(5) we infer that there exists a set $B \in \mathrm{RO}\,(X, \tau) \subset \mathrm{SC}\,(X, \tau)$ with $B \neq X$ and $\mathrm{int}\,(B) \neq \emptyset$.
(2)⇒(3). Obvious.
(3)⇒(1). Suppose there exists a set $B \in \mathrm{SC}\,(X, \tau)$ with $B \neq X$ and $\mathrm{sint}\,(B) \neq \emptyset$. From [4, Theorems 1.4(2) and 1.12] we get that B is semi-closed if and only if $\mathrm{sint}\,(\mathrm{scl}\,(B)) \subset B$. Hence $\emptyset \neq \mathrm{sint}\,(\mathrm{scl}\,(B)) \neq X$. By [33, Lemma 2.7], $\mathrm{sint}\,(\mathrm{scl}\,(B)) \in \mathrm{SO}\,(X, \tau) \cap \mathrm{SC}\,(X, \tau)$. Put $U = \mathrm{int}\,(\mathrm{sint}\,(\mathrm{scl}\,(B)))$. Clearly $U \neq \emptyset$ and $U \neq X$. We have $X \setminus U = \mathrm{cl}\,(\mathrm{scl}\,(\mathrm{sint}\,(X \setminus B)))$. and $A = X \setminus U \in \mathrm{SO}\,(X, \tau)$, since by [33, Lemma 2.2(iii)], the set $\mathrm{scl}\,(\mathrm{sint}\,(X \setminus B))$ belongs to $\mathrm{SO}\,(X, \tau)$. Also $\emptyset \neq A \neq X$. The set A cannot be a member of τ, because (X, τ) is connected. So, by Theorem 3.2(6) the space (X, τ) is p. S-disc. □

5. Mappings and p. S-disconnectedness

A function $f\colon (X,\tau) \to (Y,\sigma)$ is called *contra-continuous* [8] if the preimage $f^{-1}(V) \in \mathrm{c}\,(\tau)$ for each $V \in \sigma$.

Remark 5.1. (**a**). From [9, Theorem 5.1] and Example 3.3 we infer that there exists a subclass of not e.d. spaces (X,τ) such that any contra-continuous mapping $f\colon (X,\tau) \to (Y,\sigma)$, where (Y,σ) is T_1, is constant.
(**b**). Also with [9, Theorem 5.1] we get that if a bijection $f\colon (X,\tau) \to (\mathbb{R},\tau_e)$, τ_e the usual topology, is open and contra-continuous then (X,τ) is not p. S-disc. Therefore, there is no open and contra-continuous bijection $f\colon (\mathbb{R},\tau_e) \to (\mathbb{R},\tau_e)$ (compare Example 3.3(b)).

A metric space X is connected if and only if each continuous mapping $f : X \to \mathbb{R}$ is Darboux. This implies

Remark 5.2. From Example 3.9 we infer that there exist an s.p. S-disc. metric space X and a continuous mapping $f\colon X \to \mathbb{R}$ which is not Darboux.

A function $f\colon (X,\tau) \to (Y,\sigma)$ is called *almost continuous* (in the sense S& S) [30, Theorem 2.2] (resp. *α-continuous* [20]; *irresolute* [5]) if the preimage $f^{-1}(V) \in \tau$ (resp. $f^{-1}(V) \in \tau^{\alpha}$; $f^{-1}(V) \in \mathrm{SO}\,(X,\tau)$) for every $V \in \mathrm{RO}\,(Y,\sigma)$ (resp. $V \in \sigma$; $V \in \mathrm{SO}\,(Y,\sigma)$). α-continuous mappings are called *strongly semi-continuous* in [24]. A function $f\colon (X,\tau) \to (Y,\sigma)$ is called *pre-semi-open* [5] if $f(A) \in \mathrm{SO}\,(Y,\sigma)$ for each $A \in \mathrm{SO}\,(X,\tau)$. A bijection $f\colon (X,\tau) \to (Y,\sigma)$ is said to be a *semi-homeomorphism* (in the sense of Crossley and Hildebrand) [5], if it is pre-semi-open and irresolute. It is well known that connected spaces are preserved under semi-homeomorphims [5, Theorem 2.12] or almost continuous surjections [17, Theorem 4] or α-continuous surjections [24, Theorem 3.1]. Thus, the following is clear.

Remark 5.3. Let (X,τ) be p. S-disc. and connected, and let $f : (X,\tau) \to (Y,\sigma)$ be a semi-homeomorphism or an almost continuous surjection, or α-continuous surjection. Then (Y,σ) is connected.

For the case of semi-homeomorphism we shall show a stronger result in the sequel.

