

Admission exam - model
Written Exam for Computer Science

Subject A (30 points)

1. (5p) An integer data type represented using x bits (x is a strictly positive natural number) can hold integer values within the following interval:

- a. $[0, 2^x]$
- b. $[0, 2^{x-1}-1]$
- c. $[-2^{x-1}, 2^{x-1}-1]$
- d. $[-2^x, 2^x-1]$
- e. $[0, 10^x]$

2. (5p) Given the following subalgorithm

```
Subalgorithm f(a, b):
    If a > 1 then
        return b * f(a - 1, b)
    else
        return b * f(a + 1, b)
    EndIf
EndSubalgorithm
```

State how many times function f will be called within the following code snippet:

$x \leftarrow 4$
 $y \leftarrow 3$
 $z \leftarrow f(x, y)$

- a. 4 times
- b. 3 times
- c. infinite number of times
- d. never
- e. once

3. (5p) Let x be an integer variable that holds the smallest non-zero natural number that is a multiple of 36 and that is divisible with all prime numbers smaller than 10. Which of the following expressions holds true:

- a. $(x < 1000) \text{ and } ((x*x*x) \bmod 1000 = 0)$
- b. $(x \bmod 100 = 0) \text{ or } (x \text{ div } 100 = 0)$
- c. $(x > 1000) \&& (x \bmod 7 = 0)$
- d. $((x*x) \text{ div } 16) \bmod 2 = 1$
- e. $((x*x) \text{ div } 16) \bmod 2 = 0$

4. (5p) Consider all arrays having length $l \in \{1, 2, 3\}$ built using letters from the set $\{a, b, c, d, e\}$. How many of them are sorted strictly decreasing and have an odd number of vowels? (a and e are vowels)

- a. 14
- b. 7
- c. 81
- d. 78
- e. 0

5. (5p) Given the following code snippet

```
void positiveNumbers(int m,int a[],  
int &n, int b[]){  
    n = 0;  
    for(int i = 0; i < n; i++){  
        if (a[i] > 0){  
            n = n + 1;  
            b[n] = a[i];  
        }  
    }  
}  
  
procedure positiveNumbers (m:integer;  
a:array; var n:integer; var b:array)  
    Begin  
        n := 0;  
        for i := 1 to n do  
            if (a[i] > 0) then  
                begin  
                    n := n + 1;  
                    b[n] := a[i];  
                end;  
    End;
```

What is the result of calling $\text{positiveNumbers}(k, x, l, y)$ for $k=4$, array $x=(-1, 2, -3, 4)$, $l = -1$ and the empty array $y = ()$.

- a. $l = 3$ and $y=[2, 4]$;
- b. $l = 0$ and $y=[2, 4]$;
- c. $l = 0$ and $y=[]$;
- d. Depends on the value of k
- e. Compile-time error

6. (5p) Given the following subalgorithm:

```
Subalgorithm SA6(a):  
    If a < 50 then  
        If a mod 3 = 0 then  
            return SA6(2 * a - 3)  
        else  
            return SA6(2 * a - 1)  
        EndIf  
    else  
        return a  
    EndIf  
EndSubalgorithm
```

For which value of parameter a will the subalgorithm return 61?

- a. 16
- b. 61
- c. 4
- d. 31
- e. 51

Subject B (60 points)

1. Chocolate Tasting (25 points)

A marketing company is advertising a new brand of chocolate and intends to distribute chocolate samples to n ($10 \leq n \leq 10\,000\,000$) children who are standing in a circle. The employees know that distributing samples to each child would cost too much money. Therefore, they decide to distribute a sample to each k -th ($0 < k < n$) child. They count the children standing in the circle, and hand out a sample to each k -th child. When the count reaches the last child, it continues with the first one, and so on. All children are counted, even if they have already received chocolate. Counting stops when a chocolate sample should be given to a child who already received one.

Write a subalgorithm that determines the number of children (nr) who will not receive a chocolate sample. Input parameters are natural numbers n and k , and the output parameter is the natural number nr .

Example 1: if $n = 12$ and $k = 9$, then $nr = 8$ (the first, second, fourth, fifth, seventh, eighth, tenth, and eleventh child do not receive a sample).

Example 2: if $n = 15$ and $k = 7$, then $nr = 0$ (each child receives a chocolate sample).

2. Magic numbers (15 points)

Let us consider two natural numbers p and q ($2 \leq p \leq 10$, $2 \leq q \leq 10$). A natural number is *magic* when the set of digits used for writing its representation in base p is identical to the set of digits used for writing its representation in base q . For example, for $p = 9$ and $q = 7$, $(31)_{10}$ is a *magic* number because $(34)_9 = (43)_7$, and for $p = 3$ and $q = 9$, $(9)_{10}$ is a *magic* number because $(100)_3 = (10)_9$.

