

Geometrie Analitică

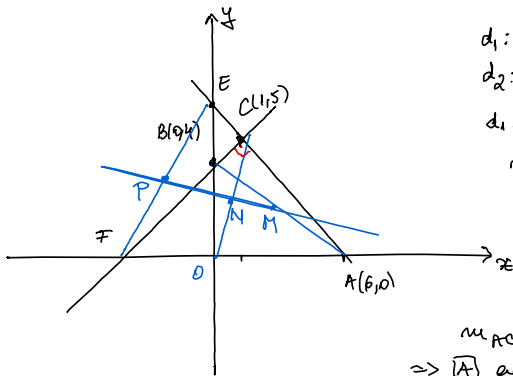
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1. Într-un reper cartezian ortonormat se dau punctele $A(6,0)$, $B(0,4)$, $C(1,5)$. Dreapta AC intersectează axa Oy în punctul E , iar dreapta BC intersectează axa Ox în punctul F . Fie M , N și P mijloacele segmentelor AB , OC și respectiv EF . Care dintre următoarele afirmații sunt adevărate?

- A $AC \perp BC$; B $P(3, -2)$;
 C $MN : x + 5y - 13 = 0$; D M, N, P sunt coliniare.



$$d_1: y = m_1 x + n_1 \quad m_1 \text{ panta dreptei } d_1$$

$$d_2: y = m_2 x + n_2 \quad m_2 \text{ panta dreptei } d_2$$

$$d_1 \perp d_2 \Leftrightarrow m_1 \cdot m_2 = -1$$

$$m_{AC} = \frac{y_C - y_A}{x_C - x_A} = \frac{y_A - y_C}{x_A - x_C}$$

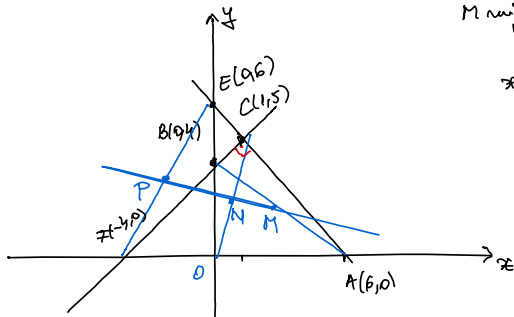
$$m_{AC} = \frac{5-0}{1-6} = \frac{5}{-5} = -1$$

$$m_{BC} = \frac{y_C - y_B}{x_C - x_B} = \frac{5-4}{1-0} = \frac{1}{1} = 1$$

$$m_{AC} \cdot m_{BC} = -1 \Rightarrow AC \perp BC$$

\Rightarrow A adevărată

B falsă



M mijlocul lui $[AB]$,

$$M(x_M, y_M),$$

$$x_M = \frac{x_A + x_B}{2}, \quad y_M = \frac{y_A + y_B}{2}$$

$$x_M = \frac{6 + 0}{2} = 3, \quad y_M = \frac{0 + 4}{2} = 2$$

$$M(3, 2)$$

N mijlocul lui $[OC]$

$$x_N = \frac{x_O + x_C}{2}, \quad y_N = \frac{y_O + y_C}{2}$$

$$x_N = \frac{0 + 1}{2} = \frac{1}{2}, \quad y_N = \frac{0 + 5}{2} = \frac{5}{2}$$

$$N\left(\frac{1}{2}, \frac{5}{2}\right)$$

$$AC: y - y_A = m_{AC}(x - x_A) \Rightarrow AC: y - 0 = -1(x - 6) \Rightarrow$$

$$\begin{cases} AC: y = -x + 6 \\ Oy: x = 0 \end{cases} \Rightarrow \begin{cases} y = 6 \\ x = 0 \end{cases} \Rightarrow CA \cap Oy = \{E\}, E(0, 6)$$

$$BC: \frac{y - y_B}{y_C - y_B} = \frac{x - x_B}{x_C - x_B} \Rightarrow BC: \frac{y - 4}{5 - 4} = \frac{x - 0}{1 - 0} \Rightarrow BC: y - 4 = x$$

$$\begin{cases} BC: y = x + 4 \\ Ox: y = 0 \end{cases} \Rightarrow \begin{cases} x = -4 \\ y = 0 \end{cases} \Rightarrow BC \cap Ox = \{F\}, F(-4, 0)$$

$$P \text{ mijlocul lui } [EF] \Rightarrow x_P = \frac{x_E + x_F}{2} = -2, \quad y_P = \frac{y_E + y_F}{2} = 3$$

