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HEAT TRANSFER IN AXISYMMETRIC STAGNATION FLOW ON A THIN CYLINDER

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Abstract. The steady axisymetric stagnation flow and heat transfer on a thin infinite cylinder of radius a is studied in this paper. Both cases of constant wall temperature and constant wall heat flux are considered. Using similarity variables the governing partial differential equations are transformed into ordinary differential equations. The resulting set of two equations is solved numerically using Runge-Kutta method combined with a shooting technique. For the special case of the Reynolds number $Re >> 1$ (boundary layer approximation), we obtained an asymptotic solution which include the Hiemenz solution. The present results are compared in some particular cases with existing results from the open literature and with the asymptotic approximation, and we found a very good agreement. It is shown that the Nusselt number and the skin friction increase and the boundary layer thickness decreases with the increase of the Reynolds number. Some graphs for the velocity and temperature profiles are presented. Also, tables with values related to the skin friction and Nusselt number are given.

1. **Introduction**

The two-dimensional orthogonal stagnation-point flow of a viscous fluid impinging on a flat wall is a very interesting problem in the history of fluid mechanics. This flow appears in virtually all flow fields of engineering and scientific interest. Hiemenz [1] was the first who derived an exact solution of the Navier-Stokes equations

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which describes the steady forced convection flow directed perpendicular (orthogonal) to an infinite flat plate. Homann [2] studied the axisymmetric stagnation flow, also against a plate, and Howarth [3] and Davey [4] extended the results to unsymmetric cases. Later, Wang [5] presented an exact solution for the steady axisymmetric stagnation-point flow on an infinite thin circular cylinder. Gorla [6] has then considered the steady boundary layer heat transfer in an axisymmetric stagnation-point flow on an infinite thin circular cylinder. Both the cases of constant wall temperature and constant wall heat flux at the surface of the cylinder were considered. Numerical results for the velocity and temperature profiles as well as for the local Nusselt number were obtained when the Reynolds number is relatively small. Further, Gorla [7] has investigated the unsteady fluid dynamic characteristics of an axisymmetric stagnation point flow on a circular cylinder performing an harmonic motion in its own plane. Also, Gorla [8] has investigated the final approach to steady state in an axisymmetric stagnation-point flow on a thin circular cylinder.

The aim of this paper is to extend the paper by Gorla [6] on heat transfer in axisymmetric stagnation point flow on a thin infinite circular cylinder to the case when the Reynolds number is large.

2. **Basic equations**

Consider the steady-state flow and heat transfer at an axisymmetric stagnation point on a thin circular cylinder of radius a placed in a viscous and incompressible fluid of ambient uniform temperature T_∞, as shown in Fig. 1. The flow is axisymmetric about z- axis and also symmetric to the $z = 0$ plane. It is assumed that both the temperature of the surface of the cylinder T_w or the heat flux from the surface of the cylinder q_w are constants. Under these assumptions, the basic equations in cylindrical co-ordinates (r, z) are:

Continuity

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

Navier Stokes

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\upsilon\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right) \tag{2}$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right) \tag{3}$$

Energy

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right) \tag{4}$$

subject to the boundary conditions of these equations

$$\begin{aligned} r=a: &\quad u=w=0 \\ &\quad T=T_w(\text{CWT})\ or\ \frac{\partial T}{\partial r}=-\frac{q_w}{k}(\text{CWHF}) \\ r\longrightarrow\infty: &\quad u=-A\left(r-\frac{a^2}{r}\right), w=2Az \\ &\quad T=T_\infty \end{aligned} \tag{5}$$

Here u and v are the velocity components along $r-$ and $z-$ axes, T is the fluid temperature, p is the pressure, ρ is the density, α is the thermal expansion coefficient, ν is the kinematic viscosity and A is a given constant.

