

COMPARATIVE STUDY BETWEEN LEMKE'S METHOD AND THE INTERIOR POINT METHOD FOR THE MONOTONE LINEAR COMPLEMENTARY PROBLEM

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Abstract. In this paper, we present two different methods in order to solve a monotone linear complementarity problem (*LCP*): a simplicial method (Lemke's method) related to the Jordan's pivot and the interior point method based on the central path. We demonstrate that a quadratic convex program (*QCP*) can be written as a (*LCP*) form and, thus, be solved by means of one of these two methods. We provide numerical simulations as well as experimental and comparative results regarding these two methods.

1. Introduction

The introduction of the polynomial-time interior point algorithm in linear programs by Karmarkar in 1984, has led many authors to generalise this algorithm in order to solve non-linear optimization problems. Successive works were devoted to solving the (*LCP*) by means of interior point methods (Kojima, Mizuno and Yoshise [5,6], Gonzaga [4], Bonnans, Gilbert and Lemarechal [2],...). Apparently, these authors were unaware of a long-time existing tool able to solve the problem (*LCP*): the Lemke's method, which is based on the principle of the simplex method introduced by Dantzig in 1951. This method converges with a finite number of iterations when the problem admits a solution. In the literature, we know that the interior point methods are very fast and more effective than the methods based on the pivot and especially if

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the problem is of big dimension. This result is completely true if we know the starting point x^0 , but in the opposite case (that is if we do not know the initial point x^0), the determination of x^0 by the interior point methods represents an inconvenience for these methods and makes them slow with regard to other algorithms. As it is well-known, though the interior point methods are robust and rapid, their major disadvantage is the determination of the initial point. Nevertheless, when a starting point is given, these methods prove to be the best, with a very fast convergence.

In this paper, we show that, in the particular cases of unknown starting points, their determination delays significantly the interior point methods and sometimes turns them slower than other classical approaches, when solving a convex quadratic problem. Our study also rivals that the evaluation of the starting point with Kojima's approach is expensive, and has been found to slower than Lemke's swiveling method, which is a simplicial method as pointed out in the literature (e.g. [1], [7]). We still underline that the interior point method is the best, faster than Lemk's method when the starting point is known.

This paper presents the two methods and well as comparative numerical simulations in order to show the importance (from theoretical and practical points of view) of the Lemke's method stability, efficiency and the longevity regarding interior points algorithms.

In the present paper, we are concerned by solving two important problems of non-linear optimisation:

1. The monotone linear complementarity problem (*LCP*)

(*LCP*) consists in finding two vectors $(x, z) \in \mathbb{R}^n \times \mathbb{R}^n$ such that

$$z = Mx + q \tag{1}$$

$$x \geq 0, z \geq 0 \tag{2}$$

$$\langle x, z \rangle = 0 \tag{3}$$

where $M \in \mathbb{R}^{n \times n}$ and $\langle x, z \rangle$ denotes the scalar product of two vectors x and z .

2. The Quadratic Convex Program (*QCP*)

$$(QCP) : \text{Min} \left\{ f(x) = \frac{1}{2} \langle Cx, x \rangle + \langle d, x \rangle : Ax \leq b \right\}$$

where $C \in \mathbb{R}^{n \times n}$ is symmetric, positive semidefinite matrix, $d \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$.

In Section 2, we present Lemke's method, a simplicial method known in the literature for solving (*LCP*) (Bazaraa, Sherali and Shetty [1], Yassine [7]). We provide the corresponding algorithm and his convergence theorem. The interior point algorithm based on the central path and its convergence properties for solving (*LCP*) are provided in Section 3. In Section 4, we give the transforming techniques for a quadratic convex program into (*LCP*) using the optimality conditions of Kuhn Tucker. Section 5 is dedicated to high-dimension numerical simulations and comparisons between numerical predictions of the two methods.

2. Lemke's method

2.1. Preliminaries. Let x_i (resp. z_i) be the component number i of vector x (resp. z). The component x_i (resp. z_i) is said basic variable if $x_i \geq 0$ (resp. $z_i \geq 0$). If x_i (resp z_i) is out of base (non-basic variable), then inevitably $x_i = 0$ (resp. $z_i = 0$).

Definition 2.1. A solution (x, z) of (*LCP*) is said feasible-complementarity solution, if it verifies the two following conditions:

- (x, z) is a feasible solution of (1) and (2)
- one and only one component of (x_i, z_i) is a basic variable for $i = 1, \dots, n$.