$$MN: \begin{vmatrix} x & y & 1 \\ x_M & y_M & 1 \\ x_N & y_N & 1 \end{vmatrix} = 0$$

$$MN: \begin{vmatrix} x & y & 1 \\ 3 & 2 & 1 \\ \frac{1}{2} & \frac{5}{2} & 1 \end{vmatrix} = 0$$

$$MN: 2x + \frac{15}{2} + \frac{y}{2} - 1 - \frac{5}{2}x - 3y = 0$$

$$MN: -\frac{1}{2}x - \frac{5}{2}y + \frac{13}{2} = 0 \Rightarrow$$

$$MN: x + 5y - 13 = 0$$

adınvānātō

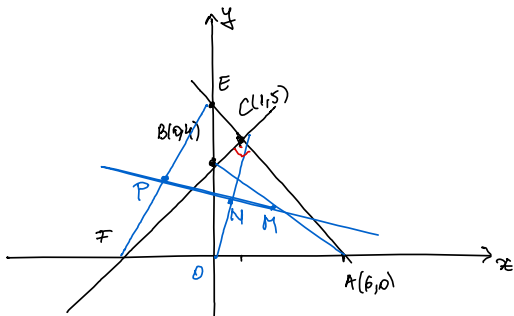
$$M, N, P \text{ coliniare} \Leftrightarrow P \in MN \Leftrightarrow x_P + 5y_P - 13 = 0 \Leftrightarrow$$

$$P(-2, 3) \quad -2 + 5 \cdot 3 - 13 = 0$$

$$13 - 13 = 0$$

adınvānātō

Rāspuns : A, C, D



$$M, N, P \text{ coliniare} \Leftrightarrow \begin{vmatrix} x_M & y_M & 1 \\ x_N & y_N & 1 \\ x_P & y_P & 1 \end{vmatrix} = 0$$

2. Se dă dreapta de ecuație $d : y = \frac{4}{3}x + 1$. Care dintre următoarele drepte d' este paralelă cu dreapta dată și se află la distanța 3 de aceasta?

A $d' : 3y = 4x + 18$;

B $d' : 3y = 4x + 12$;

C $d' : 3y = 4x - 12$;

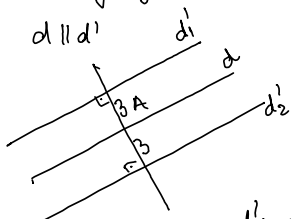
D $d' : 3y = 4x - 18$.

$d_1 : y = m_1x + m_1$

$d_2 : y = m_2x + m_2$

$d_1 \parallel d_2 \Leftrightarrow m_1 = m_2$

$d : y = \frac{4}{3}x + 1 \Rightarrow m_d = \frac{4}{3} \left. \vphantom{\frac{4}{3}} \right\} \Rightarrow m_{d'} = m_d = \frac{4}{3}$



$d' : y = \frac{4}{3}x + m \quad | \cdot 3$

$d' : 3y = 4x + 3m \Rightarrow d' : 4x - 3y + 3m = 0$

$\text{dist}(d', d) = \text{dist}(A, d')$, unde $A \in d$

$d' : ax + by + c = 0, A(x_A, y_A)$

$\text{dist}(A, d') = \frac{|ax_A + by_A + c|}{\sqrt{a^2 + b^2}}$

$A \in d \Rightarrow x = 0, y = 1 \Rightarrow A(0, 1)$

$$\text{dist}(A, d') = \frac{|4x_A - 3y_A + 3m|}{\sqrt{4^2 + (-3)^2}} = \frac{|4 \cdot 0 - 3 \cdot 1 + 3 \cdot m|}{\sqrt{16+9}} = \frac{|3m-3|}{\sqrt{25}} = \frac{3|m-1|}{5}$$

$$\text{dist}(A, d') = 3 \Rightarrow \frac{3|m-1|}{5} = 3 \Rightarrow |m-1| = 5 \begin{cases} m-1=5 \Rightarrow m=6 \\ m-1=-5 \Rightarrow m=-4 \end{cases}$$

