Games, Dynamics and Optimization 2019

COST Action CA16228 "European Network for Game Theory"

Babeş-Bolyai University Cluj-Napoca, April 9 – 11, 2019

Funded by the Horizon 2020 Framework Programme of the European Union

Welcome Address

The aim of this conference is to bring together excellent researchers interested in dynamical system approaches to Optimization and Games.

In particular, the workshop is focused on recent advances in the following areas:

- Continuous-time approaches to Optimization
- Stochastic methods with emphasis on Machine Learning
- Dynamical Systems in Games and Decision Problems
- Non-smooth/Non-convex Problems
- Variational Inequalities

The first edition of this workshop took place in Vienna, in March 2018.

The workshop is organized and sponsored by the COST Action CA16228 ”European Network for Game Theory” (GAMENET) and by Babeş-Bolyai University Cluj-Napoca, Romania. We are also thankful to the IT company FORTECH for its financial support.

The Organizers:

- Radu Ioan Boţ (University of Vienna)
- Mathias Staudigl (Maastricht University)
- Adrian Petruşel (Babeş-Bolyai University)
- Nicolae Popovici (Babeş-Bolyai Unversity)
- Yolanda Paulissen (Administrative Support, Maastricht University)

Abstracts in Alphabetic Order

The link between implicit sweeping process and quasistatic evolution problems in contact mechanics

Samir Adly

Laboratoire XLIM, Université de Limoges, France
E-mail: samir.adly@unilim.fr

Keywords: Moreau's sweeping process, evolution variational inequalities, unilateral constraints in nonsmooth mechanics, quasistatic frictional contact problems.

Abstract. In this talk, we study a new variant of the Moreau's sweeping process with velocity constraint. Based on an adapted version of the Moreau's catching-up algorithm, we show the well-posedness (in the sense existence and uniqueness) of this problem in a general framework. We show the equivalence between this implicit sweeping process and a quasistatic evolution variational inequality. It is well-known that the variational formulation of many mechanical problems with unilateral contact and friction lead to an evolution variational inequality. As an application, we reformulate the quasistatic antiplane frictional contact problem for linear elastic materials with short memory as an implicit sweeping process with velocity constraint. The link between the implicit sweeping process and the quasistatic evolution variational inequality is possible thanks to some standard tools from convex analysis and is new in the literature. Talk based on the papers [1] and [2].

References

[1] S. Adly, T. Haddad, *An implicit sweeping process approach to quasistatic evolution variational inequalities*, SIAM Journal on Mathematical Analysis, **50**(2018), no. 1, 761-778.

[2] S. Adly, T. Haddad, B.K. Le, *State-dependent implicit sweeping process in the framework of quasistatic evolution quasi-variational inequalities*, Journal of Optimization Theory and Applications, https://doi.org/10.1007/s10957-018-1427-x.

Fréchet distance approximation and Maple applications

Mira-Cristiana Anisiu[1] and Valeriu Anisiu[2]

[1]"Tiberiu Popoviciu" Institute of Numerical Analysis
Romanian Academy, Cluj-Napoca
E-mail: mira@math.ubbcluj.ro

[2]Faculty of Mathematics and Computer Science
Babeş-Bolyai University
E-mail: anisiu@math.ubbcluj.ro

Mathematics Subject Classification: 68-04, 68W30.
Keywords: Metric space, Fréchet distance, approximation, polygonal curves.

Abstract. Let (X, d) be a metric space. A curve A in X is a continuous map from the unit interval into X, i.e., $A : [0, 1] \to X$. A reparameterization α of the unit interval is a continuous, non-decreasing surjection $\alpha : [0, 1] \to [0, 1]$. For two given curves A and B in X, the *Fréchet distance* $F(A, B)$ between them is defined as the infimum over all reparameterizations α and β of $[0, 1]$ of the maximum over all $t \in [0, 1]$ of the distance in X between $A(\alpha(t))$ and $B(\beta(t))$,

$$F(A, B) = \inf_{\alpha,\beta} \max_{t\in[0,1]} \{d(A(\alpha(t)), B(\beta(t)))\}.$$

It is a good measure for the resemblence of two curves, better then the Hausdorff measure, because it takes into account the flow of the two curves.

