BOOK REVIEWS

C. E. Silva, *Invitation to ergodic theory*, Student Mathematical Library, Vol. 42, American Mathematical Society, Providence, Rhode Island 2008, ix+262 pp, ISBN: 978-0-8218-4420-5

The ergodic theory, started in 1931 by John von Neumann and G. D. Birkhoff, has its origins in the statistical physics of Boltzmann. The present book is intended to be an introduction to ergodic theory and covers topics as recurrence, ergodicity, the ergodic theorems and mixing. In order to make the book as self-contained as possible, measure theory is developed as needed in Chapters 2, *Lebesgue measure*, and 4, *The Lebesgue integral*, including an introduction to measure spaces, Carathéodori extension theorem, Lebesgue dominated convergence theorem and the Lebesgue spaces L^p.

The study of ergodic theory begins in Chapter 3, *Recurrence and ergodicity,* with the classical example of baker's transformation, doubling maps, measure-preserving transformations, ergodic transformations.

Chapter 5, *The ergodic theorem,* is devoted to the proof of Birkhoff's ergodic theorem (in fact, two proofs of this important result are given) and of the maximal ergodic theorem in L^1 and in L^p.

Chapter 6, *Mixing notions,* is concerned with the important notion of mixing, meaning that $\lim_{n\to\infty} \mu(T^{-n}(A)\cap B) = \mu(A)\mu(B)$, for all $A, B \in \mathcal{S}$, where $(X, \mathcal{S}, \mu)$ is a probability measure space and $T : X \to X$ is a measure-preserving transformation. The notion of weak-mixing, meaning that

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} |\mu(T^{-i}(A) \cap B) - \mu(A)\mu(B)| = 0,$$

is also studied.

A word must be said about the numerous examples from physics, biology, and mathematics included in the book. I do mention the nice treatment of Weyl's result on the equidistribution of numbers, with references to some results of Furstenberg on the Szemeredi theorem and the recent solution by Green and Tao of the 300-year old problem on the existence of arithmetic progressions of arbitrary length in the primes.

There are also problems and exercises completing the main text and some open questions, suggesting possible topics for further research by the reader, are included.

The book is well written and can be used for an introductory course in measure theory or in ergodic theory, or for self-study.

S. Cobzaş

Massimiliano Berti, ***Nonlinear oscillations of Hamiltonian PDEs***, Progress in Nonlinear Differential Equations and Their Applications, Vol. 74, Series Editor: Haim Brezis, Birkhäuser Verlag, Basel-Boston-Berlin, 2007, x+235 pp; ISBN- 13: 978-0-8176-4680-6, e-ISBN-13: 978-0-8176-4681-3.

In the study of complex dynamical systems, the simplest invariant manifolds are the equilibria and, next, the periodic orbits. The relevance of periodic solutions for understanding the dynamics of a finite-dimensional Hamiltonian system was highlighted at the end of the 19th century by H. Poincaré in his famous treatise on Celestial Mechanics. In spite of the fact that the set of periodic orbits has measure zero, their study is important due to the possibility, conjectured by Poincaré, of approximating arbitrarily well in long time any solution of a Hamiltonian equation by periodic solutions (with very long periods). This conjecture motivated the systematic study of periodic orbits, initiated by Poincaré and continued by the work of Lyapunov, Birkhoff, Moser, Weinstein, and others. Great progress in the understanding the complex orbit structure of Hamiltonian systems was made by Kolmogorov (1954), Arnold (1963) and Moser (1962), leading to the so called KAM theory and to small divisor theory as well.

The present book is concerned with bifurcation results of nonlinear oscillations of Hamiltonian PDEs of the form

$$u_{tt} - u_{xx} + a_1(x)u = a_2(x)u^2 + a_3(x)u^3 + \dots \tag{1}$$

Previous results about this equations referred to the "nonresonant" PDEs, that is equation (1) with non identically null term $a_1(x)$. The term $a_1(x)$ allows to verify suitable nonresonance conditions on the linear eigenfrequencies of the small oscillations, and, further, the bifurcation equation is finite-dimensional.

The main concern of the present book is to present recent bifurcation results for the completely nonresonant wave equation (1) with $a_1(x) \equiv 0$. In this case infinite-dimensional bifurcation phenomena appear jointly with small-divisor difficulties.

A good idea on the content is given by the headings of the chapters: **1.** Finite dimension; **2.** Infinite dimension; **3.** A tutorial in Nash-Moser theory; **4.** Application to the nonlinear wave equation; **5.** Forced vibrations. There are also four appendices: A. Hamiltonian PDEs; B. Critical point theory; C. Free vibrations of nonlinear wave equations: A global result; D. Approximation of irrationals by rationals; E. The Banach algebra property of $X_{\sigma,s}$.

The book is a good introduction to this fascinating and rapidly growing field of investigation, closely related to fundamental problems in mechanics and physics. It can be warmly recommended to graduate students and researchers desiring to work in nonlinear Hamiltonian PDEs or in related domains (variational techniques, critical point theory, small divisors).

Radu Precup

Patrizia Pucci and James Serrin, *The maximum principle*, Progress in Nonlinear Differential Equations and Their Applications, Vol. 73, Series Editor: Haim Brezis, Birkhäuser Verlag, Basel-Boston-Berlin, 2007, x+235 pp; ISBN: 978-3-7643-8144-8, e-ISBN: 978-3-7643-8145-5.

The maximum principle gives information about solutions of differential equations and inequalities without their explicit knowledge, being valuable tools not only for mathematicians but also for physicists, chemists, engineers, economists.

The maximum principle for elliptic partial differential equations has its origins in the maximum principle for harmonic functions proved by Gauss in 1839 on the basis of the mean value theorem. Extensions to elliptic equations and inequalities were done only at the beginning of the 20th century by Bernstein (1904), Picard (1905) and Lichtenstein (1912, 1924), with difficult proofs involving hard analysis tools as well as regularity conditions for the coefficients in the highest order term. It was Eberhard Hopf in 1927 who realized that the maximum principle can be obtained on an elementary basis. The comparison technique he invented for this purpose generated important applications in many directions. The remarkable simple proof given by Hopf to the maximum principle is given as an Appendix to Chapter 2 of the book.

The aim of the present monograph is to give a clear and thorough presentation of various maximum principles for second-order elliptic equations from their beginning in linear theory to recent work on nonlinear equations.

The maximum principles are exposed in 6 chapters of the book: **2.** *Tangency and comparison theorems for elliptic inequalities*; **3.** *Maximum principles for divergence structure elliptic differential inequalities*; **4.** *Boundary value problems for nonlinear ordinary differential equations*; **5.** *The Strong Maximum Principle and the Compact Support Principle*; **6.** *Non-homogeneous divergence structure inequalities*; **7.** *The Harnack inequality.* The first chapter, *Introduction and preliminaries*, beside some preliminary material and notation, contains the enounce of the Strong Maximum Principle and of the Compact Support Principle whose proofs are postponed to Chapter 5.

The book is clearly written, with proofs given in detail, which, although difficult, by the direct approach proposed by the authors are available to students with a basic knowledge in real analysis (including Sobolev spaces), but avoiding more advanced topics as linear operator theory, monotone operator theory, Orlicz-Sobolev spaces, or viscosity solutions, used in other treatments of the subject.

The book can be used as a good introduction to recent results in this important area of research, with the possibility for the reader to attack open problems waiting for solution.

Radu Precup