$$d_1^1: 4x - 3y + 18 = 0$$

$$\Rightarrow d_1^1: 3y = 4x + 18$$

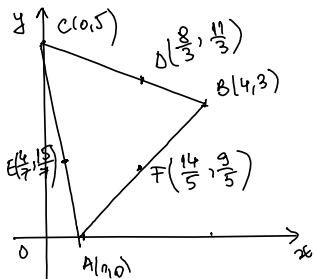
$$d_2^1: 4x - 3y - 12 = 0$$

$$\Rightarrow d_2^1: 3y = 4x - 12$$

Răspuns: \boxed{A} , \boxed{C}

3. Se dă un triunghi ABC cu vârfurile în punctele de coordonate $A(1,0)$, $B(4,3)$, $C(0,5)$ și punctele D, E, F situate pe laturile triunghiului, astfel încât $2\vec{BD} = \vec{DC}$, $3\vec{CE} = 4\vec{EA}$, $2\vec{AF} = 3\vec{FB}$. Care dintre următoarele afirmații sunt adevărate?

- A $AD: 11x - 5y - 11 = 0$; B $BE: x + 4y + 8 = 0$;
 C $CF: 8x - 7y - 35 = 0$; D AD, BE, CF sunt concurente.



$$2\vec{BD} = \vec{DC} \Leftrightarrow D \in [BC], \frac{BD}{DC} = \frac{1}{2}$$

$$\vec{BD} (x_D - x_B, y_D - y_B) \Rightarrow \vec{BD} (x_D - 4, y_D - 3)$$

$$\vec{DC} (x_C - x_D, y_C - y_D) \Rightarrow \vec{DC} (-x_D, 5 - y_D)$$

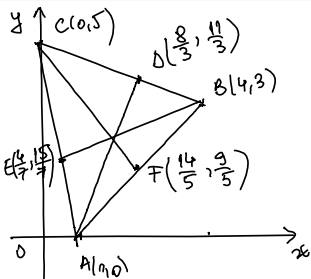
$$\begin{cases} 2(x_D - 4) = -x_D & \Rightarrow 2x_D + x_D = 8 \Rightarrow x_D = \frac{8}{3} \\ 2(y_D - 3) = 5 - y_D & \Rightarrow 2y_D + y_D = 11 \Rightarrow y_D = \frac{11}{3} \end{cases}$$

$$\frac{CE}{EA} = k = \frac{4}{3} \Rightarrow E(x_E, y_E)$$

$$x_E = \frac{x_C + kx_A}{1+k}, \quad y_E = \frac{y_C + ky_A}{1+k}$$

$$x_E = \frac{0 + \frac{4}{3} \cdot 1}{1 + \frac{4}{3}} = \frac{\frac{4}{3} \cdot \frac{3}{7}}{\frac{7}{7}} = \frac{4}{7}, \quad y_E = \frac{5 + \frac{4}{3} \cdot 0}{1 + \frac{4}{3}} = \frac{5 \cdot \frac{3}{7}}{\frac{7}{7}} = \frac{15}{7} \Rightarrow E\left(\frac{4}{7}, \frac{15}{7}\right)$$

$$\frac{AF}{FB} = \frac{3}{2} \Rightarrow x_F = \frac{x_A + \frac{3}{2}x_B}{1 + \frac{3}{2}} = \frac{1 + \frac{3}{2} \cdot 4}{1 + \frac{3}{2}} = \frac{7 \cdot \frac{2}{5}}{\frac{5}{5}} = \frac{14}{5}, \quad y_F = \frac{y_A + \frac{3}{2}y_B}{1 + \frac{3}{2}} = \frac{0 + \frac{3}{2} \cdot 3}{\frac{5}{5}} = \frac{9}{5} \Rightarrow F\left(\frac{14}{5}, \frac{9}{5}\right)$$



$$AD: \frac{x-x_A}{x_D-x_A} = \frac{y-y_A}{y_D-y_A}$$

$$AD: \frac{x-1}{\frac{8}{3}-1} = \frac{y-0}{\frac{11}{3}-0} \Rightarrow AD: \frac{x-1}{\frac{5}{3}} = \frac{y}{\frac{11}{3}} \quad | \cdot \frac{3}{5}$$

$$AD: \frac{x-1}{5} = \frac{y}{11} \Rightarrow AD: 11(x-1) = 5y$$

$$AD: 11x - 5y - 11 = 0 \quad \boxed{A} \text{ adevarat}$$

$$BE: \frac{x-x_B}{x_E-x_B} = \frac{y-y_B}{y_E-y_B}$$

$$BE: \frac{x-4}{\frac{4}{3}-4} = \frac{y-3}{\frac{15}{3}-3} \Rightarrow BE: \frac{x-4}{-24} = \frac{y-3}{-6} \quad | \cdot (-24) \Rightarrow BE: x-4 = 4(y-3)$$