However, the Fréchet distance is difficult to compute.

To find $F(A, B)$ one considers polygonal approximations of the curves A and B, as suggested in [2], and try to find algorithms which allow the approximation of the Fréchet distance.

Using the Computer Algebra System Maple (see for example [1]) we can calculate the Fréchet distance between several curves by implementing rather fast methods.

References

[1] V. Anisiu, *Calcul simbolic cu Maple*, Presa Universitară Clujeană, 2006.

[2] Th. Eiter, H. Manilla, *Computing discrete Fréchet distance*, Tech. Report CD-TR 94/64, Information Systems Department, Technical University of Vienna, 1994.

Fast proximal methods via time scaling of damped inertial dynamics

Hedy Attouch

Institut Montpelliérain Alexander Grothendieck, UMR 5149 CNRS
Université Montpellier, France

Abstract. In a Hilbert space setting, we consider a class of inertial proximal algorithms for nonsmooth convex optimization, with fast convergence properties. They can be obtained by time discretization of inertial gradient dynamics which have been rescaled in time. We will rely specifically on the recent development linking Nesterov's accelerated method with vanishing damping inertial dynamics. Doing so, we somehow improve and obtain a dynamical interpretation of the seminal papers of Güler on the convergence rate of the proximal methods for convex optimization.

References

[1] H. Attouch, Z. Chbani, J. Peypouquet, P. Redont, *Fast convergence of inertial dynamics and algorithms with asymptotic vanishing damping*, Mathematical Programming Series B, **168**(2018), no. 1-2, 123-175.

[2] H. Attouch, Z. Chbani, H. Riahi, *Fast proximal methods via time scaling of damped inertial dynamics*, 2018, HAL-01939292.

[3] O. Güler, *New proximal point algorithms for convex minimization*, SIAM Journal on Optimization, **2**(1992), no. 4, 649-664.

[4] Y. Nesterov, *Introductory lectures on convex optimization: A basic course*, volume 87 of Applied Optimization, Kluwer Academic Publishers, Boston, MA, 2004.

[5] W. Su, S. Boyd, E.J. Candès, *A Differential Equation for Modeling Nesterov's Accelerated Gradient Method: Theory and Insights,* Journal of Machine Learning Research, **17**(2016), 1-43.

Constraints qualifications for tame programs

Jérôme Bolte[1], **Antoine Hochart**[2] **and Edouard Pauwels**[3]

[1]Toulouse School of Economics, Université Toulouse 1 Capitole
E-mail: jerome.bolte@tse-fr.eu

[2]Universidad Adolfo Ibáñez, Santiago de Chile
E-mail: antoine.hochart@gmail.com

[3]Institut de Recherche en Informatique de Toulouse, Université Paul Sabatier
E-mail: edouard.pauwels@irit.fr

Abstract. In this talk we present some recent results on qualification conditions for sets defined by polynomial/definable inequality. In particular we will show that all positive diagonal perturbations, save perhaps a finite number of them, ensure that any point within the feasible set satisfies Mangasarian-Fromovitz constraint qualification. Using the Milnor-Thom theorem, we provide a bound for the number of singular perturbations when the constraints are polynomial functions. Examples show that the order of magnitude of our exponential bound is relevant. Our perturbation approach provides a simple protocol to build sequences of "regular" problems approximating an arbitrary semialgebraic/definable problem. Applications to sequential quadratic programming methods and sum of squares relaxation will be discussed.

First-order methods for the impatient: support identification in finite time with convergent Frank-Wolfe variants

Immanuel Bomze

Universität Wien, ISOR/VCOR & DS:UniVie, Vienna, Austria
E-mail: immanuel.bomze@univie.ac.at

Abstract. We focus on the problem of minimizing a non-convex function over the standard simplex. We analyze two well-known and widely used variants of the Frank-Wolfe algorithm and prove global convergence of the iterates to stationary points both when using exact and Armijo line search. Then we show that the algorithms identify the support in a finite number of iterations. This, to the best of our knowledge, is the first time a manifold identification property has been shown for such a class of methods.
This talk is based on joint work with F. Rinaldi, S. Rota Bulo.