We notice that if $q \geq 0$, then $(x, z) = (0, q)$ is a solution of (*LCP*). On the opposite, ($\exists i \in \{1, \dots, n\}$ such that $q_i < 0$), we introduce the column vector e the components of which are equal to 1, and an artificial variable z_0 initialized as:

$$z_0 = \max\{-q_i : 1 \leq i \leq n\}.$$

We consider the new system defined by: Find $(x, z, z_0) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ such that

$$(P') : \begin{cases} z - Mx - ez_0 = 0 & (4) \\ x \geq 0, z \geq 0, z_0 \geq 0 & (5) \\ \langle x, z \rangle = 0. & (6) \end{cases}$$

We notice that $x = 0$ and $z = q + ez_0$ is a feasible-complementarity solution of (P') .

Definition 2.2. (x, z, z_0) is said feasible-almost-complementarity of (P') if it verifies the three following conditions:

- (x, z, z_0) is a feasible solution of (4) and (5)
- $\exists s \in \{1, \dots, n\}$ such that x_s and z_s are out of basis ($x_s = z_s = 0$)
- z_0 is a basic variable, and $\forall i \neq s, x_i$ or z_i is a basic variable.

Remark 2.1. In entering x_s or z_s in the base, we obtain an adjacent feasible-almost-complementarity solution. Then, each feasible-almost-complementarity solution admits two adjacent solutions, one when entering as x_s in the basis and the other when entering as z_s .

2.2. Lemke's algorithm (ALGI). Initialisation stage. If $q \geq 0$, we stop: $(x, z) = (0, q)$ is a solution of (LCP) . Else, we introduce the artificial variable z_0 , we represent the problem (P') through a table and then, we choose

$$q_s = \min\{q_i : 1 \leq i \leq n\}.$$

We update the table by pivoting the line s and the column of z_0 , z_s leaves the base and z_0 enters it, then z_0 and z_i (for $i=1, \dots, n$ and $i \neq s$) are positive (basic variables). Let us put $y_s = x_s$ and go to the main stage.

Main stage: This stage is divided into three phases:

Phase 1: Let d^s the column which corresponds to the variable y_s in the current table. If $d^s = 0$, we stop: (LCP) admits no solution. Else, we determine an index r such that:

$$\frac{q_r^*}{d_r^s} = \text{Min} \left\{ \frac{q_i^*}{d_i^s} : d_i^s > 0 \forall i = 1, \dots, n \right\}$$

(the vector q^* designates the second member column).

If the basic variable of the line r is z_0 , then go to Phase 3, else go to Phase 2.

Phase 2: The basic variable of the line r is, either x_k , or z_k for some $k \neq s$.

The variable y_s enters the base and the table will define itself through the pivot of the line r and the column of y_s . If the variable, which has left the base, is z_k (resp. x_k), we put $y_s = x_k$ (resp. z_k) and return to Phase 1.

Phase 3: We pivot between the column of y_s and the line of z_0 . Then, z_0 leaves the base and y_s enters it. We obtain a solution of (LCP).

2.3. Convergence of Lemke's method. Let $M \in \mathbb{R}^{n \times n}$ be a $n \times n$ symmetric matrix and $x \in \mathbb{R}^n$ be a n -dimensional real vector.

- Definition 2.3.**
1. M is said *copositive* if and only if $\forall x \geq 0, x^t M x \geq 0$
 2. M is said *strictly copositive* if and only if $\forall x \geq 0, x \neq 0 \implies x^t M x > 0$
 3. M is said *copositive plus* if and only if it verifies the two following conditions:
 - (i): M is copositive
 - (ii): $x \geq 0$ and $x^t M x = 0 \implies (M + M^t)x = 0$.

If M is symmetric, the property (ii) becomes:

$$(ii) \quad x \geq 0 \text{ and } x^t M x = 0 \implies Mx = 0.$$

Theorem 2.1. ([1]) *We suppose that each feasible-almost-complementarity solution of (P'), is non-degenerated (each basic variable is strictly positive) and that the matrix M is copositive plus, then the algorithm (ALG1) stops after a finite number of iterations. If the system defined by (1) and (2) is consistent, then the algorithm (ALGI) stops with an optimal solution of (LCP), else, we notice that the problem (LCP) admits no solution.*

Corollary 2.1. *If the matrix M admits positive elements and if the diagonal elements are strictly positive, then the algorithm stops with a feasible complementary basic solution.*

3. Interior point algorithm

3.1. Introduction. We consider the monotone linear complementarity problem as a standard given by (1), (2) and (3). The set of all the feasible solutions is defined by:

$$S = \{(x, z) \in \mathbb{R}^{2n} : z = Mx + q, x \geq 0, z \geq 0\}$$

and its relative interior

$$S_{int} = \{(x, z) \in S : x > 0, z > 0\}.$$

Then, we suppose that the two following hypotheses are satisfied:

(H_1): M is positive semidefinite

(H_2): $S_{int} \neq \emptyset$.

