On non-integer based expansions of real numbers in a special form

Gábor Nagy

Department of Computer Algebra, Eötvös Loránd University nagy@compalg.inf.elte.hu

The research of non-integer based expansions of real numbers was started by A. Rényi and W. Parry ([3], [2]), a summary of the results of P. Erdős et al. in this field by G. Kallós can be find in [1]. The work of the aforementioned authors is about the expansions in the form:

$$x = \sum_{n=1}^{\infty} \varepsilon_n \lambda^n,$$

where $0 < \lambda < 1$ and $\varepsilon_n \in \{0, 1, \dots, \lceil \frac{1}{\lambda} \rceil\}$.

I. Kátai proposed the investigation of the expansions in the form:

$$x = \sum_{n=1}^{\infty} \varepsilon_n \lambda^n + \delta_n \omega_n,$$

where ω_n is dependent of λ and n, one can choose values for ε_n s, but the values of δ_n s are determined by an opponent. This talk is about what numbers can be represented this way in the cases $\omega_n = a\lambda^n$ and $\omega_n = (a\lambda)^n$.

References

- Kallós, G., On some problems of expansions investigated by P. Erdős et al., Annales Univ. Sci. Budapest., Sect. Comp, Vol. 45 (2016), 239-259.
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- [3] Rényi, A., Representations for real numbers and their ergodic properties, Acta Mathematica Academiae Scientiarum Hungarica, Vol. 8 (1957), 477-493.