Optimization algorithms for machine learning

Coralia Carţiş

Mathematical Institute, University of Oxford, United Kingdom
E-mail: coralia.cartis@maths.ox.ac.uk

Abstract. Optimization is a key component of machine learning application, as it helps with training of (neural net, nonconvex) models and parameter tuning. Classical optimization methods are challenged by the scale of machine learning applications and the lack of /cost of full derivatives, as well as the stochastic nature of the problem. On the other hand, the simple approaches that the machine learning community uses need improvement. Here we try to merge the two perspectives and adapt the strength of classical optimization techniques to meet the challenges of data science applications: from deterministic to stochastic problems, from small to large scale.

The forward-backward-forward method from discrete and continuous perspective for pseudo-monotone variational inequalities in Hilbert spaces

Radu Ioan Boţ[1], **<u>Ernö Robert Csetnek</u>**[2] **and P.T. Vuong**[3]

[1]University of Vienna, Faculty of Mathematics, Oskar-Morgenstern-Platz 1, A-1090 Vienna, Austria
E-mail: radu.bot@univie.ac.at

[2]University of Vienna, Faculty of Mathematics, Oskar-Morgenstern-Platz 1, A-1090 Vienna, Austria
E-mail: ernoe.robert.csetnek@univie.ac.at

[3]University of Vienna, Faculty of Mathematics, Oskar-Morgenstern-Platz 1, A-1090 Vienna, Austria
E-mail: vuong.phan@univie.ac.at

Abstract. Consider the problem:

Find $x \in C$ such that

$$\langle F(x^*), x - x^* \rangle \geq 0, \ \forall \ x \in C,$$

where C is a nonempty, convex and closed subset of the real Hilbert space H and F is pseudo-monotone.

In this talk we show that Tseng's forward-backward-forward algorithm:

$$\begin{cases} y_n = P_C(x_n - \lambda F(x_n)), \\ x_{n+1} = y_n + \lambda(F(x_n) - F(y_n)) \end{cases}, \ \forall \ n \geq 0,$$

which is usually used in the monotone case, converges also when it is applied to the solving of pseudo-monotone variational inequalities. In addition, we show that linear convergence is guaranteed under strong pseudo-monotonicity. We also associate a dynamical system to the pseudo-monotone variational inequality and carry out an asymptotic analysis for the generated trajectories. Numerical experiments show that Tseng's method outperforms Korpelevich's extragradient method when applied to the solving of pseudo-monotone variational inequalities and fractional programming problems.

Ghost penalties in nonconvex constrained optimization: Diminishing stepsizes and iteration complexity

Francisco Facchinei

Department of Computer, Control, and Management Engineering
University of Rome La Sapienza, Italy
E-mail: facchinei@diag.uniroma1.it

Abstract. We consider nonconvex constrained optimization problems and propose a new approach to the convergence analysis based on penalty functions. We make use of classical penalty functions in an unconventional way, in that penalty functions only enter in the theoretical analysis of convergence while the algorithm itself is penalty-free. Based on this idea, we are able to establish several new results, including the first general analysis for diminishing stepsize methods in nonconvex, constrained optimization, showing convergence to generalized stationary points, and the first complexity study for SQP-type algorithms.

Solving nonlinear minmax location problems with minimal time functions by means of proximal point methods via conjugate duality

Sorin-Mihai Grad

Faculty of Mathematics, University of Vienna, 1090 Vienna, Austria
E-mail: sorin-mihai.grad@univie.ac.at

Abstract. We investigate via a conjugate duality approach general nonlinear minimax location problems formulated by means of minimal time functions, necessary and sufficient optimality conditions being delivered together with characterizations of the optimal solutions in some particular instances.

A splitting proximal point method is employed in order to numerically solve such problems and their duals and we present the computational results obtained in MATLAB on concrete examples, comparing these with earlier similar ones from the literature.

This talk is based on joint work with Oleg Wilfer.

References

[1] N.T. An, D. Giles, N.M. Nam, R.B. Rector, *The log-exponential smoothing technique and Nesterov's accelerated gradient method for generalized Sylvester problems*, Journal of Optimization Theory and Applications, **168**(2016), no. 2, 559-583.

[2] S.-M. Grad, O. Wilfer, *A proximal method for solving nonlinear minmax location problems with perturbed minimal time functions via conjugate duality*, preprint.

