

## ROAMING OPTIMIZATION: A NEW EVOLUTIONARY TECHNIQUE FOR MULTIMODAL OPTIMIZATION

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**ABSTRACT.** A new evolutionary technique for multimodal optimization called *Roaming technique (RT)* is proposed. Multiple optima are detected using subpopulations evolving in isolation. A stability measure is defined for subpopulations by which they are characterized as stable or unstable. Stable subpopulations are considered to contain local optima. An external population called *the archive* is used to store the optima detected. After a number of generations the archive contains all local optima. Experimental results prove the efficiency of the algorithm.

### 1. INTRODUCTION

Most real world problems require the detection not only of one global optimum but also of the other global or local optima. In some cases the local optima may be almost as good as the global one, or they may provide the human decision maker with a better insight into the nature of the design space.

Evolutionary algorithms were initially designed to detect global optima and special techniques to avoid premature convergence to local optima have been developed. However, it is only natural due to the intrinsic parallelism of the evolutionary methods to assume that such methods should be able to detect also multiple solutions.

Several evolutionary approaches to the multimodal optimization problem have been made. Among them we mention: fitness sharing [1], crowding [2], deterministic crowding [4], Multinational genetic algorithm [6], Forking genetic algorithm [5] and the adaptive elitist-population based genetic algorithm [3].

### 2. ROAMING OPTIMIZATION

A new evolutionary technique for multimodal optimization called *Roaming technique* is proposed.

Roaming technique identifies the local optima using isolated subpopulations and stores them in an external population called *archive*.

A stability measure for subpopulations is defined. By using it, subpopulations are evaluated as stable or unstable. Unstable subpopulations evolve in isolation until they become stable. The best individual in a stable subpopulation is considered to be a potential local optimum.

The number of subpopulations is a parameter of the algorithm and it is not related to the expected number of local optima. This confers flexibility and robustness to the roaming search mechanism.

Potential optima are saved into the *archive*. Stable subpopulations are spread over the search space in order to detect other optima.

After a certain number of generations the archive contains all optima.

### 3. ROAMING TECHNIQUE

Consider the optimization problem:

$$\begin{cases} \text{maximize } eval(x), \\ x \in S, \end{cases}$$

where  $S$  is the solution space and  $eval(x)$  is the fitness value of individual  $x$ .

Let  $N$  be the number of subpopulations. At each generation  $t$  the population  $P$  is composed of  $N$  subpopulations  $P_i$ ,  $i = 1, \dots, N$ .

We may define an order relation on  $P$ .

**Definition 3.1.** We say that individual  $x$  is *better than*  $y$ , and we write  $x \succ y$ , if and only if the condition

$$eval(x) \geq eval(y),$$

holds.

**3.1. Stability measure.** A *stability measure* is introduced for determining whether a subpopulation has located a potential optimum.

By evolving subpopulation  $P_i$  for  $n$  generations a new subpopulation  $P'_i$  having the same size as  $P_i$  is obtained.

**Definition 3.2.** The number  $n$  of iterations the subpopulations evolve in isolation until their stability is measured is a parameter of the algorithm called the *iteration parameter*.

Let  $x_i^*$  be the best individual in the parent subpopulation  $P_i$ . We define an operator  $B$  as the set of individuals in the offspring in subpopulation  $P'_i$  that are better than  $x_i^*$ :

$$\begin{aligned} B : P &\longrightarrow \mathcal{P}(P) \\ B(x_i^*) &= \{x \in P'_i \mid x \succ x_i^*\}. \end{aligned}$$

Using the cardinality of the set  $B$  a stability measure  $SM(P_i)$  of subpopulation  $P_i$  may be defined.

**Definition 3.3.** Stability measure of the subpopulation  $P_i$  is the number  $SM(P_i)$  defined as

$$SM(P_i) = 1 - \frac{card B(x_i^*)}{card P_i},$$

where  $x_i^*$  is the best individual in  $P_i$  and  $card A$  represents cardinality of the set  $A$ .