The size of problem (LCP) is defined by ([5]):

$$L = E \left(\sum_{i=1}^n \sum_{j=1}^{n+1} \log(|\mathbb{M}_{ij}| + 1) + \log(n^2) \right) + 1$$

where $\mathbb{M} = [M \quad q]$ and $E(u)$ is the largest integer, not greater than $u \in \mathbb{R}_+$.

Let $H : \mathbb{R}_+ \times \mathbb{R}_+^{2n} \rightarrow \mathbb{R}^n \times \mathbb{R}^n$

$$(\mu, x, z) \rightarrow H(\mu, x, z) = (xz - \mu e, z - Mx - q)$$

for every $\mu > 0$ and $(x, z) \in \mathbb{R}_+^{2n}$, we consider the following system of equations:

$$H(\mu, x, z) = 0. \tag{7}$$

It is obvious that (x, z) is a solution of (LCP) if and only if it is a solution of the system (7) for $\mu = 0$. The Newton direction at (x, z) is defined as a solution (d_x, d_z) of the system of linear equations:

$$\begin{cases} Zd_x + Xd_z = -xz + \mu e \\ d_z = Md_x \end{cases} \tag{8}$$

where $X = \text{diag}(x_1, x_2, \dots, x_n)$ and $Z = \text{diag}(z_1, z_2, \dots, z_n)$.

By a simple calculation, we obtain:

$$\begin{cases} (M + X^{-1}Z)d_x = Ze + \mu X^{-1}e \\ d_z = Md_x. \end{cases} \quad (9)$$

Then the new point (\bar{x}, \bar{z}) will be given by:

$$(\bar{x}, \bar{z}) = (x + d_x, z + d_z). \quad (10)$$

We can easily verify that:

$$\bar{z} = M\bar{x} + q \text{ for any } (x, z) \in S \text{ and any } \mu > 0. \quad (11)$$

3.2. Centralisation measures. Note that S_{cen} is the central trajectory of (LCP):

$$\begin{aligned} S_{cen} &= \{(x, z) \in \mathbb{R}_+^{2n} : H(\mu, x, z) = 0 \text{ for } \mu > 0\} \\ &= \{(x, z) \in S_{int} : xz = \mu e \text{ for } \mu > 0\}. \end{aligned}$$

Proposition 3.1. ([5]) *If $S_{int} \neq \emptyset$, the system (7) admits a unique solution called associated center to μ , for every $\mu > 0$.*

The algorithms of central trajectory generate a sequence of points (x^μ, z^μ) verifying the following system:

$$\begin{cases} z = Mx + q \\ xz = \mu e \\ x > 0 \text{ and } z > 0. \end{cases} \quad (12)$$

In tending μ to 0, (x^μ, z^μ) tends to a solution (x^*, z^*) of (LCP) which is located at the extremity of the central trajectory.

To control the non-linearity of xz , successive points are imposed to stay in the central trajectory neighbourhood. To evaluate deviation $(x, z) \in S_{int}$ of each point for the central trajectory, we define a centralisation measure:

$$\begin{aligned} \delta(x, z) &= \text{Min}\{\|H(\mu, x, z)\| : \mu \geq 0\} = \text{Min}\{\|xz - \mu e\| : \mu \geq 0\} \\ &= \text{Min}\left\{xz - \left(\frac{x^t z}{n}\right)e : \mu \geq 0\right\}. \end{aligned}$$

For every point $(x, z) \in S_{int}$, we get: $(x, z) \in S_{int} \iff \delta(x, z) = 0$.