$$BE: x - 4y + 8 = 0 \quad \boxed{B} \text{ fals}$$

$$CF: \frac{x-x_C}{x_F-x_C} = \frac{y-y_C}{y_F-y_C} \Rightarrow CF: \frac{x}{\frac{14}{5}-0} = \frac{y-5}{\frac{9}{5}-5} \Rightarrow CF: -2x = 7(y-5)$$

$$CF: 8x + 7y - 35 = 0$$

\boxed{C} fals

T. Cerna AD, BE, CF concurente $(\Rightarrow) \frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$

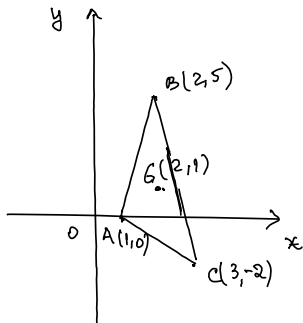
\boxed{D} adevarat

$$\frac{1}{2} \cdot \frac{4}{3} \cdot \frac{3}{2} = 1 \quad \checkmark$$

Răspuns $\boxed{A} \quad \boxed{D}$

4. Se dă un tringhi ABC cu vârfurile $A(1, 0)$, $C(3, -2)$ și $G(2, 1)$ centrul de greutate al triunghiului. Care dintre următoarele afirmații sunt adevărate?

- A $B(2, 5)$;
 B Aria triunghiului ABC este egală cu 12;
 C distanța de la punctul B la latura AC este $3\sqrt{2}$;
 D mediana din vârful B este perpendiculară pe axa Ox .



G centrul de greutate $\triangle ABC$, $G(x_G, y_G)$
 $x_G = \frac{x_A + x_B + x_C}{3}$, $y_G = \frac{y_A + y_B + y_C}{3}$

$$2 = \frac{1 + x_B + 3}{3} \Rightarrow 6 = 4 + x_B \Rightarrow x_B = 2$$

$$1 = \frac{0 + y_B - 2}{3} \Rightarrow 3 = y_B - 2 \Rightarrow y_B = 5$$

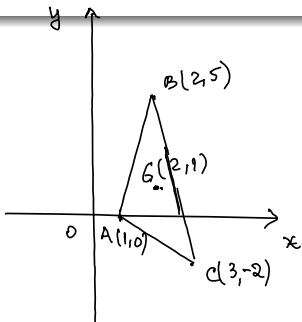
$\Rightarrow B(2, 5)$ A adevărată

$$A_{\triangle ABC} = \frac{1}{2} \begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix} =$$

$$= \frac{1}{2} \begin{vmatrix} 1 & 0 & 1 \\ 2 & 5 & 1 \\ 3 & -2 & 1 \end{vmatrix} = \frac{1}{2} |5 - 4 - 15 + 2| =$$

$$= \frac{1}{2} |-12| = 6$$

B falsă



$$AC = \sqrt{(x_A - x_C)^2 + (y_A - y_C)^2}$$

$$AC = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2}$$

$$AC = \sqrt{(3-1)^2 + (-2-0)^2} = \sqrt{2^2 + (-2)^2} = \sqrt{4+4}$$

$$A_{\Delta ABC} = 6 = \frac{AC \cdot \text{dist}(B, AC) = 2\sqrt{2}}{2}$$

$$\Rightarrow 2\sqrt{2} \cdot \text{dist}(B, AC) = 12$$

$$\text{dist}(B, AC) = \frac{12}{2\sqrt{2}} = \frac{6}{\sqrt{2}} = 3\sqrt{2}$$

e adărnăbă

BG mediană $B(2,5)$, $G(2,1)$

$$BG: \frac{x - x_G}{x_B - x_G} = \frac{y - y_G}{y_B - y_G} \Leftrightarrow BG: \frac{x - 2}{2 - 2} = \frac{y - 1}{5 - 1} \Leftrightarrow$$

$$BG: \frac{x - 2}{0} = \frac{y - 1}{4}$$

Concluzie: dacă numărătorul nul \Rightarrow
numărătorul cusp nul

$$BG: x - 2 = 0$$

$y = mx + m$ BG nu are pantă
Răspunsuri A, e, D

$$BG: x = 2 \Rightarrow BG \parallel Oy$$

D adm

MULT SUCCES LA EXAMENE!