**Proposition 3.4.** Stability measure of a subpopulation  $P$  has the following properties:

- (i)  $0 \leq SM(P) \leq 1$ ;
- (ii) If  $SM(P) = 1$  then  $x^*$  is a potential local optimum;

where  $x^*$  is the best individual in  $P$ .

*Proof.* It is obvious using stability measure definition.

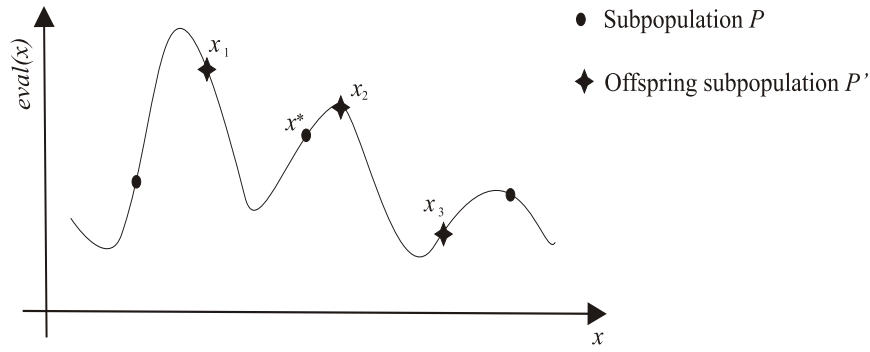


FIGURE 1. Subpopulations  $P$  and  $P'$  illustrating the stability measure concept

**Definition 3.5.** A subpopulation  $P$  is called  $\sigma$ -stable if the condition

$$(1) \quad SM(P) \geq \sigma$$

holds, where  $0 \leq \sigma \leq 1$ . A 1-stable subpopulation is called a *stable* subpopulation.

**Remarks 3.6.** The following remarks on subpopulations stability can be made:

- (i) A subpopulation  $P$  is called  $\sigma$ -unstable if it is not  $\sigma$ -stable.
- (ii) The best individual in a stable subpopulation can be considered a potential local optima.

**Example 3.7.** Suppose subpopulation  $P$  contains 3 individuals and  $x^*$  is the best of them (Figure 1). The offspring subpopulation  $P'$  contains two individuals,  $x_1$  and  $x_2$  that are better than  $x^*$ . We have

$$x_1 \succ x^*, x_2 \succ x^*, \text{ but } x^* \succ x_3.$$

Therefore

$$B(x^*) = \{x_1, x_2\}.$$

The stability measure of  $P$  is

$$\begin{aligned} SM(P) &= 1 - \frac{\text{card } B(x^*)}{\text{card } P} \\ SM(P) &= \frac{1}{3}. \end{aligned}$$

In conclusion, we can say that  $P$  is a 1-unstable subpopulation.

**3.2. Archive.** Within Roaming technique 1-unstable subpopulations evolve in isolation until they become stable. The best individual in a stable subpopulation is considered a potential local optimum. An external population called the *archive* is used to store detected potential optima.

Consider a stable subpopulation  $P$ , and  $x^*$  the best individual in  $P$ . Then  $x^*$  is considered a potential local optimum.

It is reasonable to suppose that a potential optimum can be a local optimum or can be very close to a local optimum.

Before adding a candidate solution  $x^*$  to the archive the distance between  $x^*$  and every solution  $a$  in the archive is compared with an archive parameter  $\delta$ .  $\delta$  is an algorithm parameter and is related to the minimum distance between two optima. If the condition  $d(x^*, a) < \delta$  holds then only the best fitted from  $x^*$  and  $a$  enters the archive.

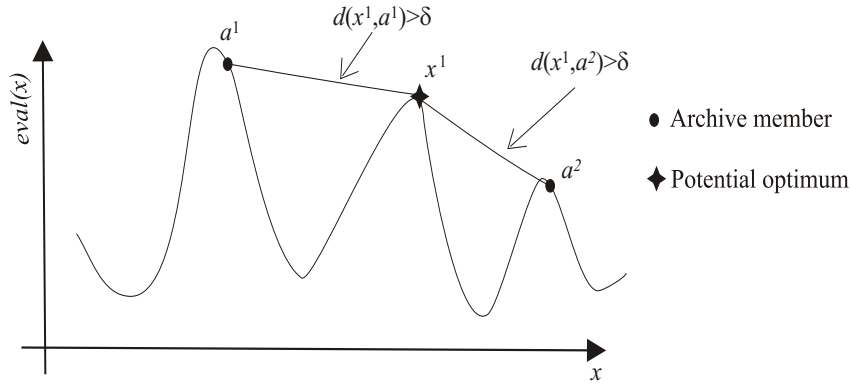


FIGURE 2. Adding a potential optima to the archive case (i):  $x^1$  is included into the archive

