ON THE EQUIDISTANT DIVISION OF TWO-DIMENSIONAL SMOOTH CURVE

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REZUMAT.- Asupra divizării echidistante a unei curbe plane netede. Această notă prezintă o metodă de divizare echidistantă a unei curbe plane netede definită prin ecuaţia sa explicită, parametrică, polară sau printr-un tabel de puncte discrete. În final este dată o aplicaţie de natură tehnică. Programarea formulei lui SIMPSON, aferentă unor integrări numerice, a fost făcută în limbajul HPL pentru un calculator de tipul HEWLETT-PACKARD 9826.

0. Introduction. Setting of Problem. For a two-dimensional smooth curve given by explicit, parametric, polar equations or by n data points, i.e.:

$$\left| \; y = f(x), \qquad x \in [x_0, x_n], \right. \tag{1}$$

$$\left| \begin{array}{l} x = x(t), \\ y = y(t), \qquad t \in [t_0, t_n], \end{array} \right. \tag{2}$$

$$\left| \; r = r(\varphi), \qquad \varphi \in [\varphi_0, \varphi_n], \right. \tag{3}$$

and respective

$$\left| \begin{array}{l} (x_0, y_0), \; (x_1, y_1), \ldots, (x_n, y_n), \\ x_0 < x_1 < \ldots < x_n, \qquad n \in N, \end{array} \right. \tag{4}$$

on search points coordinates $(\alpha_i, \beta_i), i = \overline{0,m}$ which divide this curve in m equal arcs, or the distances between (α_i, β_i) and $(\alpha_{i+1}, \beta_{i+1}), i = \overline{0,m-1}$ to be equal.

There are many technical applications of such problems. For example, in the last part of this paper is presented equidistant division of basis ellipse from gearings with elliptical gear wheels.

1. BRIEF THEORETICAL SPECIFICATIONS. SOLUTION OF PROBELM.

Let a plane smooth curve describe by the equations (1), (2) or

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(3) be given. For a value of x (t or φ) may be obtained corresponding value of the function. If the curve is given by n data points (see (4)), first it is necessary an interpolation proceeding and for a value x the ordinate y is obtained as the value of the interpolating polinom at x.

Let ℓ be the length of the curve. We have:

$$\ell = \int_{x_0}^{x_n} \sqrt{1 + [f'(x)]^2} \cdot dx = \int_{T_0}^{T_n} \sqrt{[x'(t)]^2 + [y'(t)]^2} \cdot dt = \\ = \int_{\varphi_0}^{\varphi_n} \sqrt{r^2 + [dr/d\varphi]^2} \cdot d\varphi, \tag{5}$$

and respective:

$$\ell = \sum_{k=0}^{n-1} \int_{x_k}^{x_{k+1}} \sqrt{1 + [f'_{s,k}(x)]^2} \cdot dx, \tag{6}$$

where $f_{s,k}$ are elementary functions of cubic spline interpolation for n data points (4), [1], [2], [3], [4]. If it is necessary m division arcs, then length of elementary arc (10) is $(1/m)$. Now we determine coordinates of the division points (α_i, β_i), $i = \overline{0,m}$ The first point of division (α_0, β_0) coincides with (x_0, y_0) and the last one (α_m, β_m) coincides with (x_n, y_n). The estimation of definite integrals and solution of transcendent equation are maked with numerical methods.

Thus, for integration we have 1/3 SIMPSON rule, [1], [3]:

$$\int_b^a f(x) \cdot dx - (h/3) \cdot \{f(a) + 4 \cdot f(a + h) + 2 \cdot f(a + 2h) + \\ + 4 \cdot f(a + 3h) + \ldots + 4 \cdot f[a + (p - 1)h] + f(a + ph)\}, \tag{7}$$

where $h = (b - a)/p$.