Definition 3.1. Let $\alpha > 0$. We call α -center neighbourhood, the set

$$S_\alpha = \{(x, z) \in S_{int} : \delta(x, z) \leq \frac{x^t z}{n} \alpha\}.$$

Theorem 3.1. ([5]) Let $0 < \alpha < 0.1$ and $\delta = \frac{\alpha}{1-\alpha}$. We assume that $(x, z) \in S_\alpha$ and $\mu = (1 - \frac{\delta}{\sqrt{(n)}}) \frac{x^t z}{n}$ then (\bar{x}, \bar{z}) , given by (10), verifies:

$$(\bar{x}, \bar{z}) \in S_\alpha \tag{13}$$

$$(\bar{x}^t \bar{z}) \leq \left(1 - \frac{\delta}{6\sqrt{(n)}}\right) x^t z. \tag{14}$$

3.3. Interior point algorithm (ALG2).

Initialisation: (see Appendix 1): Let $0 < \alpha < 0.1$ and $\delta = \frac{\alpha}{1-\alpha}$.

We suppose that the initial point $(x^1, z^1) \in S_{int}$ are known, such that

$$\delta(x^1, z^1) \leq \frac{(x^1)^t z^1}{n} \alpha \text{ and } (x^1)^t z^1 \leq 2^{0(L)}, k = 1.$$

Stage 1: If $(x^k)^t z^k \leq 2^{-2L}$, we stop: $(x^*, z^*) = (x^k, z^k)$ is a solution of (LCP)

Stage 2: $\mu = (1 - \frac{\delta}{\sqrt{(n)}}) \frac{(x^k)^t z^k}{n}$ and $(x, z) = (x^k, z^k)$.

Stage 3: We determine the Newton's direction (d_x, d_z) defined by (9) and $(x^{k+1}, z^{k+1}) = (\bar{x}, \bar{z})$ defined by (10). $k \leftarrow k + 1$ and return to Stage 1.

Theorem 3.2. ([5,6]) The algorithm (ALG2) generates a sequence (x^k, z^k) verifying:

$(x^k, z^k) \in S_\alpha$ and $(x^{k+1}, z^{k+1}) \leq (1 - \frac{\delta}{6\sqrt{(n)}}) (x^k)^t z^k$ for $k = 1, \dots$, The sequence (x^k, z^k) converges to (x^*, z^*) solution of (LCP) after, at the most, $O(n^{0.5L})$ iterations.

4. Transformation of a convex quadratic program into a monotone linear complementarity problem

In this paragraph, we firstly present the transformation of convex quadratic program (based on the optimality conditions of Kuhn Tucker) into a complementarity linear problem. We distinguish the two following cases, may there be or not positivity constraints of the variable x components. We also provide the conditions ensuring

the convergence of Lemke's algorithm for the solving of these monotone linear complementarity problems.

4.1. Transformation with positivity constraints. Let us consider the following problem:

$$(QCP_1) : \text{Min}\{f(x) = \frac{1}{2} \langle x, Cx \rangle + \langle d, x \rangle : Ax \leq b, x \geq 0\}$$

where $C \in \mathbb{R}^{n \times n}$ is symmetric, positive definite; $A \in \mathbb{R}^{m \times n}$; $x, d \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$.

The conditions of Kuhn Tucker's related to the problem (QCP_1) are written as follows:

x is a solution of (QCP_1) if and only if there exists $u \in \mathbb{R}^m$ and $v \in \mathbb{R}^n$ such that

$$(*) : \begin{cases} Cx + d + A^t u - v = 0 \\ \langle u, b - Ax \rangle = \langle v, x \rangle = 0 \\ Ax \leq b, x \geq 0, u \geq 0, v \geq 0. \end{cases}$$

Let

$$q = \begin{bmatrix} b \\ d \end{bmatrix} \in \mathbb{R}^{n+m}, z = \begin{bmatrix} u \\ x \end{bmatrix} \in \mathbb{R}^{n+m} \text{ and } M = \begin{bmatrix} 0 & -A \\ A^t & C \end{bmatrix} \in \mathbb{R}^{(n+m) \times (n+m)}.$$

It is readily verified that the quadratic program solving (QCP_1) is equivalent for solving the following linear complementarity problem:

$$(LCP_1) : \text{Find } z \in \mathbb{R}^{(n+m)} \text{ such that } : z \geq 0, Mz + q \geq 0 \text{ and } z^t(Mz + q) = 0.$$

Theorem 4.1. *If C is symmetric, positive definite, then M is copositive plus and Lemke's algorithm converges to a solution of (LCP) .*

Proof. Let $z = \begin{bmatrix} x \\ y \end{bmatrix} \geq 0$ where $x \in \mathbb{R}^m$ and $y \in \mathbb{R}^n$, then $z^t = [x^t \ y^t] \geq 0$.

$$z^t M z = [x^t \ y^t] \cdot \begin{bmatrix} 0 & -A \\ A^t & C \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = y^t C y.$$

$z \geq 0 \implies y \geq 0 \implies y^t C y \geq 0$ (by assumption, C is symmetric, positive definite) $\implies z^t M z \geq 0 \implies M$ is copositive.

