Appendix 3A – Curriculum for the written test in Mathematics

NOTE. The curriculum coincides with the current one for the baccalaureate exam, with the exception of contents related to Financial Mathematics, in tenth grade.

(According to OM 6.156 from 31.08.2023 and OM nr. 4.430 from 29.08.2014)

NINTH GRADE

Sets and elements of mathematical logic

- The set of real numbers: algebraic operations with real numbers, ordering of real numbers, the absolute value of a real number, under- and over-estimates, integer part, fractional part of a real number; operations with intervals of real numbers.
- Proposition, predicate, quantifiers.
- Elementary logical operations (negation, conjunction, disjunction, implication, equivalence) correlated to set operations and relations (complementarity, intersection, union, inclusion, equality); proof by contradiction.
- Mathematical induction.

Sequences

- Ways of defining a sequence, bounded sequences, monotone sequences.
- Special sequences: arithmetic and geometric progressions, the general term formula given a particular term and the ratio, the sum of the first *n* terms of a progression.
- The condition that *n* numbers form an arithmetic or geometric progression, for $n \geq 3$

Functions; reading graphs

- Cartesian coordinate system, cartesian product; graphical representation of a cartesian product of numerical sets; algebraic conditions for points in quadrants; lines in plane of the form $x = m$ or $y = m$, with $m \in \mathbb{R}$.
- Functions: definitions, examples, examples of correspondences that are not functions, ways of describing a function, reading graphs. Equality of two functions, the image of a set through a function, the graph of a function, restrictions of a function.
- Numerical functions $(F = \{f: D \to \mathbb{R}, D \subseteq \mathbb{R}\})$; geometric representation of the graph of a function: intersections with the coordinate axes, graphical solutions of equations and inequalities of the type $f(x) = g(x), (\leq, <, \geq, >)$; properties of numerical functions introduced by reading graphs; boundedness, monotonicity; other properties; even/odd functions, symmetry about lines of the form $x = m$, $m \in \mathbb{R}$, periodicity
- Composition of functions; examples with numerical functions.

Linear functions

- Definition; graphical representation of the function $f: \mathbb{R} \to \mathbb{R}, f(x) = ax + b$, with $a, b \in \mathbb{R}$, the intersection of the graph with the coordinate axes, the equation $f(x) = 0$.
- Graphical interpretation of the algebraic properties of a function: monotonicity and sign; the study of monotonicity by the sign of the difference $f(x_1) - f(x_2)$ (or by the study of the ratio $\frac{f(x_1) - f(x_2)}{x_1 - x_2}$, $x_1, x_2 \in \mathbb{R}, x_1 \neq x_2$).
- Inequalities of the form $ax + b \leq 0$ $(<, >, \geq)$ on $\mathbb{R}$ or on intervals of real numbers.

- Relative position of two lines, systems of equations of the type $\begin{cases} ax + by = c \\ mx + ny = p \end{cases}, a, b, c, m, n, p$ real numbers.
- Systems of linear inequalities.

Quadratic functions

- Graphical representation of the function $f: \mathbb{R} \to \mathbb{R}, f(x) = ax^2 + bx + c$, with $a, b, c \in \mathbb{R}$ and $a \neq 0$, intersection of the graph with the coordinate axes, the equation $f(x) = 0$, symmetry about lines of the type $x = m$, with $m \in \mathbb{R}$.
- Viète's relations, solving systems of the form $\begin{cases} x + y = s \\ xy = p \end{cases}$, with $s, p \in \mathbb{R}$.

