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Laminar flows of non-Newtonian fluids driven by power-law shear over a porous stretching flat sheet

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Section: Novel numerical and analytical for the solution of porous media equation in multidimensional and complex geometry

Abstract *The boundary-layer flows of non-Newtonian power-law fluids adjacent to a stretching plane surface driven by an outer power-law shear in the presence of suction or injection are investigated. The boundary-layer equations are reduced to an ordinary differential equation with algebraical boundary condition at far field. A number of solutions with algebraical decaying behaviour are captured numerically. It is found that such solutions are possible if and only if* $f_w \geq f_w^{min}$ *for properly given values of* α *and* β*. We further notice that for properly prescribed values of* f_w *and* α*, solutions can always be found in the range* $0 \leq \beta \leq \beta_{max}$*, where* β_{max} *corresponds to* $\lim_{\eta \to 0} f''(\eta) = 0$*. Besides, it is found that in the range* $-1/2 < \alpha < 0$*, both the suction and the injection solutions could be available. While when* $-1 < \alpha < -1/2$*, only the solutions of suction type are possible. Furthermore, it is found that no solution is possible when the wall stretching is applied to the porous wall in the case of* $\alpha = -1$*.*

Summary

Here we consider the boundary layer flows of non-Newtonian power-law fluids driven over a stretching permeable flat surface by a outer power shear, defined by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\left(\frac{\partial \tau_{xy}}{\partial y}\right), \tag{2}$$

where x, y are Cartesian coordinates with the x-axis extending along the length of the wall and the y-axis in the wall-normal direction, u and v are the velocity components in the x- and y-directions, ρ and τ_{xy} are the density and shear stress, respectively. The shear tensor is defined by the Ostwald-de Wäle model

$$\tau_{ij} = 2K(2D_{kl}D_{kl})^{(\kappa-1)/2}D_{ij}, \tag{3}$$

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where

$$D_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{4}$$

denotes the rate of stretching tensor, K is the consistency coefficient and κ is the power-law index. The appropriate boundary conditions are

$$u = C \cdot x^c, \quad v = D \cdot x^d \quad \text{at} \quad y = 0, \quad u \to \hat{\beta} y^\alpha \quad as \quad y \to \infty, \tag{5}$$

where C, D, c, d, α and $\hat{\beta}$ are constants. The shear tensor used here is given by

$$\tau_{xy} = -K\left(-\frac{\partial u}{\partial y}\right)^\kappa, \tag{6}$$

where the shear rate $\partial u/\partial y$ is assumed to be nonpositive in the whole boundary layer since the velocity component u decreases monotonically as y enlarges from the stretching surface to the edge of boundary layer. It is worth mentioning that the flows to be discussed are strictly the zero pressure gradient flows. These are different in structure from those given by Andersson and Bech [10], who considered non-Newtonian power-layer flows induced by a stretching wall with no external power shear being applied.

In search of similarity solutions, we define a function $f(\eta)$ and its variable η as follows:

$$\psi(x, y) = A \cdot x^{a+b} \cdot f(\eta), \quad \eta = B \cdot y \cdot x^{-b}, \tag{7}$$

where $\psi(x, y)$ is the stream function satisfying $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, A, B, a and b are constants to be determined. From Eq.(7), we obtain

$$u = \frac{\partial\psi}{\partial y} = A \cdot B \cdot x^{a-b} f'(\eta), \tag{8}$$

$$v = -\frac{\partial\psi}{\partial x} = ABb\, y\, x^{a-b-1} f'(\eta) - A\, a x^{a-1} f(\eta). \tag{9}$$

Here a, b, c and d are given by

$$a = \frac{1+\alpha}{1+\kappa+\alpha(2-\kappa)}, \quad b = \frac{1}{1+\kappa+\alpha(2-\kappa)},$$
$$c = a - b, \quad d = a - 1. \tag{10}$$

And the constants A and B are determined as

$$A = \left(\frac{K}{\rho}\right)^{\frac{1}{\kappa+1}} C^{\frac{2\kappa-1}{\kappa+1}}, \quad B = \left(\frac{K}{\rho}\right)^{-\frac{1}{1+\kappa}} C^{\frac{2-\kappa}{\kappa+1}}. \tag{11}$$

We then obtain the reduced similarity equation as:

$$\kappa(-f'')^{\kappa-1} f''' + a f f'' - (a-b) f'^2 = 0, \tag{12}$$

subject to the boundary conditions

$$f(0) = f_w, \quad f'(0) = 1, \quad f'(\infty) \to \beta\eta^\alpha, \tag{13}$$

where $\beta = \hat{\beta}/(C \cdot B^\alpha)$ and $f_w = -D/(A \cdot a)$ is the suction/injection coefficient.

The shooting technique will be then used here to give the results. An infinity number of solutions with algebraical decaying behaviour are obtained numerically. We will further give an analysis on the existence of these solutions for suction/injection parameter f_w, as well as the exponents α and β. Besides, an investigation will be provided for the solutions' behaviour.

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