Summarizing, a solution  $x^*$  is added to the archive if and only if one of the following situations arise:

(i) the distance to all other solutions in the archive is higher than  $\delta$ , meaning that  $x^*$  represents a new optimum for the archive;

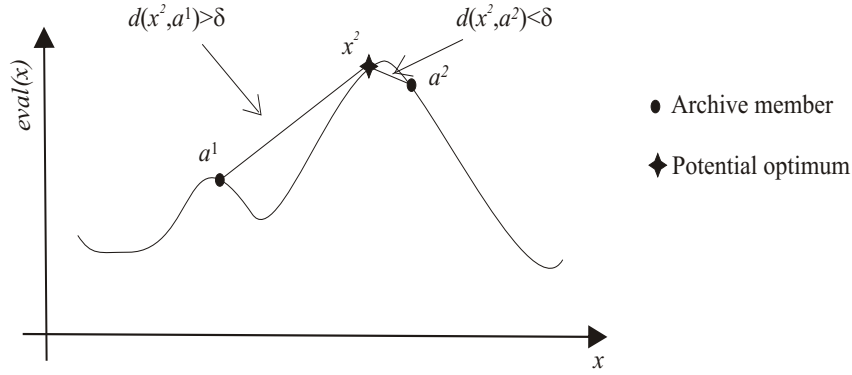


FIGURE 3. Adding a potential optima to the archive case (ii):  $x^2$  is included into the archive

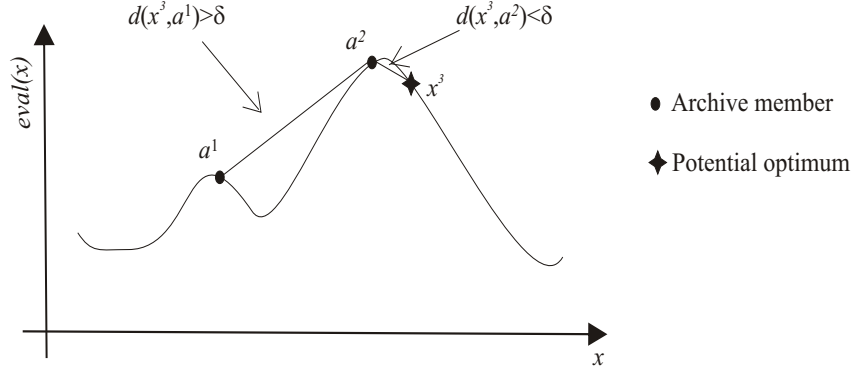


FIGURE 4. Adding a potential optima to the archive case (ii):  $x^3$  is not included into the archive

(ii)  $x^*$  is better than a solution  $a$  in the archive that is at a distance smaller than  $\delta$  from  $x^*$ , in which case  $x^*$  replaces  $a$ .

Figures 2 3 and 4 illustrate the two archive addition cases. In the first case (Figure 2) the potential optimum  $x^1$  is added to the archive. This is the case when the distance to all individuals in the archive is higher than  $\delta$ . In the second case (fig. 3) there is an individual  $a^2$  in the archive within the distance  $\delta$ .  $x^2$  is

better than  $a^2$ , therefore  $x^2$  is added to the archive. Figure 4 describes another situation that can arrive, in which the potential optima  $x^3$  is not added to the archive.

**Remark 3.8.** The archive is used exclusively to store the potential optima. Solutions preserved in the archive do not participate in further selection processes. Thus the Roaming technique can not be considered as an elitist one.