Geometrical interpretation of algebraic properties of quadratic functions

- Monotonicity; study of monotonicity by the sign of the difference $f(x_1) - f(x_2)$ or by the rate of increase/decrease $\frac{f(x_1) - f(x_2)}{x_1 - x_2}$, $x_1, x_2 \in \mathbb{R}, x_1 \neq x_2$, point of extremum, vertex of the parabola
- Position of the parabola with respect to the *Ox* axis, the sign of the function, inequalities of the type $ax^2 + bx + c \leq 0$ $(\geq, <, >)$, $a, b, c \in \mathbb{R}, a \neq 0$, analyzed on $\mathbb{R}$ or on intervals of real numbers, geometrical interpretation: images of intervals (projections of parts of the parabola onto the *Oy* axis)
- Relative position of a line with respect to a parabola: solving systems of the form $\begin{cases} mx + n = y \\ ax^2 + bx + c = y \end{cases}, a, b, c, m, n \in \mathbb{R}$.

Vectors in plane

- Directed line segment, vectors, collinear vectors.
- Operations with vectors: addition (the triangle rule, the parallelogram rule), properties of vector addition; multiplication by a scalar, properties of scalar multiplication; condition of collinearity, decomposition into two non-collinear vectors.

Collinearity, concurrence, parallelism – vector calculus in plane geometry

- Position vector of a point.
- Position vector of the point that divides a line segment into a given ratio, Thales' theorem (conditions of parallelism).
- Position vector of the center of gravity of a triangle (concurrence of the medians of a triangle).
- Theorem of Menelaus, theorem of Ceva.

Trigonometry topics

- Trigonometric circle, definition of trigonometric functions $\sin: [0, 2\pi] \to [-1, 1]$, $\cos: [0, 2\pi] \to [-1, 1]$, $\text{tg}: [0, \pi] \setminus \left\{\frac{\pi}{2}\right\} \to \mathbb{R}$, $\text{ctg}: (0, \pi) \to \mathbb{R}$.
- Definition of trigonometric functions $\sin: \mathbb{R} \to [-1, 1]$, $\cos: \mathbb{R} \to [-1, 1]$, $\text{tg}: \mathbb{R} \setminus D \to \mathbb{R}$, with $D = \left\{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\right\}$, $\text{ctg}: \mathbb{R} \setminus D \to \mathbb{R}$, with $D = \{k\pi \mid k \in \mathbb{Z}\}$.
- Reduction to the first quadrant; trigonometric formulas: $\sin(a + b)$, $\sin(a - b)$, $\cos(a + b)$, $\cos(a - b)$, $\sin 2a$, $\cos 2a$, $\sin a + \sin b$, $\sin a - \sin b$, $\cos a + \cos b$, $\cos a - \cos b$ (sum-to-product trigonometric identities).

Applications of trigonometry and of the scalar product of two vectors to plane geometry

- Scalar product of two vectors: definition, properties. Applications: law of cosines, conditions of perpendicularity, solving a right triangle.
- Vector and trigonometric applications to geometry: law of sines, solving an arbitrary triangle.
- Computation of the radius of the inscribed and of the circumscribed circle of a triangle, computation of the lengths of important lines in a triangle, computation of some areas.

TENTH GRADE

Number sets

- Real numbers: properties of rational, irrational and real powers of a nonzero positive number, rational approximations of real numbers.
- Radicals of order n ($n \in \mathbb{N}$ and $n \geq 2$) of a number, properties of radicals.
- Logarithms, properties of logarithms, computations involving logarithms, applications of logarithms.
- The set $\mathbb{C}$. Complex numbers in algebraic form, the conjugate of a complex number, operations with complex numbers. Geometric interpretation of addition, subtraction and multiplication by a real number of complex numbers.
- Solving in $\mathbb{C}$ quadratic equations with real coefficients. Biquadratic equations.