This formula, applyed on each interval $[\alpha_i, \alpha_{i+1}]$, $i = \overline{0,m-1}$, with

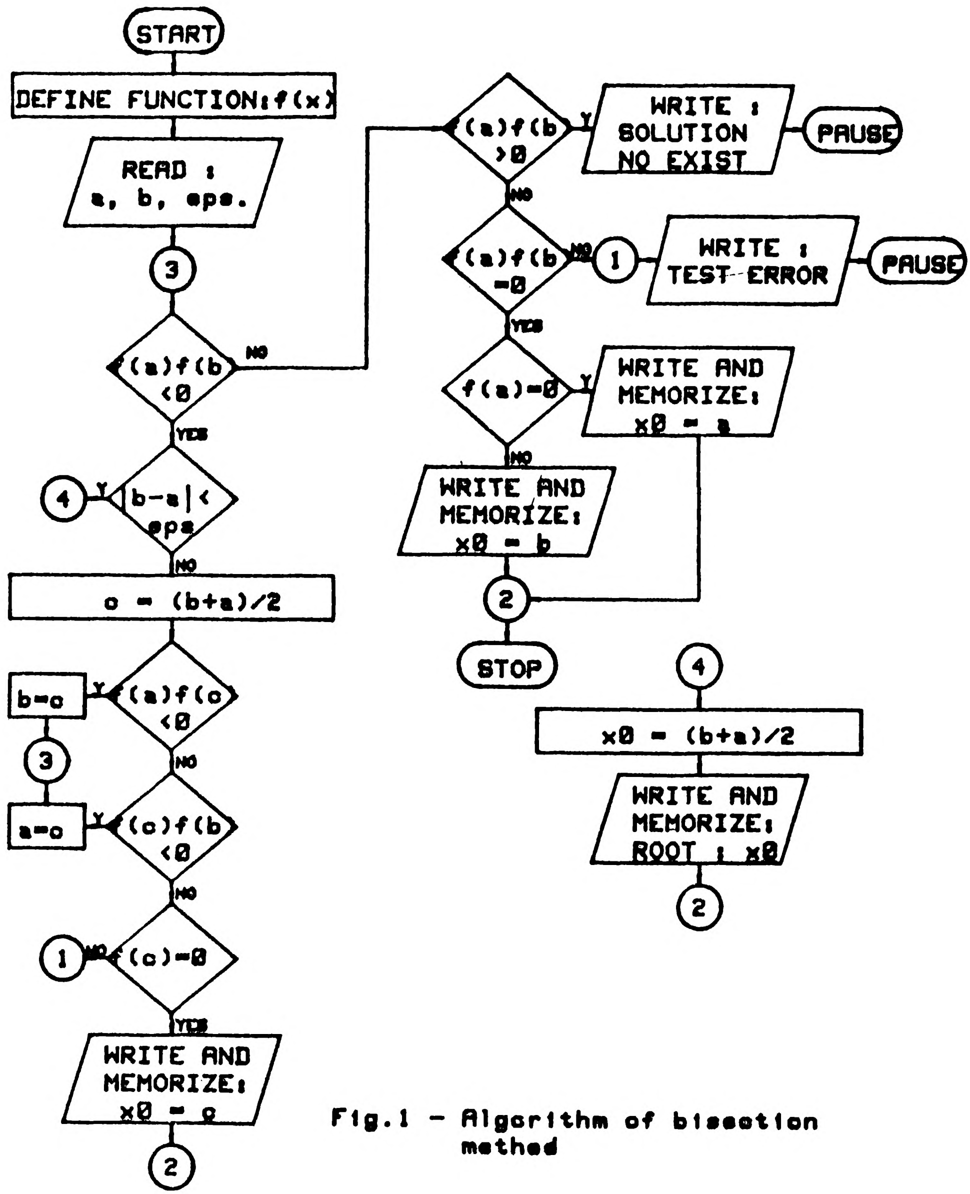

Fig.1 - Algorithm of bisection method

$$\int_{\alpha_i}^{\alpha_{i+1}} f(x) \cdot dx = 10, \qquad i = \overline{0, m-1}$$

and α_{i+1} unknown (initial value is $\alpha_0 = x_0$ and maximum value is $\alpha_m = x_n$), becomes:

$$(h/3) \cdot \{f(\alpha_i) + 4 \cdot f(\alpha_i + h) + 2 \cdot f(\alpha_i + 2h) + 4 \cdot f(\alpha_i + 3h) + \ldots + \\ + 4 \cdot f[\alpha_i + (p - 1)h] + f(\alpha_i + ph)\} - 10 = 0, \qquad (8)$$

where $h = (\alpha_{i+1} - \alpha_i)/p$, $\alpha_0 = x_0$, $\alpha_m = x_n$, $i = \overline{0, m-1}$ and α_{i+1} is unknown value.