In order to solve Eqs. (1) - (4), we introduce the following similarity variables

$$u=-Aa\eta^{-1/2}f(\eta),\ w=2Af\text{'}(\eta)z,\ \eta=\left(\frac{r}{a}\right)^2, \tag{6}$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}(\text{CWT}),\quad \theta(\eta)=\frac{2(T-T_\infty)}{(aq_w/k)}(\text{CWHF})$$

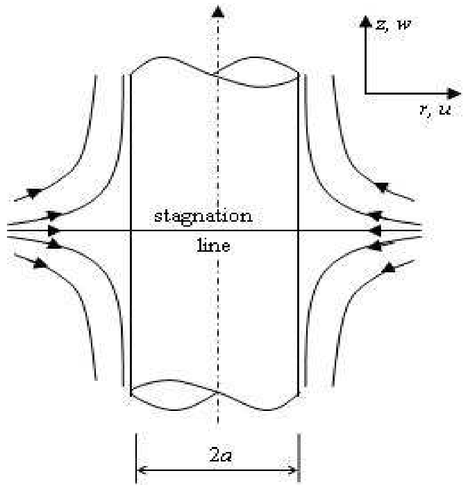

FIGURE 1. The coordinate axis

Substituting (6) into Eqs. (2) and (4), we get the following ordinary differential equations

$$\eta f''' + f'' + \mathrm{Re}(1 + ff'' - f'^2) = 0 \tag{7}$$

$$(\eta\theta')' + \mathrm{Pr}\,\mathrm{Re}\, f\ \theta' = 0 \tag{8}$$

subject to the boundary conditions (5) which become

$$f(1) = 0,\ f'(1) = 0,\ f'(\infty) = 1 \tag{9}$$

$$\theta(1) = 1,\ \theta(\infty) = 0\ \text{(CWT)}$$

$$\theta'(1) = -1,\ \theta(\infty) = 0\ \text{(CWHF)}$$

where Re is the Reynolds number and Pr is the Prandtl number which are defined

$$\mathrm{Re} = \frac{Aa^2}{2\nu}, \mathrm{Pr} = \frac{\nu}{\alpha} \tag{10}$$

The physical quantities of interest in this problem are the skin friction coefficient C_f, the Nusselt numbers for the wall constant temperature case Nu and for the constant wall heat flux case Nu^*. It is easily to show that these quantities can be expressed as

$$\operatorname{Re} C_f = -f''(1),\ Nu = -2\theta'(1)\ (\text{CWT}),\ Nu^* = \frac{2}{\theta(1)}\ (\text{CHF}) \tag{11}$$

Case $Re >> 1$

We consider now the boundary layer approximation ($Re >> 1$) of the problem under consideration. In this respect, we introduce the following new variables:

$$\begin{aligned} \xi &= \operatorname{Re}^{1/2}(\eta - 1),\ f(\eta) = \operatorname{Re}^{-1/2} F(\xi), \\ \theta(\eta) &= \Theta(\xi)\ (CWT),\ \theta(\eta) = \operatorname{Re}^{-1/2} \Theta(\xi)\ (\text{CHF}) \end{aligned} \tag{12}$$

Substituting (12) into Eqs. (7) and (8), we obtain:

$$\left(1 + \operatorname{Re}^{-1/2} \xi\right) F''' + 1 + FF'' - F'^2 + \operatorname{Re}^{-1/2} F'' = 0 \tag{13}$$

$$\left(1 + \operatorname{Re}^{-1/2} \xi\right) \Theta'' + \Pr F\Theta' + \operatorname{Re}^{-1/2} \Theta' = 0 \tag{14}$$

along with the boundary conditions

$$\begin{aligned} &F(0) = 0, F'(0) = 0, F'(\infty) = 1 \\ &\Theta(0) = 1, \Theta(\infty) = 0(CWT) \\ &\Theta'(0) = -1, \Theta_0(\infty) = 0(CWHF) \end{aligned} \tag{15}$$

We notice that for $Re \longrightarrow \infty$, that corresponds to the boundary layer approximation, Eq. (13) - (15) reduce to the Hiemenz equations that describe the stagnation point flow on a plate, see Hiemenz [1]. Equations (13) - (15) were solved analytically using the following series expansions:

$$\begin{aligned} F &= F_0 + \operatorname{Re}^{-1/2} F_1 + \operatorname{Re}^{-1} F_2 + ... \\ \Theta &= \Theta_0 + \operatorname{Re}^{-1/2} \Theta_1 + \operatorname{Re}^{-1} \Theta_2 + ... \end{aligned} \tag{16}$$