Theorem 5.4. *Let $f : (X,\tau) \to (Y,\sigma)$ be a continuous surjection and (Y,σ) be a p. S-disc. connected space. Then, there is a proper subset of X which is open and disconnected (in (X,τ)).*

Proof. From Theorem 4.9 we infer that there exists an open and disconnected proper subset S of (Y,σ). So, $f^{-1}(S)$ is an open and disconnected proper subset of (X,τ). □

Corollary 5.5. *Let $f : (X, \tau) \to (Y, \sigma)$ be a continuous surjection and (Y, σ) be a connected and S-disconnected space. Then, there is an open disconnected subset of (X, τ).*

Proof. Remark 3.6 and Theorem 5.4. □

Theorem 5.6. *Let $f : (X, \tau) \to (Y, \sigma)$ be a homeomorphism and (Y, σ) be p. S-disc. and connected. Then $X = A \cup B$, where $A \cap B = \emptyset$, A is an open disconnected subset of (X, τ), and $B \in \mathrm{c}(\tau) \setminus \tau$.*

Proof. We apply Theorems 5.4 and an obvious fact that there is no open bijection $f : (X, \tau) \to (Y, \sigma)$, where (X, τ) is disconnected and (Y, σ) is connected. □

Theorem 3.2 is followed by the series of results given below, concerning preimages and images of p. S-disc. spaces under some well known types of functions. Straightforward proofs are omitted.

Theorem 5.7. *Let (X, τ) be connected, (Y, σ) be s.p. S-disc., and let $f : (X, \tau) \to (Y, \sigma)$ be an irresolute surjection. Then (X, τ) is p. S-disc.*

A function $f \colon (X, \tau) \to (Y, \sigma)$ is called *completely continuous* [2] (resp. *an R-map* [3]; *α-irresolute* [19]) if the preimage $f^{-1}(V) \in \mathrm{RO}\,(X, \tau)$ (resp. $f^{-1}(V) \in \mathrm{RO}\,(X, \tau)$; $f^{-1}(V) \in \tau^{\alpha}$) for every $V \in \sigma$ (resp. $V \in \mathrm{RO}\,(Y, \sigma)$; $V \in \sigma^{\alpha}$).

Theorem 5.8. *Let (X, τ) be not e.d. and connected, (Y, σ) be p. S-disc., and let a surjection $f : (X, \tau) \to (Y, \sigma)$ fulfil one of the following conditions:*

1. *f is irresolute and almost continuous;*
2. *f is irresolute and it is an R-map;*
3. *f is irresolute and α-continuous.*

Then (X, τ) is p. S-disc.

Remark 5.9. If (X, τ) is e.d. and connected, if (Y, σ) is p. S-disc., then it is clear by [13, Theorem 2.9(b)] and [11, Lemma 1(i)] (for the case (3)) that there is no surjection $f : (X, \tau) \to (Y, \sigma)$ fulfilling (1) or (2) or (3) of Theorem 5.8.

Obviously, (2) is a particular case of (1) in Theorem 5.8. Since each continuous function is almost continuous, each completely continuous function is an R-map and each α-irresolute function is α-continuous, therefore the next corollary is obvious. None of these three implications is reversible, see respectively: [30, Example 2.1], [26, Example 4.6], and [19, Example 1].

Corollary 5.10. *Let (X, τ) be not e.d. and connected, (Y, σ) be p. S-disc., and a surjection $f : (X, \tau) \to (Y, \sigma)$ fulfils one of the following conditions:*

(1') *f is irresolute and continuous;*
(2') *f is irresolute and completely continuous;*

(3') *f is irresolute and α-irresolute.*

Then (X,τ) is p. S-disc.

Remark 5.11. (a). [7, Example 7.1] shows that there exists an irresolute mapping, which is not almost continuous and hence: not an R-map, not continuous, and not completely continuous.
(b). [7, Example 7.2] guarantees the existence of not irresolute mapping, which is continuous (hence almost continuous).
(c). Notions of irresolutness and α-continuity are independent of each other, see [25, Example 3.11 and Theorem 3.12]. In [10] the author has shown that concepts of irresolutness and α-irresolutness are independent of each other.

Example 5.12. Let $X = \{a,b\} = Y$, $\tau = \big\{\emptyset, X, \{a\}\big\}$, and $\sigma = \big\{\emptyset, Y, \{b\}\big\}$. Let $f : (X,\tau) \to (Y,\sigma)$ be the identity function. then f is an R-map, but it is not irresolute.

Example 5.13. Let $X = \{a,b,c\} = Y$, $\tau = \big\{\emptyset, X, \{b\}, \{a,c\}\big\}$, and $\sigma = \big\{\emptyset, Y, \{b\}\big\}$. Then, the identity function $f : (X,\tau) \to (Y,\sigma)$ is completely continuous and not irresolute.

The result from Theorem 5.8 for the case (2) may be strengthened (see Theorem 5.20 below).

A function $f\colon (X,\tau) \to (Y,\sigma)$ is said to be *almost open* [30] (resp. *R-open*; *α-open* [20]) if the image $f(U) \in \sigma$ (resp. $f(U) \in \mathrm{RO}\,(Y,\sigma)$; $f(U) \in \sigma^{\alpha}$) for every $U \in \mathrm{RO}\,(X,\tau)$ (resp. $U \in \mathrm{RO}\,(X,\tau)$; $U \in \tau$).