We have, moreover, $M + M^t = \begin{bmatrix} 0 & 0 \\ 0 & 2C \end{bmatrix} \implies (M + M^t)z = 2Cy$, then, $z^t.M.z = 0 \implies y^t.C.y = 0 \implies C.y = 0 \implies (M + M^t).z = 0 \implies M$ is copositive plus.

According to Theorem 2.1, *ALG1* converges to a solution of (*LCP*). \square

4.2. No-constraint transformation of positivity. Considering the following problem:

$$(QCP_2) : \text{Min}\{f(x) = \frac{1}{2} \langle x, Cx \rangle + \langle d, x \rangle : Ax \leq b\}$$

where $C \in \mathbb{R}^{n \times n}$ symmetric, positive definite; $A \in \mathbb{R}^{m \times n}$; $x, d \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$.

Kuhn Tucker's optimality conditions related to the problem (*QCP*₂) are written:

x is a solution of (*QCP*₂) if and only if there exists $u \in \mathbb{R}^m$ such that

$$(**) : \begin{cases} Cx + d + A^t u = 0 \\ \langle u, b - Ax \rangle = 0 \\ Ax \leq b, u \geq 0 \end{cases}$$

which are equivalent in solving the following *LCP*:

$$(LCP_2) : \text{Find } u \in \mathbb{R}^m \text{ such that } : u \geq 0, Mu + q \geq 0 \text{ and } \langle u, Mu + q \rangle = 0$$

where $M = AC^{-1}A^t$ and $q = AC^{-1}d + b$.

Remark 4.1.

- u^* is a solution of the problem *LCP*₂ if and only if $x^* = (-C^{-1}A^t u^* - C^{-1}d)$ is a solution of the problem (*QCP*₂).
- Given that $M = M^t = AC^{-1}A^t$ is positive definite, Lemke's algorithm leads to a solution of (*LCP*₂) (then of (*QCP*₂)) or concludes on the vacuity of the solution set of (*LCP*₂) (consequently that of (*QCP*₂)).
- In case 4.2., the transformation requires that C is positive definite. Moreover, applying Lemke's algorithm needs to calculate C^{-1} . These are the drawbacks when solving (*QCP*₂) through Lemke's algorithm.

- In case 4.1., such drawbacks do not exist. On the opposite, we have to work in \mathbb{R}^{n+m} (instead of \mathbb{R}^m in 4.2.). Lemke's algorithm would be more expensive when n is quite big.

5. Numerical experiments

In this section, we present the comparative numerical results between the two methods for the (*LCP*) problem. The numerical simulations are applied to quadratic problems. In our numerical applications, the matrix C is always definite positive (chosen in a random way) to ensure the convergence of Lemke's method. In the Table I, the first column represents the problem dimension ($M \in \mathbb{R}^{n \times n}$ and $q \in \mathbb{R}^n$), the second provides (resp. the third) the *CPU* calculation time in seconds for the Lemke's method to be carried out (resp. interior point algorithm).

N	Lemke	Interior Point
10	5	23
20	8	34
50	14	56
100	32	148
200	74	289
400	126	518
500	159	665
1000	334	875

Table I. Calculation time of two methods (in seconds).

According to the numerical results, the following remarks can be make:

- Lemke's method efficiency, stability and robustness compared with the interior point method, should be underlined.
- The interior point method becomes low for too small values of α (see Section 3.3.) (lower than 0.01) or too big (upper than 0.08).
- Interior point method difficulty lies in the determination of the initial point x^0 (initial stage).

- In some cases, and for a fixed value of α , the interior point method diverges leading to change the α value in order to obtain convergence on an optimal solution. This numerical instability does not exist in Lemke's method.

N	Interior Point method by knowing the initial point x^0
10	1
20	2
50	4
100	9
200	16
400	31
500	42
1000	78

Table II. Calculation time (in second) of the Interior Point Method by Knowing x^0 .

The result of the table (*Table II.*) show clearly that if we know x^0 then the interior point method is much faster and more effective than the Lemkes' method.

