Is the Minty variational problem the dual of an optimization problem?

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Abstract

Let E be a normed vector-space, E^* its topological dual, $K \subset E$ a nonempty convex set and $T : E \to E^*$. We say that $\bar{u} \in K$ is a solution of the variational inequality guverned by T if it satisfies

$$\langle T(\bar{u}), v - \bar{u} \rangle \geq 0, \ \forall v \in K. \tag{VI}$$

We make the assumption that T is pseudomonotone, which means that for any $u, v \in E$, we have

$$\langle T(u), v - u \rangle \geq 0 \Rightarrow \langle T(v), v - u \rangle \geq 0.$$

Furthermore, we assume T is hemicontinuous, which means that

$$t \mapsto \langle T((1-t)u + tv), w \rangle, \ t \in [0, 1]$$

is continuous at 0 for every $u, v, w \in E$. It is well known that, if T is pseudomonotone and hemicontinuous, then (VI) is equivalent to the following problem, termed the dual (also called the Minty) variational inequality: find $\bar{u} \in K$ such that

$$\langle T(v), \bar{u} - v \rangle \leq 0, \ \forall v \in K. \tag{DVI}$$

Our aim is to provide a general duality theory which justifies the term (DVI) for this problem, and at the same time links (VI)-(DVI) to the classical concept of duality in vector optimization.