NATURAL CONVECTION OF NANOFLUIDS ABOUT A VERTICAL CONE EMBEDDED IN A POROUS MEDIUM: PRESCRIBED HEAT GENERATION/ABSORPTION SOURCE

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Section: **Natural and forced convection in porous media**

Abstract
The analysis and simulation of natural convection in saturated porous media have many important engineering and geophysical applications. In this paper, natural convection heat transfer over a vertical cone in a Darcy porous medium saturated with a nanofluid subject to heat generation/absorption is studied. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. The governing partial differential equations are transformed into a set of ordinary differential equations using appropriate similarity transformations and then numerically solved using the Runge-Kutta method. The influence of parametric variation of the heat generation/absorption parameter on the two important parameters of heat and mass transfer, reduced Nusselt number and reduced Sherwood number, is investigated. The results show that an increase in the heat generation/absorption parameter decreases the reduced Nusselt number whereas increases the reduced Sherwood number.

Keywords: Nanofluids, Natural Convection, Porous Media, Heat generation/absorption, Brownian motion, Thermophoresis.

Nomenclature

D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
f	rescaled nano particle volume fraction
Le	Lewis number
Nb	Brownian motion parameter
Nr	buoyancy ratio
Nt	thermophoresis parameter
P	pressure
Ra_x	local Rayleigh number
S	dimensionless stream function

Greek symbols

μ	viscosity of fluid
α_m	thermal diffusivity of porous media
β	volumetric expansion coefficient of fluid
γ	cone angle
η	dimensionless distance
θ	dimensionless temperature
τ	parameter defined by equation (6)

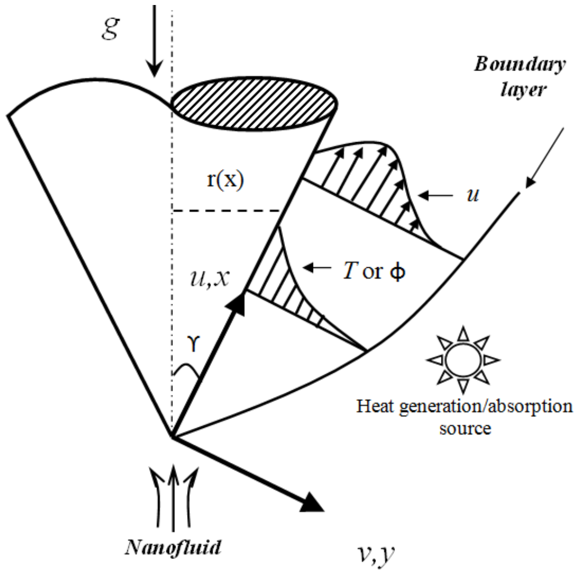
Introduction
The study of natural convection heat transfer in a porous medium is gaining a lot of attention because of its many engineering applications such as thermal energy storage, groundwater systems, flow through filtering media, and crude oil extraction [1]. Among these applications, the thermal convection in a fluid saturated porous medium with internal energy sources is very important in the theory of thermal ignition and in problems dealing with chemical reactions and those concerned with dissociating fluids [2]. Possible heat generation effects in the porous space may alter the temperature distribution; consequently the particle deposition rate in nuclear reactors, semiconductor wafers and electronic chips [3].

The problem of natural convection around a cone embedded in a porous medium at high Rayleigh numbers has been numerically analyzed by Cheng et al. [4]. Then, Yih [5] numerically investigated the effect of uniform lateral mass flux on natural convection around a cone embedded in a saturated porous medium using Keller box method. However, the flow and heat transfer of natural convection over embedded bodies in porous medium has been studied in large amount of papers, but

few papers considered the nanofluid flow and heat transfer. The nanofluid flow and heat transfer in porous medium is completely a new problem. Hence, only has few works been done in this area. The consideration of additional heat transfer mechanisms in the convective heat transfer problems was taken one step further by Buongiorno in 2006 [6]. He introduced slip mechanisms for nanoparticles. From this point of view, the natural convective boundary-layer flow over a vertical plate embedded in a porous medium which is saturated by a nanofluid has been investigated by Nield and Kuznetsov [7]. The mixed convective boundary layer flow over a vertical wedge embedded in a porous medium saturated with a nanofluid subject to natural convection dominated regime has been analyzed by Gorla *et al.* [1]. Later, Rashad et al. [8] investigated the natural convection boundary layer of a non-Newtonian fluid about a permeable vertical cone embedded in a porous medium saturated with a nanofluid. They neglected the heat generation/absorption effect in their model.
In the present study, the effect of heat generation/absorption parameter on the heat and mass transfer over a vertical cone embedded in a porous medium saturated with a nanofluid has been theoretically examined. To the best of author's knowledge this is the first time which the effect of heat generation/absorption in the presence of dynamic effects of nanofluids is analyzed.

