

Project code: PN-II-ID-PCE-2011-3-0024
Contract: 157/05.10.2011, financed by UEFISCDI
Project title: *The structure and sensitivity of the solution sets of variational inequalities, optimization and equilibrium problems under generalized monotonicity*
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SCIENTIFIC RESEARCH REPORT No. 1
covering the period of 05.10.2011 – 15.12.2011

I. Research Team

Prof. Dr. **Gabor KASSAY** (project leader)
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II. Project summary and objectives

(a) The project is mainly motivated by the growing literature in scalar and vector optimization problems, variational inequalities, and equilibrium problems, which neatly shows that these fields are appropriate for applying the modern tools of variational analysis. The following objectives have been proposed in the funding application:

- O1** *Studying condition numbers and metric regularity within parametric variational inequalities and parametric equilibrium problems*
- O2** *Identifying classes of generalized monotone operators for which local and global monotonicity are equivalent and deduce injectivity results*
- O3** *Studying the structure of the solution sets for generalized monotone operators*
- O4** *Characterizing the subdifferential for certain classes of generalized monotone operators*
- O5** *Approaching the sum problem for maximal monotone operators*
- O6** *Constructing algorithms for variational inequalities and equilibrium problems*
- O7** *Extending the proximal point algorithm for equilibrium problems to reflexive Banach spaces*
- O8** *Characterizing generalized convex vector functions by scalarization*
- O9** *Studying the structure of the solution sets of vector variational inequalities and equilibrium problems*

(b) All objectives planned for the period 05.10.2011 – 15.12.2011 have been achieved as follows:

O1: 1 paper in preparation*

O3: 1 paper in preparation*

O5: 1 published paper [A1 in Section III.(a)]

O8: 1 paper in preparation*

*see Section III.(b)

III. Scientific results

(a) Published/submitted papers

1 paper has been published in an ISI journal during the period 05.10.2011 – 15.12.2011.

Ref.	Article	Objective
A1	Szilard Laszlo and Boglarka Burjan-Mosoni: <i>About the maximal monotonicity of the generalized sum of two maximal monotone operators</i> , Set-Valued Var. Anal. , 20 (2012), 355-368, doi:10.1007/s11228-011-0202-z [JCR Science Edition 2010 IF: 0.333, JCR Science Edition 2013 IF: 0.918]	O5

In the paper **A1** [*About the maximal monotonicity of the generalized sum of two maximal monotone operators*, **Set-Valued Var. Anal.**, 20 (2012), 355-368] **Sz. Laszlo** and **B. Burjan-Mosoni** have obtained a closedness type condition, which ensures the maximal monotonicity of the generalized sum $S + A * TA$, where S and T are strongly representable operators. This condition is less restrictive than the interior type conditions known in the literature. The authors' technique is based on Fenchel conjugates and stable, strong duality. The operators considered are strongly representable monotone operators in arbitrary Banach spaces. Since the subdifferential of a convex, proper and lower semicontinuous function is a strongly representable operator in arbitrary Banach spaces, the results of the authors cover all the results of this kind involving subdifferentials of convex functions. Also, any maximal monotone operator defined in a reflexive Banach space is strongly representable, hence the results of authors cover all the results of this kind known in reflexive Banach spaces. As particular cases, beside the new results are obtained some known ones. First, is settled in what circumstances the Fenchel conjugate of a generalized infimal convolution is exact. This provides conditions that ensure the maximal monotonicity of the sum of two monotone operators. Interior and closedness type conditions are formulated in parallel, that ensure that the conjugate of the generalized infimal convolution of the representative functions of the maximal monotone operators S and T be a strong representative function of the generalized sum.

(b) Work in progress

Preliminary results have been obtained in the period of 05.10.2011 – 15.12.2011, namely:

- **G. Kassay** with M. Bianchi (University “Cattolica del Sacro Cuore” of Milan, Italy) and R. Pini (University Bicocca of Milan, Italy) studied an inverse function result and obtained some applications to sensitivity of generalized equations (within objective **O1**);
- **G. Kassay, C. Pinte**a and **Sz. Laszlo** studied the global injectivity of certain functions defined on open convex domains (within objective **O3**);
- **N. Popovici** studied the characterization of explicitly C-quasiconvex functions defined by means of a convex cone C whose interior may be empty (within objective **O8**).

Project leader,
Prof. Dr. Gabor Kassay