We remark that, in general, the function f has one of the following forms:

$$\sqrt{1 + [f'(x)]^2} \to f(x), \qquad \sqrt{[x'(t)]^2 + [y'(t)]^2} \to f(t),$$

$$\sqrt{r^2 + (dr/d\varphi)^2} \to f(\varphi), \qquad \sqrt{1 + [f'_{s,k}(x)]^2} \to f(x) \text{ and}$$

$$[t_0, t_n] \to [x_0, x_n], \ [\varphi_0, \varphi_n] \to [x_0, x_n] \text{ or } [x_k, x_{k+1}] \to [x_0, x_n]$$

For equation (8) we present a numerical solution by using bisection method where $\alpha_i < \alpha_{i+1} < \alpha_m = x_n$. In Fig. 1 is given the algorithm of this method.

The number of points, p, for SIMPSON rule depends on the ε-precision supplyed by the user. If for the estimation of the curve length ℓ, we impose the precision $\varepsilon/2$ and for each interval $[\alpha_i, \alpha_{i+1}]$, the precision $\varepsilon/(2m)$ both from the estimation of number p and for the solution of the equation (8), the total error will be:

$$\varepsilon/2 + m \cdot \varepsilon/(2 \cdot m) = \varepsilon \qquad (9)$$

which may be considered as error for entire proceeding (7)-(8).

CO-ORDINATES OF DIVISION POINTS :

X(0.000)= 97.594	Y(0.000)= 0.000
X(1.000)= 96.368	Y(1.000)= 12.641
X(2.000)= 92.777	Y(2.000)= 24.822
X(3.000)= 87.055	Y(3.000)= 36.160
X(4.000)= 79.517	Y(4.000)= 46.382
X(5.000)= 70.497	Y(5.000)= 55.322
X(6.000)= 60.305	Y(6.000)= 62.899
X(7.000)= 49.213	Y(7.000)= 69.084
X(8.000)= 37.451	Y(8.000)= 73.875
X(9.000)= 25.217	Y(9.000)= 77.283
X(10.000)= 12.682	Y(10.000)= 79.322
X(11.000)=-.000	Y(11.000)= 80.000
X(12.000)=-12.682	Y(12.000)= 79.322
X(13.000)=-25.217	Y(13.000)= 77.283
X(14.000)=-37.451	Y(14.000)= 73.875
X(15.000)=-49.213	Y(15.000)= 69.084
X(16.000)=-60.305	Y(16.000)= 62.899
X(17.000)=-70.497	Y(17.000)= 55.322
X(18.000)=-79.517	Y(18.000)= 46.382
X(19.000)=-87.055	Y(19.000)= 36.160
X(20.000)=-92.777	Y(20.000)= 24.822
X(21.000)=-96.368	Y(21.000)= 12.641
X(22.000)=-97.594	Y(22.000)= 0.000