Mathematical Formulations

Consider the two-dimensional problem and steady natural convection boundary layer flow past along a vertical cone which is placed in a porous medium saturated with nanofluid in the presence of heat generation/absorption. The coordinate system is chosen such that the x-axis is aligned with the flow over the cone. The scheme of physical model and the coordinate system have been shown in Figure 1. Although there are three distinct boundary layers namely, hydrodynamic boundary layer, thermal boundary layer, and nanoparticle concentration boundary layer over the cone, only one boundary layer has been shown in this figure to avoid congestion. It is assumed that the temperature at the surface of the cone is constant, and also the nanoparticle volume fraction (φ) at the wall surface (y=0) takes the constant value of φ_w. In the porous medium the heat is either generated or absorbed with the rate of Q_0 where Q_0 is negative in the case of heat absorption and positive in case of heat generation. The ambient values of T and φ are denoted by T_∞ and φ_∞, respectively. The flow in the porous medium with porosity of ε and permeability of K is considered as Darcy flow. It is also assumed that the nanofluid and porous medium are homogeneous and in local thermal equilibrium.



By employing the Oberbeck-Boussinesq approximation and applying the standard boundary layer approximations, the basic steady conservation of total mass, momentum, thermal energy, and nanoparticles for nanofluids in the Cartesian coordinate system of x and y are as follows [8] and [9],

$$\frac{\partial (ru)}{\partial x}+\frac{\partial (rv)}{\partial y}=0, \tag{1}$$

$$\frac{\partial p}{\partial y}=0, \tag{2}$$

$$\frac{\mu}{K}u=\left[\begin{array}{l}-\left(\frac{\partial p}{\partial x}-(1-\phi_\infty)\beta g\rho_{f_\infty}\cos\gamma\,(T-T_\infty)\right)\\ -(\rho_P-\rho_{f_\infty})\,g\cos\gamma\,(\phi-\phi_\infty)\end{array}\right], \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\nabla^2T+\tau\left[D_B\frac{\partial\varphi}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{Q_0}{(\rho c_p)_f}(T-T_\infty), \tag{4}$$

$$\frac{1}{\varepsilon}\left[u\frac{\partial\phi}{\partial x}+v\frac{\partial\phi}{\partial y}\right]=D_B\frac{\partial^2\phi}{\partial y^2}+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2T}{\partial y^2}, \tag{5}$$

α_m and τ, in the above equations, are defined as:

$$\alpha_m=\frac{k_m}{(\rho c)_f},\qquad \tau=\frac{\varepsilon(\rho c)_p}{(\rho c)_f}, \tag{6}$$

Based on the problem description, the boundary conditions are:

$$v=0,\qquad T=T_w,\qquad \phi=\phi_w,\qquad at\quad y=0, x\geq 0 \tag{7}$$

$$u=v=0,\quad T\to T_\infty,\quad \phi\to\phi_\infty,\quad at\quad y\to\infty \tag{8}$$

Equations (2) and (3) are simplified using cross-differentiation, and the continuity equation will also be satisfied by introducing a stream function, (ψ):

$$u=\frac{1}{r}\frac{\partial\psi}{\partial y},\qquad v=-\frac{1}{r}\frac{\partial\psi}{\partial x}, \tag{9}$$

Here can local Rayleigh number Ra_x be introduced as:

$$Ra_x = \frac{(1-\phi_\infty)\,\rho_{f\infty}\beta g K x \cos\gamma\,(T_w - T_\infty)}{\mu\,\alpha_m}, \tag{10}$$

By introducing the similarity variable η as,

$$\eta = \frac{y}{x} Ra_x^{\frac{1}{2}}, \tag{11}$$

and the dimensionless similarity quantities S, θ, and f as,

$$S = \frac{\psi}{\alpha_m\, r\, Ra_x^{\frac{1}{2}}}, \qquad f = \frac{\phi-\phi_\infty}{\phi_w-\phi_\infty}, \qquad \theta = \frac{T-T_\infty}{T_w-T_\infty}, \tag{12}$$

and also by considering,

$$r = x\sin\gamma \tag{13}$$

Eqs. (1)-(5) can be written as following three ordinary differential equations,

$$S'' - \theta' + Nr.f' = 0, \tag{14}$$

$$\theta'' + \frac{3}{2} S\theta' + Nb.f'.\theta' + Nt\,(\theta')^2 + \lambda.\theta = 0, \tag{15}$$