Functions and equations

- The power function with natural exponent: $f: \mathbb{R} \to D, f(x) = x^n, n \in \mathbb{N}, n \geq 2$ and the radical function: $f: \mathbb{R} \to D, f(x) = \sqrt[n]{x}, n \in \mathbb{N}, n \geq 2$, where $D = [0, +\infty)$ for even n and $D = \mathbb{R}$ for odd n
- The exponential function $f: \mathbb{R} \to (0, +\infty), f(x) = a^x, a \in (0, +\infty), a \neq 1$ and the logarithmic function $f: \mathbb{R} \to (0, +\infty), f(x) = \log_a x, a \in (0, +\infty), a \neq 1$.
- Injectivity, surjectivity, bijectivity; invertible functions: definition, graphical properties, necessary and sufficient conditions for a function to be invertible.
- Trigonometric and inverse trigonometric functions.
- Solving equations using properties of functions:
  - Equations containing radicals of order 2 or 3.
  - Exponential and logarithmic equations.
  - Trigonometric equations: $\sin x = a, \cos x = a, a \in [-1, 1], \operatorname{tg} x = a, \operatorname{ctg} x = a, a \in \mathbb{R}$, $\sin f(x) = \sin g(x)$, $\cos f(x) = \cos g(x), \operatorname{tg} f(x) = \operatorname{tg} g(x)$, $\operatorname{ctg} f(x) = \operatorname{ctg} g(x)$. (Note: here tg=tan, and ctg=cot.)

Note: For all types of functions, the following will be studied: intersection with the coordinate axes, the equation $f(x) = 0$, graphical representation by plotting points, symmetry, reading graphically the algebraic properties of functions: monotonicity, bijectivity, invertibility, sign, convexity.

Counting

- Finite ordered sets. Number of functions.
- Permutations: the number of ordered sets obtained when ordering a finite set with n elements.
  the number of bijective functions $f: A \to B$, where A and B are finite sets.
- Arrangements
  - the number of ordered sets of k elements, $k \leq n$, that can be obtained from the n elements of a finite set.
  - the number of injective functions $f: A \to B$, where A and B are finite sets.
  - Combinations – the number of subsets of k elements, with $0 \leq k \leq n$, of a finite set with n elements. Properties: formula for complementary combinations, the number of subsets of a set with

n elements.
- Newton's binomial theorem.

Geometry

- Cartesian coordinate system, coordinates of a vector in plane, coordinates of the vector sum, coordinates of the product of a vector and a real number, cartesian coordinates of a point in plane, distance between two points in plane
- Equations of a line in plane, given a point and a direction, and given two distinct points
- Conditions of parallelism, conditions of perpendicularity of two lines in plane; computation of distances and areas

ELEVENTH GRADE

MATRIX CALCULUS AND SYSTEMS OF LINEAR EQUATIONS TOPICS

Permutations

- The concept of permutation, operations, properties.
- Inversions, the sign of an inversion.

Matrices

- Matrix tables. Matrices, sets of matrices.
- Operations with matrices: addition, subtraction, multiplication by a scalar, properties.

Determinants

- Determinant of order n, properties

Systems of linear equations

- Invertible matrices in $M_n(\mathbb{C}), n \leq 4$.
- Matrix equations.
- Linear systems of at most 4 unknowns, Cramer's Rule, rank of a matrix.
- Study of compatibility and solvability of systems: Kronecker-Capelli property, Rouché property, Gauss method.
- Applications: equation of a line determined by two distinct points, area of a triangle and collinearity of three points in plane.

MATHEMATICAL ANALYSIS TOPICS

Limits of functions

- Elementary notions of sets of points on the real line: intervals, boundedness, neighborhoods of a point, line segments, the $+\infty$ and $-\infty$ symbols.
- Real functions of a real variable: polynomial function, rational function, power function, radical function, logarithmic function, exponential function, trigonometric and inverse trigonometric functions.
- The limit of a sequence using neighborhoods, convergent sequences.
- Monotonicity, boundedness, limits; Weierstrass property. Significant examples: $(a^n)_{n \geq 0}$, $(n^a)_{n \geq 0}$, $\left(\left(1 + \frac{1}{n}\right)^n\right)_{n \geq 1}$ (without proof), the number e; the limit of the sequence $(1 + u_n)^{\frac{1}{u_n}}$, $u_n \to 0, u_n \neq 0$, for any natural number n.
- Operations with sequences that have limits.
- Limits of functions: graphical interpretation of the limit of a function at a point, using neighborhoods, one-sided limits.