Substituting (16) into (13) - (15), we get the following three sets of equations:

first order approximation:

$$F_0''' + F_0 F_0'' - F_0'^2 + 1 = 0 \tag{17}$$

$$\Theta_0'' + \Pr F_0 \Theta_0' = 0$$

$$F_0(0) = 0,\ F_0'(0) = 0,\ F_0'(\infty) = 1$$

$$\Theta_0(0) = 1,\ \Theta_0(\infty) = 0\ \text{(CWT)}$$

$$\Theta_0'(0) = -1,\ \Theta_0(\infty) = 0\ \text{(CWHF)}$$

second order approximation:

$$F_1''' + F_0 F_1'' - 2F_0' F_1' + F_0'' F_1 + F_0'' + \xi F_0''' = 0 \tag{18}$$

$$\Theta_1'' + \Pr(F_0 \Theta_1' + F_1 \Theta_0') + \Theta_0' + \xi \Theta_0'' = 0$$

$$F_1(0) = 0,\ F_1'(0) = 0,\ F_1'(\infty) = 0$$

$$\Theta_1(0) = 0,\ \Theta_1(\infty) = 0\ \text{(CWT)}$$

$$\Theta_1'(0) = 0,\ \Theta_1(\infty) = 0\ \text{(CWHF)}$$

third order approximation:

$$F_2''' + F_0 F_2'' - 2F_0' F_2' + F_0'' F_2 + F_1'' + F_1'' F_1 - F_1'^2 + \xi F_1''' = 0 \tag{19}$$

$$\Theta_2'' + \Pr(F_0 \Theta_2' + F_1 \Theta_1' + F_2 \Theta_0') + \Theta_1' + \xi \Theta_1'' = 0$$

$$F_2(0) = 0,\ F_2'(0) = 0,\ F_2'(\infty) = 0$$

$$\Theta_2(0) = 0,\ \Theta_2(\infty) = 0\ \text{(CWT)}$$

$$\Theta_2'(0) = 0,\ \Theta_2(\infty) = 0\ \text{(CWHF)}$$

3. **Results and discussions**

Equations (7) - (8) subject to boundary conditions (9) were solved numerically for different values of the Prandtl number ($Pr = 0.01,\ 0.1,\ 1,\ 10,\ 100$) and some values of Reynolds number, $Re = 0.01,\ 0.1,\ 0.2,\ 1,\ 10,\ 20,\ 50,\ 100$ using Runge-Kutta method combined with a shooting technique. Some values related to the Nusselt numbers and skin friction are given in Table 1 for $Pr = 7$. Results reported by Wang

[5] are also included in this table. It is seen that there is a very good agreement between the present results and those reported by Wang [5]. We are, therefore, confident that our results are very accurate. The validity of the results are also illustrated in Figs. 2 to 4.

Figures 5 to 9 show the dimensionless velocity and temperature profiles for different values of the Reynolds and Prandtl numbers. Thus, it is seen that for a fixed value of the Parndtl number, the velocity profiles increase with the increase of the Reynolds number. However, the temperature profiles decrease with increase of the Reynolds number in the both cases of constant wall temperature and constant heat flux from the plate, respectively, see Figs. 5 to 7. Further, Figs. 8 and 9 show that for the both cases of constant wall temperature and constant heat flux from the plate, temperature profiles decreases with the Parndtl number when the Reynolds number is fixed. As expected the thickness of the temperature boundary layer decreases when the Parndtl number increases.

Finally, Figs. 10 and 11 show the variation of the Nusselt number with the Parndtl number in both cases of constant wall temperature and constant heat flux from the surface for a fixed value of the Reynolds number. The increase of the Nusselt number with the Reynolds number is in agreement with the results given in Table 1.