$$f'' + \frac{3}{2} Le.S.f' + \frac{Nt}{Nb}\theta'' = 0, \tag{16}$$

subject to the following boundary conditions,

$$\eta = 0: \qquad S=0, \qquad \theta=1, \qquad f=1 \tag{17}$$

$$\eta \to \infty: \qquad S'=0, \qquad \theta=0, \qquad f=0 \tag{18}$$

where

$$\left.\begin{aligned} Nr &= \frac{(\rho_p-\rho_{f\infty})(\phi_w-\phi_\infty)}{(1-\phi_\infty)\,\rho_{f\infty}\beta\,(T_w-T_\infty)},\\ Nb &= \frac{\varepsilon\,(\rho c)_p D_B\,(\phi_w-\phi_\infty)}{(\rho c)_f\,\alpha_m},\\ Nt &= \frac{\varepsilon\,(\rho c)_p D_T\,(T_w-T_\infty)}{(\rho c)_f\,\alpha_m T_\infty},\\ \lambda &= \frac{\mu Q_0 x}{(1-\phi_\infty)\,K.\cos\gamma.\rho_{f\infty}^2 c_p\beta g\,(T_w-T_\infty)},\\ Le &= \frac{\alpha_m}{\varepsilon D_B}, \end{aligned}\right\} \tag{19}$$

In the above equations where Q_0 varies with the inverses of position (1/x), the obtained equation are in the form of fully similarity solution, or else they are in the form of local similarity. Two important heat transfer parameters, local Nusselt number (Nu_x) and local Sherwood number (Sh_x), are defined as [7]:

$$Nu = \frac{q_W x}{k\,(T_W - T_\infty)}, Sh = \frac{q_m x}{D_B\,(\phi_W-\phi_\infty)}, \tag{20}$$

where q_w and q_m are the wall heat flux and mass flux, respectively. Using similarity transforms introduced in Eq. (12), the reduced Nusselt number, -θ'(0), and reduced Sherwood number, -f'(0), are obtained as follows:

$$\begin{aligned} Nu_x Ra_x^{-\frac{1}{2}} &= Nu_r = -\theta'(0),\\ Sh_x Ra_x^{-\frac{1}{2}} &= Sh_r = -f'(0) \end{aligned} \tag{21}$$

Solution

Consider the system of equations (14)-(16) subject to the boundary conditions (17) and (18). The system of equations is numerically solved using an efficient, iterative fourth-order Runge-Kutta method starting with an initial guess. In this method, every n^{th}-order equation is converted to n first order equations. Therefore, the system of ordinary differential equation is converted into a system of first order equations. Then, an iteration method is applied on the mentioned first order differential equations system. The critical value of this method is the selection of appropriate asymptotic value of η_∞. In the present study η_∞ is taken to be 10, which ensures the asymptotic convergence of the solution. Then, the η direction is divided into 300 nodal points so that the results become accurate. A maximum relative error of 10^{-5} is used as the stopping criteria for the iterations. As a test of the accuracy of the solution, the value of Nur is compared with the reported value by Cheng et al. [4] and Yih [5] in table 1 when the effects of nanofluid parameters and heat generation/absorption parameter are neglected in this study.

Table 1: Comparison of results for reduced Nusselt number

Cheng et al. [4]	Yih [5]	Present Results (Nt=Nb=Nr=0)
0.7685	0.7686	0.76859

Results and discussion

Figure 2 shows the effect of heat generation /absorption on the velocity and nanoparticle concentration profiles for fixed values of Nr, Nb, Nt and Le. As seen, an increase in the heat generation/ absorption parameter increases the velocity profile, whereas decreases the nanoparticle concentration profile. Figure 3 demonstrates the variation of temperature profiles for different values of heat generation/absorption parameter. The results depict that increase of heat generation/absorption parameter increases the value of temperature profile.

It is worth mentioning that by integrating Eq. (14), the obtained equation is clearly shown a direct dependency between the velocity, temperature and nanoparticle concentration.

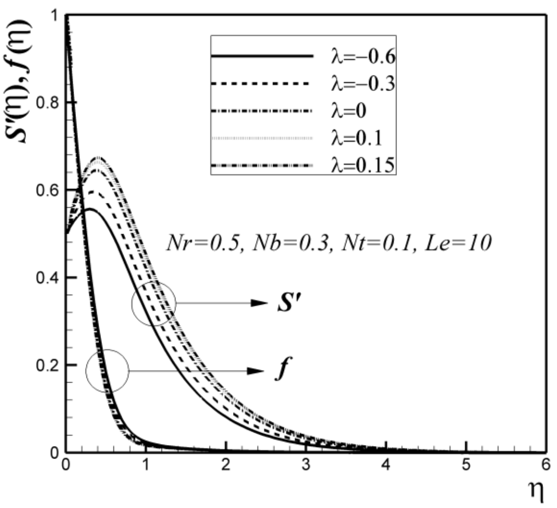


Figure 2: Velocity and nanoparticle concentration profiles for various values of λ

