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RADIATION AND VARIABLE VISCOSITY EFFECTS IN FORCED CONVECTION FROM A HORIZONTAL PLATE EMBEDDED IN A POROUS MEDIUM

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Abstract. Radiation and temperature dependent viscosity effects on forced convection boundary layer flow over a horizontal plate embedded in a fluid-saturated porous media is studied in this paper. Darcy's law model, Rosseland model for radiation and an inverse proportional law for temperature dependent viscosity have been considered. The transformed ordinary differential equations are solved numerically, and a very good agreement between the present results and those reported for particular situations were found.

1. Introduction

Many technological applications in geophysics and conservation energy systems, thermal insulations, cooling, water waste disposal, petroleum industry involve mathematical models related to flows in fluid-saturated porous media. Recent monographs by Ingham and Pop [1,2,3], Pop and Ingham [4], Bejan et al.[5] and Vafai [6] give an excellent summary of the work on the subject.

It is well known that viscosity of many fluids depends strongly by temperature and this change influence also the flow. Water's viscosity decreases by about 240 percent when temperatures varies form 10^oC ($\mu = 0.0131$ g/cm.s) to 50^oC ($\mu = 0.00548$ g/cm.s) (see Ling and Dybs [7]), where μ is the dynamic viscosity of water. Thus, one can make significant errors when such viscosity variations are not considered.

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When technological processes take place at high temperatures (metal and glass cooling) thermal radiation effects start to play an important role and cannot be neglected (see Modest [8]). Previous works in this area were done by Ling and Dybs [7] who examined the effect of variable viscosity on forced convection past a horizontal flat plate embedded in a fluid-saturated porous medium. Postelnicu et al. [9] considered also in addition to the variable viscosity in this problem the internal heat generation effects. For viscous fluids, Kafoussias and Williams [10] and Kafoussias et al. [11] studied the combined free and forced convection on an isothermal vertical flat plate with temperature dependent viscosity while, Soundalgekar et al. [12] and Ali [13] considered the same problem for moving surfaces. The combined effects radiation and variable viscosity were also considered by Elbashbesy and Dimian [14] on the flow over a wedge.

2. **Basic Equations**

Consider the steady forced convection flow adjacent to a heated horizontal flat plate, which is embedded in an opaque fluid-saturated porous medium of ambient temperature T_∞ and velocity U_∞ as shown in Figure 1. It is assumed that the temperature of the plate is constant T_w ($T_w > T_\infty$) and there is a radiation heat transfer effect modeled by the Rosseland approximation. Following Ling and Dybs [7] we consider the temperature dependent dynamic viscosity, μ, given by:

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \gamma(T - T_\infty)] \tag{1}$$

where μ_∞ is the dynamic viscosity of the ambient fluid and γ is a constant.

Under the boundary-layer and Boussinesq approximations the governing boundary layer equations can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\frac{\partial}{\partial y}(\mu u) = 0 \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho_\infty c_p}\frac{\partial q^r}{\partial y} \tag{4}$$

where x and y are the Cartesian co-ordinate along the plate and normal to it, respectively, u and v are the velocity components along x and y -axes, T is the temperature, α is the effective thermal diffusivity of the porous medium ρ_∞ is the ambient density and c_p is the specific heat at constant pressure. We assume that the radiation heat flux, q^r, is given by, see Modest[8],

$$q^r = -\left(\frac{4\sigma}{3\chi}\right)\frac{\partial T^4}{\partial y} \tag{5}$$

where σ is the Stefan-Boltzman's constant and χ is the average absorption coefficient in Rosseland approximation. The boundary conditions of equations (2)-(4) are:

$$\begin{aligned} v = 0, T = T_w \qquad &\text{at} \qquad y = 0 \\ u = U_\infty, T \to T_\infty \qquad &\text{as} \qquad y \to \infty \end{aligned} \tag{6}$$

Equation (1) is written, for convenience, as

$$\frac{1}{\mu} = a(T - T_e) \tag{7}$$

where $a = \gamma/\mu_\infty$ is a constant with $a > 0$ for liquids and $a < 0$ for gases, see Soundalgekar et al.[12], and $T_e = T_\infty - (1/\gamma)$ is a reference temperature.