**3.3. Roaming subpopulations.** Consider a potential optimum  $x_i^*$  has been added to the archive. To avoid the search process to get stuck, the search performed by the subpopulation  $P_i$  has to be redirected towards other regions of the search space. In this respect stable subpopulations are selected to be spread in the search space in order to detect new optima.

Subpopulations selected for spreading are called *Roaming Subpopulations*. Selection is performed using subpopulations stability measure.

A parameter  $RS \in [0, 1]$  is used as a threshold in order to select the stable subpopulations. If for subpopulation  $P$  we have  $SM(P) > RS$  then it is selected as a roaming subpopulation. Actually all  $RS$ -stable subpopulations are selected. The  $RS$ -unstable subpopulations are called *non-roaming subpopulations*.

Roaming subpopulations are spread in the search space in order to detect other optima. The subpopulation roaming is realized using variation operators acting on subpopulations. We have used a mutation operator for subpopulations that applies strong mutation to each individual of the subpopulation. To ensure complete change of the individuals of the subpopulations, the mutation rate should be close to 1. Other types of genetic operators for subpopulations can be defined.

**3.4. Computing next generation.** Let  $P'_i(t)$  be the offspring subpopulation from  $P_i(t)$ . The next generation  $P(t+1)$  is composed of the roamed subpopulations and the offspring  $P'_i(t)$  of all non-roaming subpopulations  $P_i(t)$ .

**3.5. Roaming algorithm.** Roaming technique evolves several subpopulations in order to detect the local optima of a multimodal problem.

A measure for the stability of a subpopulation is used. Unstable subpopulations evolve in isolation until they become stable. The best individual of a stable subpopulation is considered to be a potential local optima. Potential local optima are saved into an external population called *archive*.

The algorithm stops after a given number of generations. At the end the archive contains detected local optima.

Figure 5 illustrates one iteration of the algorithm for a population  $P(t)$  composed of five subpopulations.

Roaming algorithm may be outlined as follows:

#### Roaming algorithm

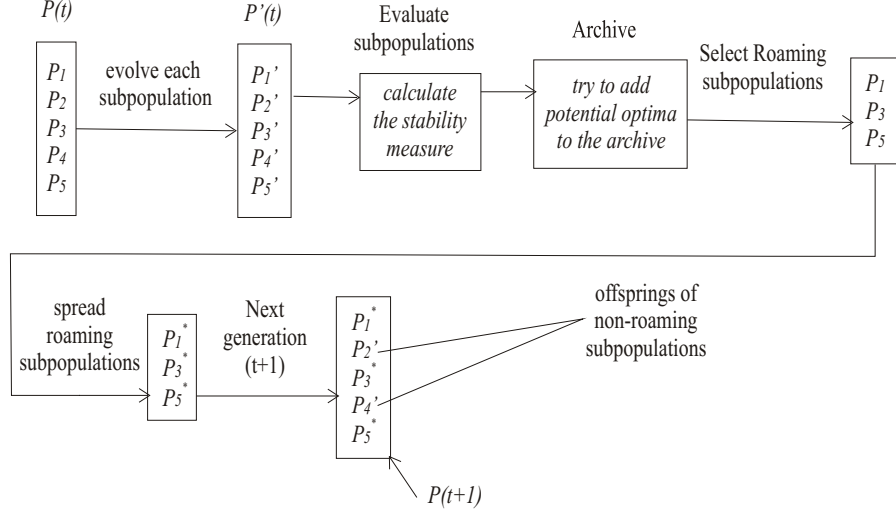


FIGURE 5. An iteration of the algorithm

**Input:**  $N$  - subpopulations number  
 $Popsiz$  - subpopulation size  
 $Ngen$  - maximum number of generations  
 $n$  - iteration parameter  
 $\delta$  - archive parameter  
 $RS$  - roaming threshold  
 $p_c, p_m$  - crossover probability and mutation rate

**Output:** *Archive* - the set of local optima

- Step 1. *Initialization:*  
a)  $t := 0$ ;  
b) Initialize  $P_i(t)$  for each  $i = 1, \dots, N$  by randomly generating  $popsiz$  number of individuals;  
c) *Archive*  $:= \emptyset$ .
- Step 2. Evaluate each individual  $x$  in each subpopulation  $P_i(t)$  by computing its fitness  $eval(x)$ ;
- Step 3. Evolve each subpopulation  $P_i(t)$  for  $n$  iterations applying selection, recombination and mutation. Let  $P'_i(t)$  be the resulting offspring subpopulation.
- Step 4. Evaluate each individual  $x$  in the offspring subpopulation  $P'_i(t)$  by calculating its fitness  $eval'(x)$ .