η_∞	Re	f"(1)	$\theta'(1)$	$\theta(1)$
320	0.01	0.313605	-0.320451	3.120599
80	0.1	0.615487	-0.615504	1.624684
35	0.2	0.786053	- 0.780247	1.281645
		0.78605*		
11	1	1.484185	- 1.450720	0.689313
		1.484185*		
3.5	10	4.162922	- 4.013979	0.249129
		4.16292*		
2	20	5.779734	- 5.560052	0.179855
1.75	50	8.985168	- 8.624974	0.115942
1.5	100	12.596429	-12.077699	0.082797

Wang[5]

Table 1. Values of the skin friction, $f''(1)$, Nusselt numbers, ($\theta'(1)$ for constant temperature case and $\theta(1)$ for the constant wall heat flux case), and boundary layer thickness, η_∞, for Prandtl number, $Pr = 7$ and different values of the Reynolds number, Re.

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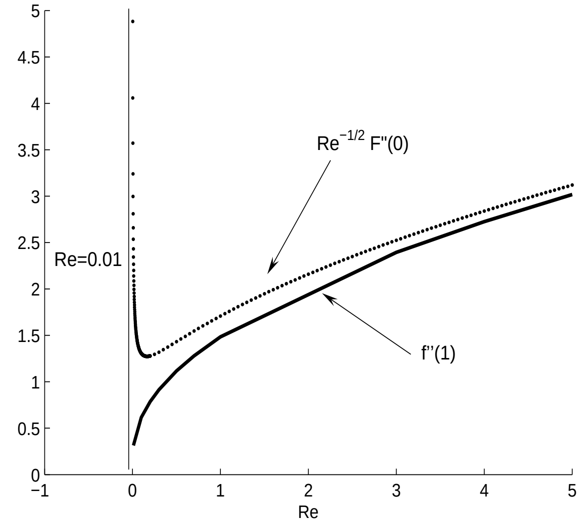

Figure 2. Validity range of the asymptotic approximation for velocity in the case $Re >> 1$.

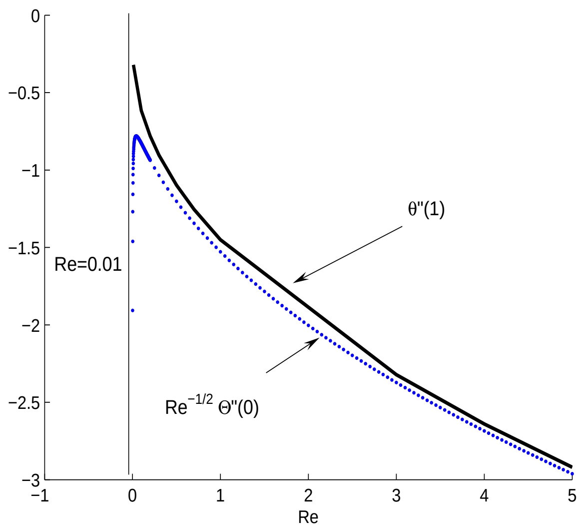

Figure 3. Validity range of the asymptotic approximation for temperature (CWT) in the case $Re >> 1$.

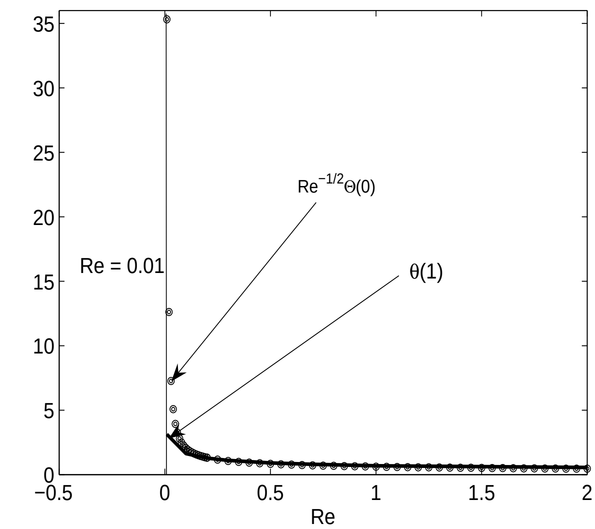

FIGURE 4. Validity range of the asymptotic approximation for temperature (CWHF) in the case $Re >> 1$.