Further, introducing the stream function,ψ, defined as usual by $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, and using (7) in (2) equations (2)-(4) become:

$$\frac{T - T_e}{T_\infty - T_e} = \frac{1}{U_\infty}\frac{\partial \psi}{\partial y} \tag{8}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho_\infty c_p}\frac{16\sigma}{3\chi}\left(T^3\frac{\partial T}{\partial y}\right) \tag{9}$$

These partial differential equations can now be reduced to ordinary differential equations by introducing the following similarity variables

$$\psi = (\alpha U_\infty x)^{1/2} f(\eta), \qquad \theta(\eta) = \frac{T - T_w}{T_\infty - T_w}, \qquad \eta = (U_\infty x/\alpha)^{1/2}\,\frac{y}{x} \tag{10}$$

Using transformations (10) in equations (8) - (10) we get:

$$f' = \frac{\theta + \theta_e}{1 + \theta_e} \tag{11}$$

$$\left(\left(1 + \frac{4}{3}N\left(\theta_w + (1 - \theta_w)\theta\right)^3\right)\theta'\right)' + \frac{1}{2}f\theta' = 0 \tag{12}$$

$$f'(0) = 0, \qquad \theta(0) = 0, \qquad \theta(\infty) = 1 \tag{13}$$

where primes denote differentiation with respect to η and the parameter θ_e is the dimensionless reference temperature given by

$$\theta_e = \frac{T_w - T_e}{T_\infty - T_w} = \frac{1}{\gamma(T_\infty - T_w)} \tag{14}$$

In the energy equation (12) θ_w and N are the wall temperature and the radiation parameters defined as:

$$\theta_w = \frac{T_w}{T_\infty}, \qquad N = \frac{4\sigma T_\infty^3}{k\chi} \tag{15}$$

Using the energetic balance at the plate we deduce the convection heat transfer coefficient, h, defined as:

$$-k\left[\frac{\partial T}{\partial y}\right]_{y=0} + q^r = h(T_w - T_\infty) \tag{16}$$

and the local Nusselt number, Nu_x, which is given by:

$$\frac{Nu_x}{Pe_x^{1/2}} = \theta'(0)\left(1 + \frac{4}{3}N\theta_w^3\right) \tag{17}$$

where $Pe_x = U_\infty x/\alpha$ is the local Pclet number. We mention that for $N = 0$ (radiation is absent) equations (11) - (13) reduce to those obtained by Ling and Dybs [7].

3. **Results and Discussions**

Equations (11) and (12), subject to the boundary conditions (13) have been solved numerically using a 4th Runge-Kutta method coupled with a shooting technique for some values of the parameters θ_e, N and θ_w. In the particular case $N = 0$ (i.e. radiation is absent) the results for the dimensionless heat transfer at the plate, $-\theta'(0)$, were compared with those obtained by Ling and Dybs [7], see Table 1. It is seen that these results are in a very good agreement (see Table 1). The results obtained in the presence of radiation ($N \neq 0$) are shown in Table 2 for some values of the radiation parameter N and temperature parameter θ_w.

i) Influence of the parameter θ_e.

The dimensionless temperature and viscosity profiles $f(\eta)$ and $\theta(eta)$ are shown in Figures 2 and 3 for some values of the parameter θ_e when $N = 1$ and 10, and

$\theta_w = 1.5$. We can see from these figures that the thickness of the thermal boundary layer and viscous boundary layer increase with the decreasing of the parameter θ_e. It should also be noticed that for $\theta_e >> 1$, the predicted temperature profiles are close to those when viscosity is constant. We also notice that for the constant viscosity ($\theta_e \to \infty$) the value of $-\theta'(0) = 0.564$, agrees with the value reported by Bejan [15]. When $\theta_e \to \infty$ the velocity profiles $f'(\eta)$ are convergent to the constant profile $f'(\eta) = 1$ corresponding to the constant viscosity case (see Figure 3).

ii) Influence of the parameter N.

Figures 4 and 5 show that increasing of the radiation parameter N leads to an increasing of the thermal and viscous boundary layers. The influence of the radiation parameter N is higher for small values of θ_e (i.e. the radiation effects are more pregnant if the dependence of the viscosity with the temperature is stronger) .

iii) Influence of the parameter θ_w

Figures 6 and 7 present temperatures and velocity profiles for different values of the parameter θ_w. It is seen that the thermal and viscous boundary layer increase with the increasing of θ_w, the effect being more pregnant for small values of θ_e.

TABLE 1. Values of $-\theta'(0)$ for different values of θ_e and $N = 0$

θ_e	Ling and Dybs [7]	Present results
0	0.332	0.3320
0.05	0.347	0.3474
0.10	0.361	0.3606
0.25	0.392	0.3916
0.5	0.426	0.4260
1.00	0.465	0.4649
2.00	0.500	0.5004
5.00	0.533	0.5333
10.00	0.548	0.5476
∞	0.564	0.5641

