THE ONSET OF CONVECTION IN THE UNSTEADY THERMAL BOUNDARY LAYER ABOVE A SINUOIDALLY HEATED SURFACE EMBEDDED IN A POROUS MEDIUM.

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ABSTRACT

In this paper we study the linear stability of a thermal boundary layer in a semi-infinite porous medium. This unsteady boundary layer is induced by varying the temperature of the horizontal boundary sinusoidally in time about the ambient temperature of the porous medium. Thus instability, if it occurs, will happen in those regions where cold fluid lies above hot fluid, and this is not necessarily a region which includes the bounding surface.

We perform a linearized stability analysis. Squire's theorem holds, meaning that two-dimensional disturbances may be considered. Monochromatic cells of wavenumber, k, are considered and a parabolic system describing the time-evolution of small-amplitude disturbances has been obtained. These equations have been solved in two ways to find the critical Darcy-Rayleigh number as a function of the wavenumber. The first method involves solving the parabolic governing equations directly using the Keller box method, iterating on the Darcy-Rayleigh number until no mean growth occurs. The second method involves using a Floquet theory to determine how the neutral stability curve changes with the imposed period of the disturbance. A Fourier decomposition of the governing equations yields a large system of ordinary differential equations of eigenvalue form, where the Darcy-Rayleigh number is the eigenvalue. These equations were solved using a shooting method coupled with a fourth order Runge-Kutta scheme.

We find that the most dangerous disturbance has a period which is twice that of the underlying basic state. This is the case which was found when using the Keller-box method. Cells which rotate clockwise tend to rise upwards from the surface and weaken, but they then induce an anticlockwise cell near the surface at the end of one forcing period which is otherwise identical to the clockwise cell found at the start of that forcing period.