Write a subalgorithm that for given bases p and q determines the array x containing all *magic* numbers that are strictly positive and strictly smaller than a given natural number n ($1 < n \leq 10000$). The input parameters for the subalgorithm are p and q (the two bases) and the value n . The output parameters are the array x and its length k . **Example:** if $p = 9$, $q = 7$ and $n = 500$, array x will contain $k = 11$ elements: $(1, 2, 3, 4, 5, 6, 31, 99, 198, 248, 297)$.

3. Search (10 points)

Given the following subalgorithm:

```
1: Subalgorithm search(x, n, val):
2:     If n = 0 then
3:         return (x[0] = val)
4:     else
5:         return search(x, n - 1, val)
6:     EndIf
7: EndSubalgorithm
```

What code must be added and where so that the function above determines whether element val is contained within the n element array x (with n a strictly positive natural number)?

4. Control digit (10 points)

Consider the following subalgorithm that determines the control digit of a natural number that has at least 2 digits.

```
1: Subalgorithm controlDigit(x):
2:     While x > 9 execute:
3:         s ← 0
4:         While x > 0 execute:
5:             s ← s + x MOD 10 { x mod 10 is the remainder of dividing x by 10}
6:             x ← x DIV 10 { x div 10 is the quotient of dividing x by 10}
7:         EndWhile
8:         x ← s
9:     EndWhile
10:    return x
11: EndSubalgorithm
```

Replace the body of this subalgorithm with maximum 2 instructions so that the subalgorithm has the same effect.

Note:

1. All subjects are mandatory.
2. Solutions must be written on the exam sheets in detailed form (drafts are not graded).
3. Default 10 points.
4. Working time is 3 hours.

Evaluation

Granted points	10 points
SUBJECT A	30 points
A. 1. Answers b,c	5 points
A. 2. Answer c	5 points
A. 3. Answers c,d	5 points
A. 4. Answer a	5 points
A. 5. Answer c	5 points
A. 6. Answers a, b, d	5 points
SUBJECT B	60 points
B. 1. Chocolate Tasting	25 points
• V1: correct determination of nr (by using $nr = n - n/gcd(n, k)$)	25 points
• V2: correct determination of nr (simulation, circular list).....	15 points
B. 2. Magic numbers.....	15 points
– property of <i>magic number</i>	
• V1: by using characteristic vectors associated to the sets of number digits corresponding to two representations (base p and base q , respectively)	10 points
• V2: other versions.....	maxim 5 points
– construction of vector x	5 points
B. 3. Search	10 points
– identification of condition($x[n] = val$)	5 points
– return the truth value of composed condition from row 5.....	5 points
B. 4. Control digit	10 points
– control digit can be computed as $nr \bmod 9$	10 points

Solving – Subject B.1.: Chocolate tasting.....25 points

```
#include <iostream>
using namespace std;
//****************************************************************************
Subiectul I.1. Degustare de ciocolata
*****
//greatest common divisor of 2 numbers a and b
int cmmdc(int a, int b){
    if ((a == b) && (a == 0))
        return 1;
    if (a * b == 0)
        return a + b;
    while (b != 0){
        int c = b;
        b = a % b;
        a = c;
    } //while
    return a;
}

int degustareCiocolata(int n, int k){
    return n - n / cmmdc(n, k);
}
```

Solving – Subject B.2.: Magic numbers.....15 points

```
#include <iostream>
using namespace std;

bool nrMagic(int x, int p, int q){
    int cifre[10] = { 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 };
    int copie = x;
    while (copie != 0){ //stabilim multimea cifrelor lui x in baza p
        int uc = copie % p; //uc - ultima cifra (in baza p)
        cifre[uc] = 1;
        copie = copie / p;
    }
    copie = x;
    while (copie != 0){ //determinam cifrele lui x in baza q
        int uc = copie % q;
        if (cifre[uc] == 0) //daca cifra curenta (in baza q) nu e folosita in baza p
            return false;
        cifre[uc]++;
        copie = copie / q;
    }
    for (int i = 0; i < 10; i++){
        if (cifre[i] == 1) //daca cifra i e folosita in baza p, dar nu e folosita in baza q
            return false;
    }
    return true;
}

void sirNrMagice(int p, int q, int n, int &k, int sir[]){
    k = 0;
    for (int i = 1; i < n; i++){
        if (nrMagic(i, p, q))
            sir[k++] = i;
    }
}
```

Solving – Subject B.3.: Search..... 10 points

Row 5 must be changed: return ((x[n] = val) and search(x, n - 1, val))

```
1: Subalgorithm search(x, n, val):
2:     If n = 0 then
3:         return x[0] = val
4:     else
5:         return ((x[n] = val) and search(x, n - 1, val))
6:     EndIf
7: EndSubalgorithm
```

Solving - Subject B. 4. Control digit..... 10 points

```
1: Subalgorithm controlDigit(x):
2:     Return x mod 9
3: EndSubalgorithm
```