Appendix 1

Initialisation. Let

$$M' = \begin{bmatrix} 0 & -e^t \\ e & M \end{bmatrix} \in \mathbb{R}^{(n+1) \times (n+1)} \text{ and } q' = [q_0 \quad q] \in \mathbb{R}^{(n+1)}$$

where

$$q_0 = \frac{2^{L^*} \cdot (n+1)}{n^2}, \quad L^* = \sum_{i=1}^n \sum_{j=1}^{n+1} \log(|M_{ij}| + 1) + \log(n^2) \text{ and } M = [M \quad q].$$

We consider the following *LCP*:

$$(LCP') : \begin{cases} z' = M'x' + q' \\ x'z' = 0 \\ (x', z') \geq 0 \end{cases}$$

where $(x', z') = (x_0, x, z_0, z) \in \mathbb{R}_+^{2(n+1)}$.

Assumptions (H_1) and (H_2) , its size is:

$$L' = E \left(\sum_{i=1}^{n+1} \sum_{j=1}^{n+2} \log(|\widetilde{M}_{ij}| + 1) + \log((n+1)^2) \right) + 1$$

where $\widetilde{M} = [M' \quad q']$. Putting

$$\begin{aligned} x_0^1 &= 2^{2L^*}, x^1 = \left(\frac{2L^*}{n^2}\right).e, z_0^1 = x_0^1.e + Mx^1 + q = 2^{2L^*}.e + \left(\frac{2L^*}{n^2}\right).Me + q \\ x'_1 &= (x_0^1, x^1) \text{ and } z'_1 = (z_0^1, z^1) \end{aligned}$$

We denote by $S', S'_{int}, S'_{cen}, S'_\alpha$, the solutions set of (LCP') , its relative interior, its central trajectory and its α -center neighbourhood, respectively.

Lemme 5.1. ([5])

1.

$$0 < \left(\frac{15}{16}\right).2^{2L^*}.e \leq 2^{2L^*}. \left(1 - \frac{1}{n^4}\right).e \leq z^1 \leq 2^{2L^*}. \left(1 + \frac{1}{n^4}\right)e \leq \left(\frac{17}{16}\right).2^{2L^*}.e$$

2.

$$(x'^1, z'^1) \in S'_{int}.$$

Lemme 5.2. ([5])

1.

$$(x'^1)^t z'^1 \leq 2^{2L} \leq 2^{2L'}$$

2.

$$(x'^1, z'^1) \in S'_{0,1}.$$

Theorem 5.1. ([5]) Suppose that the (LCP) has a solution. Then $\bar{x}_0 = 0$ for any solution $(\bar{x}_0, \bar{x}, \bar{z}_0, \bar{z})$ of the (LCP') .

According to Theorem 5.1, (x'^1, z'^1) can be useful as an initial point to the algorithm $(ALG2)$. We calculate the solution $(\bar{x}_0, \bar{x}, \bar{z}_0, \bar{z})$ of (LCP') such that $\bar{x}^t \bar{z} < 2^{-2L'}$.

If $\bar{x}_0 = 0$ then (\bar{x}, \bar{z}) is a solution of (LCP) , else the above-mentioned theorem ensures that (LCP) admits no solution.

References

- [1] Bazararaa, M.S., Sherali, H.D., Shetty, C.M., *Nonlinear Programming, Theory and Algorithms*, Sec. ed. John Wiley and Sons, New York, 1993.
- [2] Bonnans, J.F., Gilbert, J.C., Lemarechal, C., *Optimisation Numérique, Aspects théoriques et pratiques*, Springer-Verlag, 1997.
- [3] Bonnans, J.F., Gonzaga, C.C., *Convergence of interior point algorithms for the monotone linear complementarity problem*, Mathematics of Operations Research, vol. 21, 1996, 1-25.
- [4] Gonzaga, C.C., *The largest step path following algorithm for monotone linear complementarity problems*, Mathematical Programming, vol. 76, 1997, 309-332.
- [5] Kojima, M., Mizuno, S., Yoshise, A., *A polynomial-time algorithm for a class of linear complementarity problems*, Mathematical Programming, vol. 44, 1989, 1-26.
- [6] Kojima, M., Mizuno, S., Yoshise, A., *A $O(\sqrt{n})L$ iteration potential reduction algorithm for linear complementarity problems*, Mathematical Programming, vol. 50, 1991, 331-342.
- [7] Yassine, A., *Etude comparative de méthodes numériques en optimisation non convexe, Théorie, Algorithmes et Applications*, Thèse de Doctorat de l'Université Joseph Fourier (Grenoble I), 1989.

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