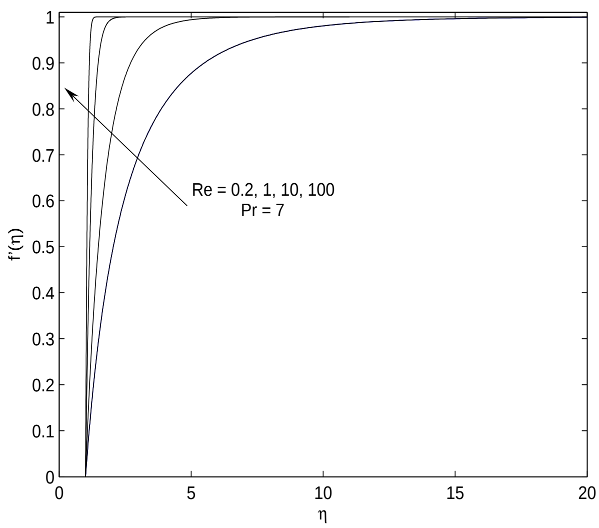

FIGURE 5. Dimensionless velocity profiles for $Pr = 7$ and $Re = 0.2, 1, 10, 100$.

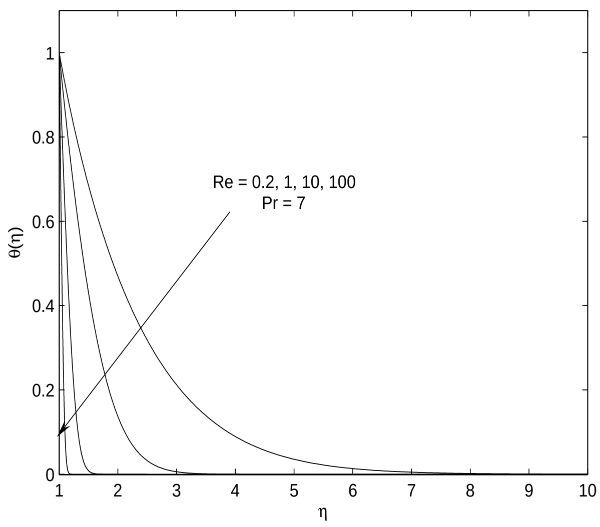

FIGURE 6. Dimensionless temperature profiles for $Pr = 7$ and $Re = 0.2, 1, 10, 100$ in the constant wall temperature case.

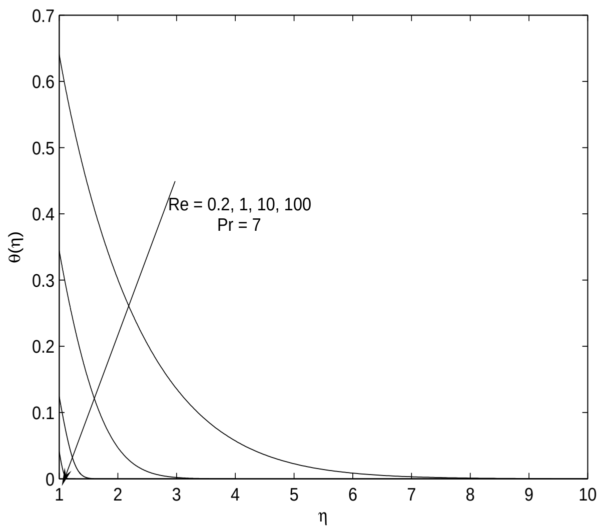

FIGURE 7. Dimensionless temperature profiles for $Pr = 7$ and $Re = 0.2, 1, 10, 100$ in the constant wall heat flux case.

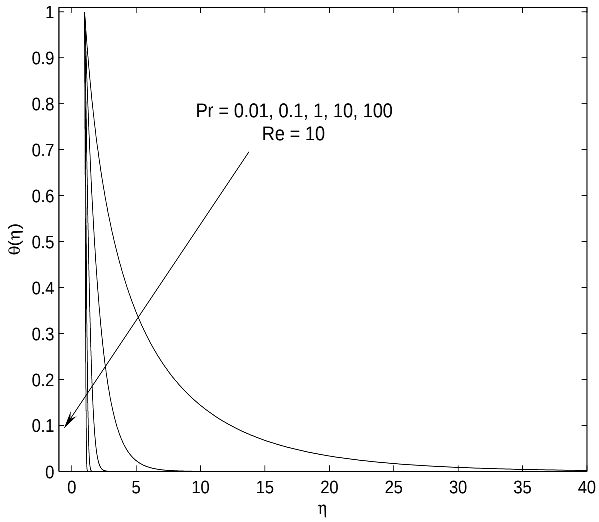

FIGURE 8. Dimensionless temperature profiles for $Pr = 0.01$, 0.1, 1, 10, 100 and $Re = 10$ for the constant wall temperature case.