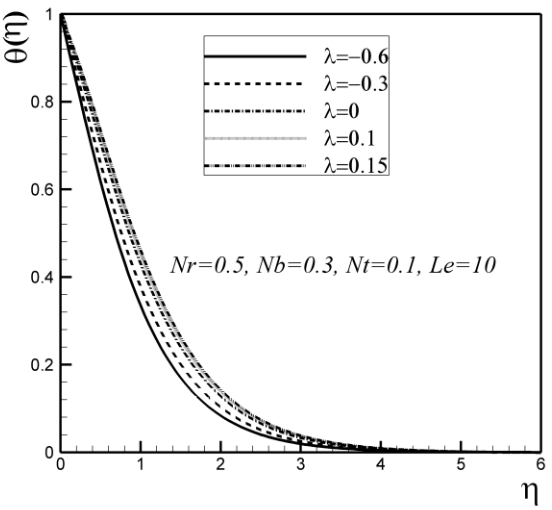


Figure 3: Temperature profiles for various values of λ

Figures 4 and 5 are plotted for different values of λ (i.e. $-0.3<\lambda<0.1$) and fixed values of *Nr*, *Nb*, *Nt* and *Le* to display obviously the effect of heat generation parameter (λ) on the reduced Nusselt number and the reduced Sherwood number. Figures 4 and 5 reveal that increase of heat generation/absorption parameter decreases the magnitude of reduced Nusselt number, whereas it increases the magnitude of reduced Sherwood number. Increase of heat generation parameter decreases the magnitude of temperature gradient on the wall surface, which results in the decrease of reduced Nusselt number. Also, increase of heat generation/absorption parameter increases the magnitude of concentration gradient on the wall and decreases the concentration boundary layer thickness, which these results in the increase of reduced Sherwood number.

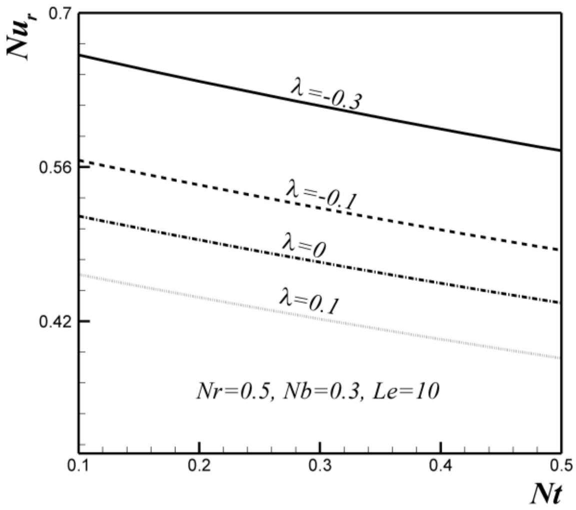


Figure 4: Variation of Nu_r as a function of *Nt* and λ

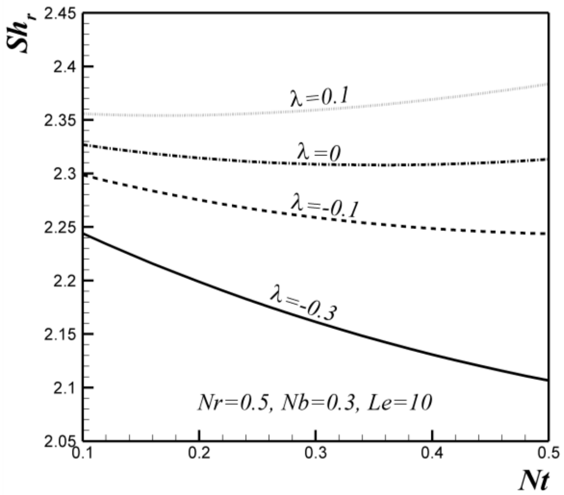


Figure 5: Variation of Sh_r as a function of *Nt* and λ

Conclusion

In this paper, a combined similarity and numerical approach have been used to theoretically investigate the natural convection from a vertical cone which is embedded in a nanofluid-saturated porous medium in the presence of internal heat generation/absorption. In the modeling of nanofluid, thermophoresis and Brownian motion effects have been taken into account. The results show that increase of heat generation/absorption parameter decreases the reduced Nusselt number whereas increases the reduced Sherwood number. The increase of thermophoresis parameter decreases the reduced Nusselt and Sherwood numbers.

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