Degree bounds for generators of invariant rings

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Let $G \subset \operatorname{GL}(V)$ be a finite group where V is a finite dimensional vector space over a field \mathbb{F} . The action of G on V extends naturally to an action on the coordinate ring $\mathbb{F}[V]$. The invariant ring $\mathbb{F}[V]^G$ is the subring of $\mathbb{F}[V]$ consisting of those polynomials which are fixed under this action of G. We know from a classic result of Hilbert that this invariant ring is finitely generated. However the task of actually determining a minimal set of generators is computationally very difficult, and until now in most cases is practically unfeasible. This motivates the attempt to find an a priori bound on the maximal degree of these generators, which is denoted by $\beta(G)$. Already Emmy Noether showed that $\beta(G) \leq |G|$. Later it turned out that this bound is sharp only for cyclic groups and it can be substantially smaller in general.

In our talk we will present some new results about these degree bounds obtained in joint work with Mátyás Domokos and István Szöllősi.