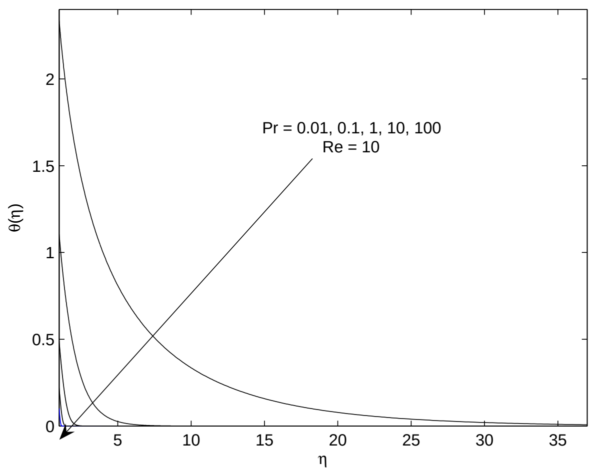

FIGURE 9. Dimensionless temperature profiles for $Pr = 0.01$, 0.1, 1, 10, 100 and $Re = 10$ for the constant wall heat flux case.

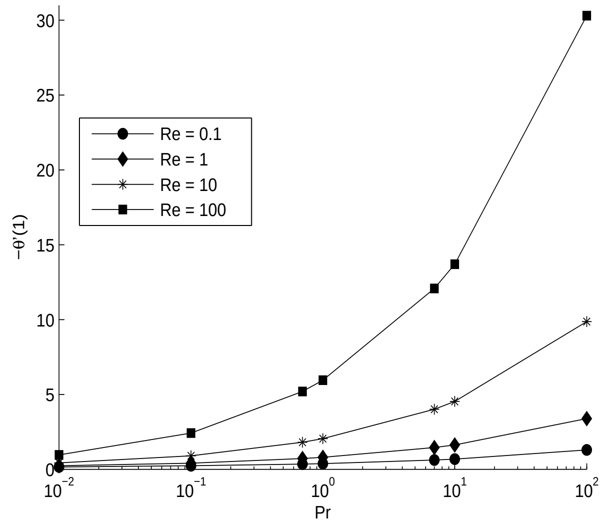

FIGURE 10. Variation of the Nusselt number with Prandtl number for $Re = 0.1, 1, 10, 100$ in the case of constant wall temperature.

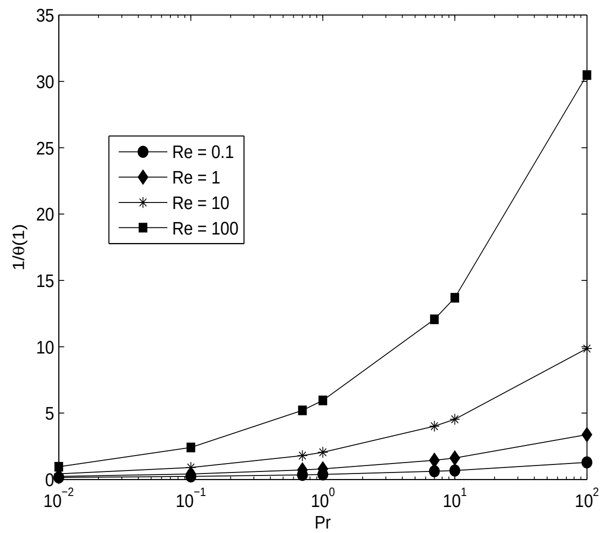

FIGURE 11. Variation of Nusselt number with Prandtl number for $Re = 0.1, 1, 10, 100$ in the constant wall heat flux case.

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