TABLE 2. Values of $-\theta'(0)$ for different values of θ_e, N and θ_w

θ_e	N	$\theta_w = 1.1$	$\theta_w = 1.5$	$\theta_w = 2$
0	1	0.190643	0.114643	0.067368
	5	0.099127	0.052877	0.029892
	10	0.071675	0.037508	0.021103
0.1	1	0.207738	0.126601	0.075515
	5	0.108174	0.058657	0.033674
	10	0.078238	0.041640	0.023791
1	1	0.269628	0.168878	0.103644
	5	0.140838	0.078948	0.046644
	10	0.101924	0.056126	0.032998
10	1	0.318507	0.201712	0.125149
	5	0.166585	0.094628	0.056516
	10	0.120587	0.067311	0.040002
∞	1	0.328301	0.208257	0.129416
	5	0.171739	0.097749	0.058472
	10	0.124323	0.069537	0.041389

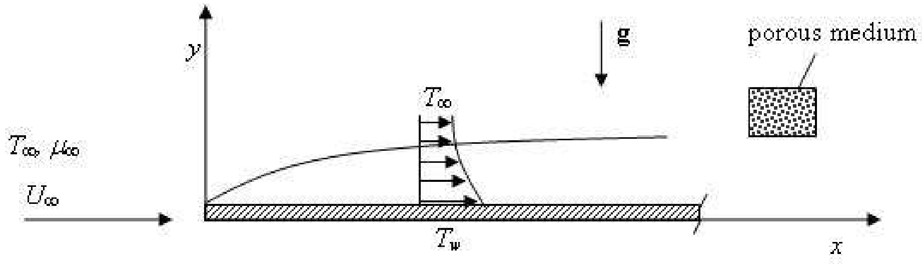

FIGURE 1. Physical model and co-ordinate system

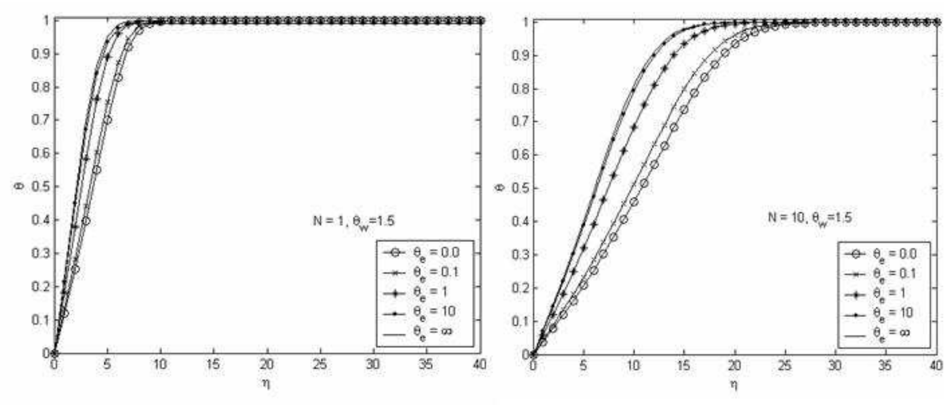

FIGURE 2. Variation of dimensionless temperature θ for different values of the parameter θ_e

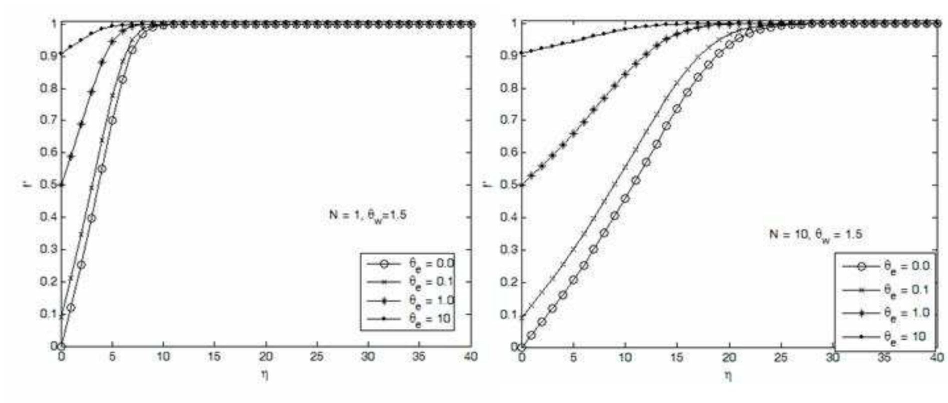

FIGURE 3. Variation of dimensionless velocity $f'(\eta)$ for different values of the parameter θ_e