Theorem 5.14. *Let (X,τ) be p. S-disc., (Y,σ) be not e.d. and connected, and let a bijection $f : (X,\tau) \to (Y,\sigma)$ fulfil one of the following conditions:*

(a) *f is pre-semi-open and almost open;*
(b) *f is pre-semi-open and R-open;*
(c) *f is pre-semi-open and α-open;*

Then (Y,σ) is p. S-disc.

Proof. Apply respective parts of Theorem 3.2 (obviously: (b)⇒(a)). □

Remark 5.15. By the same reasoning as mentioned in Remark 5.9, there is no bijection between a p. S-disc. space (X,τ) and an e.d. connected space (Y,σ) fulfilling (a) or (b) or (c) of Theorem 5.14.

Remark 5.16. (a). [23, Example 1.8] shows that there exists an almost open function (in fact, R-open), which is not pre-semi-open.

(b). Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, X\}$, and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$. The mapping $f : (X, \tau) \to (Y, \sigma)$ defined as follows: $f(a) = a$, $f(b) = f(c) = b$, is almost open, but it is not R-open.

Example 5.17. Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, X, \{b\}, \{a, c\}\}$, and $\sigma = \{\emptyset, Y, \{b\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ as in Remark 5.16(b). Then, f is pre-semi-open and not almost open (hence not R-open).

Example 5.18. Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, X, \{b\}, \{b, c\}\}$, and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$. The identity function $f : (X, \tau) \to (Y, \sigma)$ is pre-semi-open and not α-open.

Example 5.19. Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, X, \{a\}\}$, and $\sigma = \{\emptyset, Y, \{a\}, \{b, c\}\}$. We define a mapping $f : (X, \tau) \to (Y, \sigma)$ as follows $f(a) = f(b) = a$, $f(c) = b$. Then, f is α-open and not pre-semi-open.

Theorem 5.20. *Let (X, τ) be connected, (Y, σ) be p. S-disc. and connected, and a surjection $f : (X, \tau) \to (Y, \sigma)$ be an R-map. Then (X, τ) is p. S-disc.*

Proof. By Theorem 4.12(2) there exist $U_1, V_1 \in \mathrm{RO}\,(Y, \sigma) \setminus \{\emptyset\}$ such that $Y = \mathrm{cl}_Y(U_1) \cup V_1$ and $\mathrm{cl}_Y(U_1) \cap V_1 = \emptyset$. Clearly $\mathrm{cl}_Y(U_1)$ is regular closed in (Y, σ). It is obvious that the set $f^{-1}(\mathrm{cl}_Y(U_1))$ is regular closed in (X, τ). So, we have $X = f^{-1}(Y) = \mathrm{cl}_X(\mathrm{int}_X(f^{-1}(\mathrm{cl}_Y(U_1)))) \cup f^{-1}(V_1)$, where $U = \mathrm{int}_X(f^{-1}(\mathrm{cl}_Y(U_1))) \in \tau \setminus \{\emptyset\}$, $V = f^{-1}(V_1) \in \tau \setminus \{\emptyset\}$, and $\mathrm{cl}_X(U) \cap V = \emptyset$. This proves that (X, τ) is p. S-disc., since, by hypothesis, it is connected (Theorem 4.12(3)). □

Theorem 5.21. *Let (X, τ) be a connected p. S-disc. space and $f : (X, \tau) \to (Y, \sigma)$ be a semi-homeomorphism. Then (Y, σ) is connected and p. S-disc.*

Proof. Since (X, τ) is connected and p. S-disc., by [5, Theorem 2.12] and Theorem 4.14(3) respectively, (X, τ) is connected and there exists a set $B \in \mathrm{SC}\,(X, \tau)$ with $B \neq X$ and $\mathrm{sint}_X(B) \neq \emptyset$. By [5, Theorem 2.12] the space (Y, σ) is connected. Obviously, $f(B) \neq Y$. Recall that for every semi-homeomorphism $f : X \to Y$ and any $B \subset X$ we have $f(\mathrm{sint}_X(B)) = \mathrm{sint}_Y(f(B))$ [5, Corollary 1.2]. So, $\mathrm{sint}_Y(f(B)) \neq \emptyset$. It is not difficult to see that each bijective pre-semi-open map preserves semi-closed sets. Therefore $f(B) \in \mathrm{SC}\,(Y, \sigma)$ and applying once more Theorem 4.14(3) we finish the proof. □

Corollary 5.22. *Let $f : (X, \tau) \to (Y, \sigma)$ be a homeomorphism, (X, τ) be connected and p. S-disc. Then, (Y, σ) is connected and p. S-disc.*

Proof. [5, Theorem 1.9] and Theorem 5.21. □

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