- Step 5. For each subpopulation  $P_i(t)$  calculate:
- a) The best individual  $x_i^*$ ;
  - b) The stability measure  $SM(P_i(t))$  using Definition 3.3.
- Step 6. For each subpopulation  $P_i(t)$  having  $SM(P_i(t))=0$  try to add  $x_i^*$  to the *Archive*.
- Step 7. For each  $i = 1, \dots, N$  do if  $SM(P_i(t)) \geq RS$  then consider  $P_i(t)$  to be a roaming subpopulation;
- Step 8. Migrate all roaming subpopulations using strong mutation with  $rate = 1$
- Step 9. Set  $P(t+1) = \{P_i(t) \mid P_i(t) \text{ is a Roaming Subpopulation}\} \cup \{P'_i(t) \mid P_i(t) \text{ is not a Roaming Subpopulation}\}$ ;  $t = t + 1$ . If  $t < Nrgen$  then go to step 2, else stop.

#### 4. EXPERIMENTAL RESULTS

Roaming Algorithm has been tested for several standard functions. In this section the following functions are considered:

- $f_1(x) = e^{-2 \ln(2) \left(\frac{x-0.1}{0.8}\right)^2} \sin^6(5\pi x)$ ,  $0 \leq x \leq 1$ ,
- $f_2(x) = \sum_{j=1}^5 j \cos((j+1)x + j)$ ,  $0 \leq x \leq 10$ ,
- $f_3(x_1, x_2) = 1 + \sin^2 x_1 + \sin^2 x_2 - 0.1 \cdot e^{(-x_1^2 - x_2^2)}$ ,  $-5 \leq x_1 \leq 5$ ,  $-5 \leq x_2 \leq 5$ .

The parameters used to run the algorithm for functions  $f_1$ ,  $f_2$  and  $f_3$  are presented in Table 1.

Evolution of the archive content for function  $f_1$  the is presented in Figures 6 and 7. Roaming algorithm detects the peaks of the function at early stage of the search process. This can be noticed also for the functions  $f_2$  and  $f_3$  in Figures 8 and 10 where the archive content after 10 generations is presented.

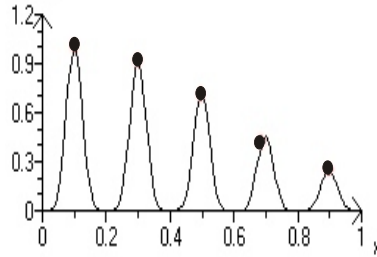
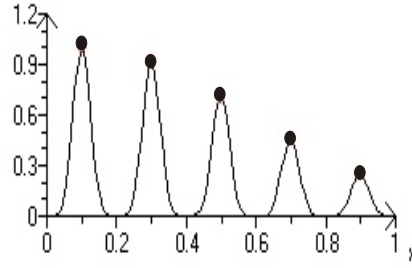


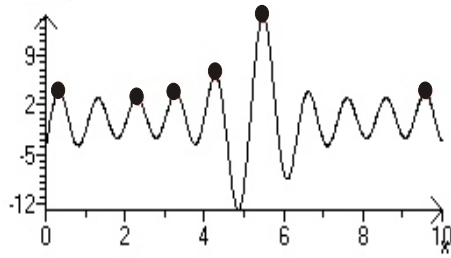
FIGURE 6. Function  $f_1$  - the archive after 10 generations

Figures 9 and 11 present the final archive for functions  $f_2$  and  $f_3$ . The number of optima stored in the archive for each function is presented in Table 2.



