On semilinear integro-differential inclusions in Banach spaces under nonlocal conditions

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Abstract

In this paper we deal with a nonlocal Cauchy problem governed by the following semilinear integro-differential inclusion:

$$\begin{cases} x'(t) \in A(t)x(t) + F(t, x(t), Kx(t)) \text{ a.e. } \ t \in [0, b] \\ x(0) = g(x) \end{cases}$$

On the linear part of the inclusion we assume that: $\{A(t) : \ t \in [0, b]\}$ is a family of linear operators (not necessarily bounded), $A(t) : D(A) \subset E \to E, \ t \in [0, b], \ D(A)$ is a dense subset of the Banach space E not depending on t and $T : \Delta = \{(t, s) : \ 0 \leq s \leq t \leq b\} \to \mathcal{L}(E)$ is a continuous evolution operator generated by this family.

Given a continuous function $k : \Delta \to \mathbb{R}^+$, we consider the Volterra-type integral operator $K : C([0, b], E) \to C([0, b], E)$ where

$$Kx(t) = \int_0^t k(t, s)x(s)ds, \ \forall \ t \in [0, b], \ x \in C([0, b], E).$$

We prove the existence of *mild solutions* of the problem.