- Computation of limits for the functions studied; exceptions to computations of function limits: $\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0 \cdot \infty, 1^{\infty}, \infty^{0}, 0^{0}$.
- Asymptotes to the graphs of the functions studied: vertical asymptotes, oblique asymptotes.

Continuity
- Continuity of a function at a point in the domain of definition, continuous functions, graphical interpretation of the continuity of a function, the study of continuity at points on the real line for the functions studied, operations with continuous functions.
- Darboux property, the sign of a continuous function on an interval of real numbers, existence of solutions in $\mathbb{R}$ of some equations.

Differentiability
- Tangent line to a curve, derivative of a function at a point, differentiable functions, operations with differentiable functions, computation of first and second order derivatives of the functions studied.
- Differentiable functions on an interval: points of extremum of a function, Fermat's theorem, Rolle's theorem, Lagrange's theorem and their geometric interpretation, the corollary of Lagrange's theorem referring to the derivative of a function at a point.
- The role of the first derivative in the study of functions: monotonicity of functions, points of extremum.
- The role of the second derivative in the study of functions: concavity, convexity, inflection points.

Graphical representation of functions
- Graphical representation of functions.
- Solving equations graphically, using graphs of functions to determine the number of solutions of some equations.
- Graphical representation of conics (circle, ellipse, hyperbola, parabola).
- L'Hospital's rules.

TWELVETH GRADE

ALGEBRA TOPICS
Groups
- Laws of composition (algebraic operations), table of operation, closed subset.
- Groups, examples: numerical groups, matrix groups, permutation groups, the additive group of integers modulo n.
- Subgroups.
- Finite groups, table of operation, order of an element.
- Group homomorphisms and isomorphisms.

Rings and fields
- Rings, examples: numerical rings $(\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C})$, $\mathbb{Z}_n$, matrix rings, real function rings.
- Division rings, fields, examples: numerical fields $(\mathbb{Q}, \mathbb{R}, \mathbb{C})$, $\mathbb{Z}_p, p$ prime.
- Homomorphisms of rings and of fields.

Rings of polynomials with coefficients in a field $(\mathbb{Q}, \mathbb{R}, \mathbb{C}, \mathbb{Z}_p, p$ prime)
- Algebraic form of a polynomial, polynomial function, operations (addition, multiplication, scalar multiplication).
- Division algorithm; polynomial division, division by $X - a$, Horner's scheme.
- Divisibility of polynomials, Bézout's theorem; g.c.d. and l.c.m. of polynomials, polynomial decomposition

into irreducible factors.
- Roots of polynomials, Viète's relations.
- Solving algebraic equations with coefficients in $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$, finding complex solutions of the equation $z^n = a, a \in \mathbb{C}, n \in \mathbb{N}^*$, biquadratic equations, reciprocal polynomial equations.

MATHEMATICAL ANALYSIS TOPICS

- Problems leading to the concept of integral.

Primitives (antiderivatives)

- Primitives of a function defined on an interval. Indefinite integral of a function, properties of indefinite integrals, linearity. Common primitives

Definite integrals

- Partitions of an interval $[a, b]$, the norm of a partition, intermediary points, Riemann sums, geometric interpretation. Definition of the integrability of a function on an interval $[a, b]$.
- Properties of the definite integral: linearity, monotonicity, additivity with respect to the interval of integration
- Leibniz-Newton formula.
- Integrability of continuous functions, mean value theorem, geometric interpretation, existence of primitives for continuous functions.
- Computational methods for definite integrals: integration by parts, integration by substitution. Computing integrals of the form $\int_a^b \frac{P(x)}{Q(x)} dx, \deg Q(x) \leq 4$, by the method of partial fraction decomposition.

Applications of definite integrals

- Area of a plane surface.
- Volume of a rotational body.
- Computation of some limits of sequences using the definite integral.

Note: The terms "property" or "rule" are used to emphasize the fact that they refer to a mathematical result used in applications, whose proof is not included in the curricula.

Bibliography

School textbooks and auxiliary materials approved by the Ministry of National Education.