FIGURE 7. Function  $f_1$  - the archive after 75 generationsTABLE 1. Parameters used for  $f_1, f_2$  and  $f_3$ 

Parameter	$f_1$	$f_2$	$f_3$
Subpopulation number	15	15	10
Subpopulation size	10	10	10
Number of generations	75	75	50
$\delta$	0,1	0,3	2
Iteration parameter	1	1	1
$RS$	0,8	0,8	0,8
Crossover probability	0,5	0,5	0,5
Mutation rate	0,05	0,05	0,05

FIGURE 8. Function  $f_2$  - the archive after 10 generations

**4.1. Parameters Setting.** Roaming technique uses specific parameters such as the  $RS$  threshold,  $\delta$  and the number of subpopulations.

The  $RS$  parameter is chosen so that only stable subpopulations are selected to roam. Actually all  $RS$ -stable subpopulations are selected to roam.

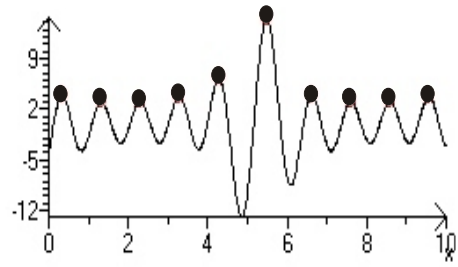
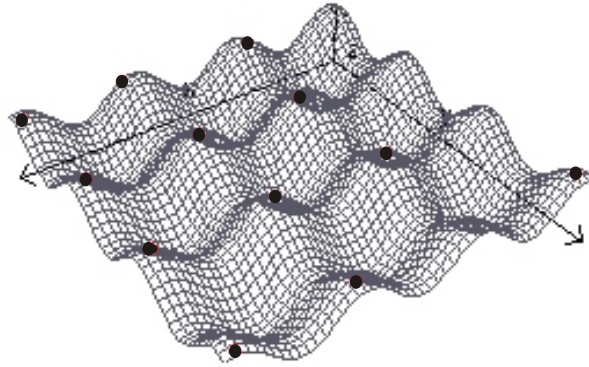
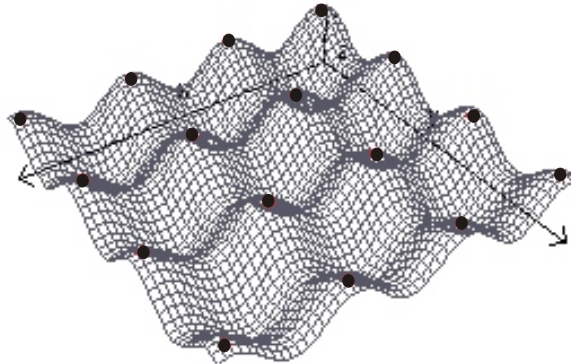
FIGURE 9. Function  $f_2$  - the archive after 75 generationsFIGURE 10. Function  $f_3$  - the archive after 10 generationsFIGURE 11. Function  $f_3$  - the archive after 50 generations

TABLE 2. Number of peaks detected for functions  $f_1$ ,  $f_2$  and  $f_3$ 

Function	Number of generations	Number of detected peaks
$f_1$	75	5
$f_2$	75	10
$f_3$	50	16

The value of  $\delta$  depends on the distribution of the local optima in the fitness landscape. For evenly distributed landscapes, the choice of  $\delta$  does not represent a problem. However it is almost impossible to choose a suitable value for  $\delta$  in the case of the functions with unevenly spaced optima. A mechanism to avoid the use of this parameter is the object of current work.

The algorithm works for any number of subpopulations. If a small number of subpopulations it is used then in order to detect more optima the number of generations has to be increased. Convergence speed depends on the subpopulation number.

## 5. CONCLUSIONS AND FUTURE WORK

A new evolutionary technique for multimodal optimization called Roaming is proposed. The technique uses a number of roaming subpopulations in order to detect multiple optima. A measure for the stability of a sub-population is introduced in order to assess whether a subpopulation has located an optimum or not.

Subpopulations evolve in isolation until an optimum is detected. Detected optima are saved into an archive and the corresponding subpopulations are spread towards new promising regions of the search space.

Numerical examples are presented to illustrate the efficiency of the technique proposed.

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