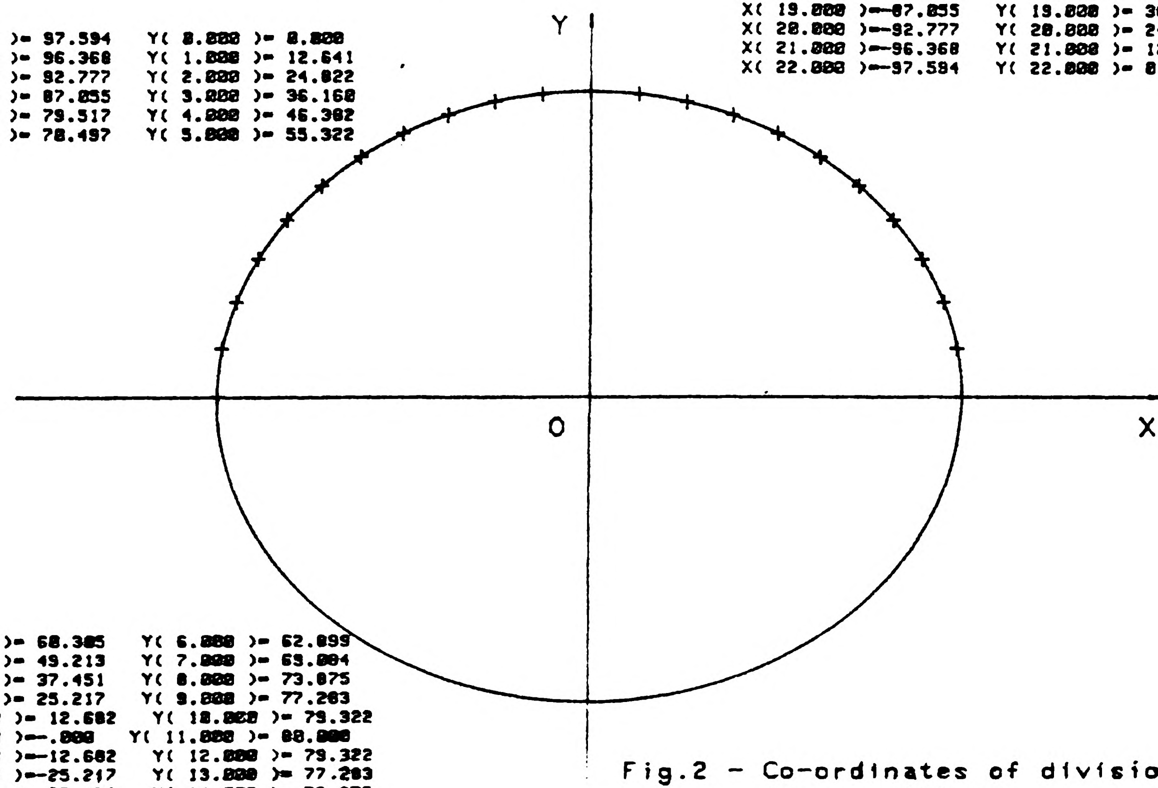

Fig.2 - Co-ordinates of division points for an ellipse

A subroutine for SIMPSON rule is presented by HPL, [5], program given in Table 1.

In this method the stop criterion is to obtain a maximum number of divisions for interval [a,b] or the difference between two successive values is less than ε-tolerance supplyed by the user.

All parameters supplyed as input data for subroutine must be initialized in driver programme:

The user may by joint subroutine with any your programme and solution for integral value is stored in *V* variable.

Parameters and used variables:

a) Input data:

P - maximum numbers of iterations;

A - lower bounds of definite integral;

B - upper bounds of definite integral;

$f(x) \to Y$ - which may be given by the user;

E - error tolerance (difference between two successive calculated values of integral.

b) Output data:

P, A, B, f(x), E (see a));

V - approximative value for definite integral: $\int_a^b f(x) \cdot dx$.

c) Working variables:

D - domain of abscissae;

J, ($J < P$) - index of iterations;

Q - precedent value of integral (used for to stop criterion);

N - current number of intervals (*N* = 2^*J*);

C - size of interval [*C* = (*B* - *a*)/*N*];

X - function argument;

Y - function value {*y* = *f*(*x*)];

R - the first and the last value of the integral.

Table 1 - HPL Programme of SIMPSON rule

```
0: "SIMPSON":0->i;0->V;sfg3;if A>B;prt"error limits";ret
1: A->X;gsb"Function";V+Y->X;gsb"Function";V+Y->V;V+Y->R
2: if (J+1->J)>P;Q->V;ret
3: 2^J->N;((B-A)/N->C)+A->X;gsb"Function";V+4Y->V;A->D
4: if (D+2C->D)<B;gto 8
5: CV/3->V;if flg3;cfg3;gto 7
6: if abs(Q-V)<E;ret
7: V->Q;R->V;gto 2
8:  D->X;gsb"Function";V+2Y->V;D+C->X;gsb"Function";4+4Y->V;
   gto4
9: "Function"
10: 72.5*V(1-X*X/(1041312))->Y;ret
```

2. EXAMPLE. The described proceeding has been applicated for equidistant division of basis ellipse from gearings with elliptical gear wheels in paper industry (gearings axis has been situated in focus of ellipse). Such an example of division is given in Fig. 2, where it is shown coordinates of division points.

R E F E R E N C E S

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