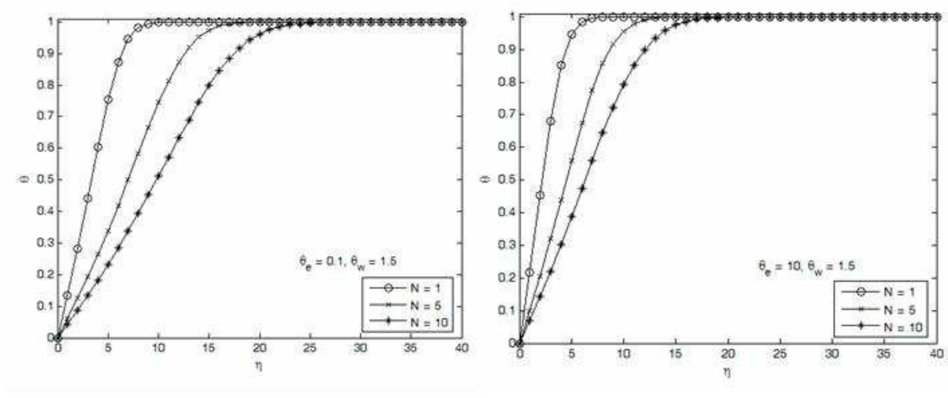

FIGURE 4. Variation of dimensionless temperature $\theta(\eta)$ for different values of the parameter N

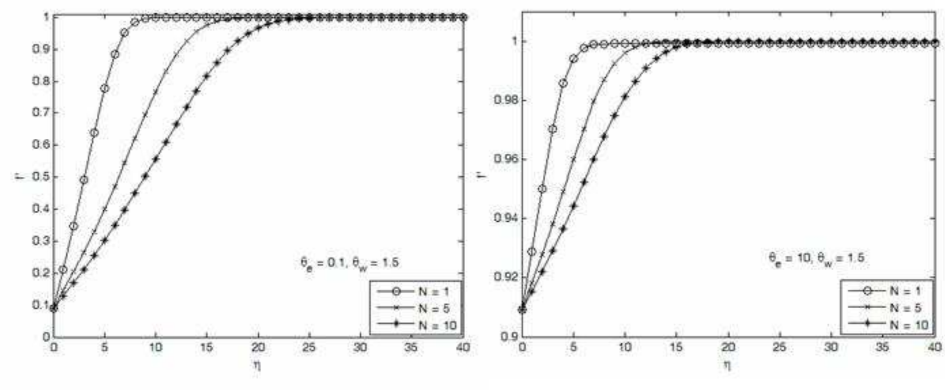

FIGURE 5. Variation of dimensionless velocity $f'(\eta)$ for different values of the parameter N

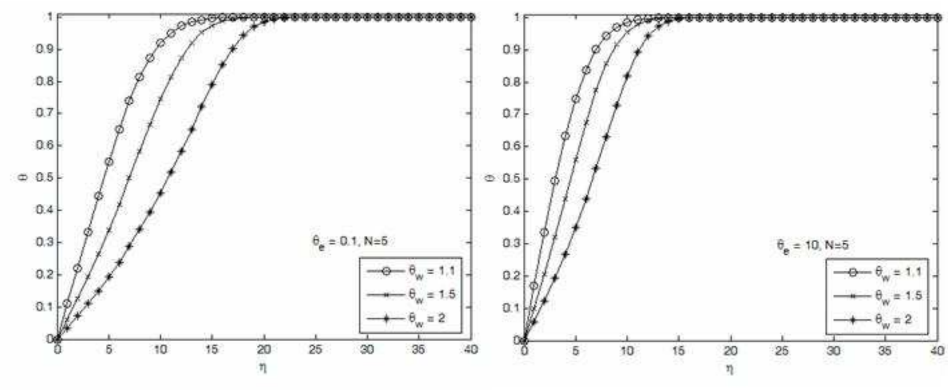

FIGURE 6. Variation of dimensionless temperature θ for different values of the parameter θ_w

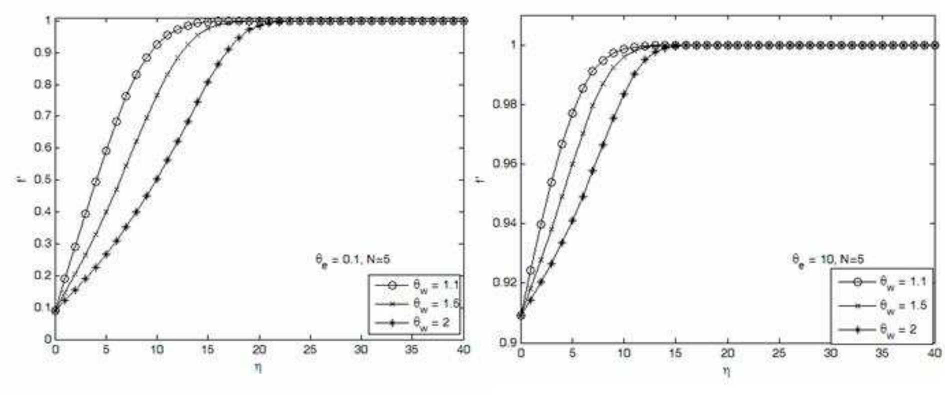

FIGURE 7. Variation of dimensionless velocity $f'(\eta)$ for different values of the parameter θ_w

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