# Şiruri

1. Consideram sirul $(a_n)_{n\in\mathbb{N}}$ definit prin

$$a_0 = 0,\ a_1 = 1,\ a_n = \frac{a_{n-1} + a_{n-2}}{2}, \quad \forall n \geq 2.$$

   a) Este sirul monoton?
   b) Dar convergent?
   c) Calculati $\lim\limits_{n\to\infty} a_n$

2. (*admitere 2021 - simultan 2 probleme*)

Pentru orice $n \in \mathbb{N}$ se noteaza

| | |
|---|---|
| $I_n = \displaystyle\int_0^1 \frac{x^{2n}}{1+x^2}\,\mathrm{d}x$ | $I_n = \displaystyle\int_1^{\mathrm{e}} (\ln x)^n \mathrm{d}x$ |

a) Aratati ca sirul $(I_n)_{n\in\mathbb{N}}$ verifica relatia de recurenta

| | |
|---|---|
| $I_{n+1} + I_n = \dfrac{1}{2n+1},\ \forall n \in \mathbb{N}$ | $I_{n+1} + (n+1)I_n = \mathrm{e},\ \forall n \in \mathbb{N}$ |

b) Justificati ca $\lim\limits_{n\to\infty} I_n = 0$

c) Calculati apoi $\lim\limits_{n\to\infty} nI_n$

d) Calculati si $\lim\limits_{n\to\infty} \dfrac{I_n}{I_{n+1}}$

3. Calculati urmatoarele limite

   a) $\lim\limits_{n\to\infty} \sqrt{n}\left(7^{\sqrt{n+1}-\sqrt{n}} - 1\right)$
   b) $\lim\limits_{n\to\infty} n\left(\sqrt[7]{\frac{n+1}{n}} - 1\right)$
   c) $\lim\limits_{n\to\infty} \frac{1\cdot3\cdot5\cdot\ldots\cdot(2n-1)}{2\cdot4\cdot6\cdot\ldots\cdot(2n)}$