[3] N.M. Nam, N.T. An, J. Salinas, *Applications of convex analysis to the smallest intersecting ball problem*, Journal of Convex Analysis, **19**(2012), no. 2, 497-518.

Three (small) examples of problems of ”optimal curves” which are still open

Jean-Baptiste Hiriart-Urruty

Institut de Mathématiques, Université Paul Sabatier
118, Route de Narbonne, 31062 Toulouse Cedex 9, France
www.math.univ-toulouse.fr/∼jbhu/

Keywords: Conjectures, mathematical challenges, curve or shape optimization.

Abstract. The so-called open problems, the conjectures... have always been driving forces in the advance of knowledge in mathematics, but also in their learning, at all levels: colleges, high schools, mathematical training in universities. To propose such questions, in the form of challenges ” *Who will do the best?*” is even a suggested activity in the teaching methods because it is very formative. Added to this is the need for a demonstration or proof of an advanced assertion: to propose the best known solution does not mean that there is no better one; a solution is the best when it has been proved that it is... Number Theory like Geometry are major problem areas containing problems for which we can propose the best known solutions, but without knowing, for as much, if they are the best possible, in other words without being able to prove the optimality of these solutions (in a sense to be defined).

We propose in this talk three problems of geometry (with variants), with a variational flavor, of the type ”*Find the curve of minimum length such as...*”; the first one is somehow classical, the next two are more original. In all three cases, we indicate how far mathematicians have been able to go; none of them has been fully resolved, as yet. Of course, we do not expect the listener to solve these problems... But what we propose can be used as material for a research process, between students, friends, colleagues, on ”*how to bring and/or improve*” a proposed answer to a given problem, each one making his imagination and mathematical ability work.

Games, graphs and dynamics

Josef Hofbauer

Department of Mathematics, University of Vienna, Austria
E-mail: Josef.Hofbauer@univie.ac.at

Abstract. To a game in normal form one can associate directed graphs that describe the best or better reply structure. How much does such a graph tell us about game dynamics, such as replicator dynamics or best response dynamics?

A primal-dual dynamical approach to a nonsmooth convex minimization

Radu Ioan Boţ[1], **Ernö Robert Csetnek**[2] **and**
Szilárd Csaba László[3]

[1]University of Vienna, Faculty of Mathematics, Oskar-Morgenstern-Platz 1,
A-1090 Vienna, Austria
E-mail: radu.bot@univie.ac.at.

[2]University of Vienna, Faculty of Mathematics, Oskar-Morgenstern-Platz 1,
A-1090 Vienna, Austria
E-mail: er-noe.robert.csetnek@univie.ac.at.

[3]Technical University of Cluj-Napoca, Department of Mathematics,
Memorandumului 28, Cluj-Napoca, Romania
E-mail: szilard.laszlo@math.utcluj.ro

Abstract. In this paper we investigate a dynamical system in connection to a complexly structured minimization problem. More precisely, the objective function of the minimization problem considered is the sum of a smooth convex function and the sum of two nonsmooth convex functions, where one of them is composed with a continuous linear operator. We show the existence and uniqueness of strong global solutions in the framework of the Cauchy-Lipschitz Theorem and prove convergence for the generated trajectories to the minima of the above mentioned minimization problem but also for its Fenchel dual.

"Lion and man" Revisited I

Genaro López-Acedo

Dept. Análisis Matemático, Fac. Matemáticas
Universidad de Sevilla, Apdo. 1160, 41080-Sevilla, Spain
E-mail: glopez@us.es

Abstract. In these talks we analyze a discrete pursuit-evasion game, based on Rado's lion and man celebrated problem, defined on a geodesic space. In the first part we prove that in uniformly convex bounded domains the lion always wins and, using ideas stemming from proof mining, we extract a uniform rate of convergence for the successive distances between the lion and the man.

References

[1] S. Alexander, R. Bishop, R. Ghrist, *Total curvature and simple pursuit on domains of curvature bounded above*, Geom. Dedicata, **149**(2010), 275-290.

[2] U. Kohlenbach, G. López-Acedo, A. Nicolae, *A quantitative analysis of the "Lion-Man" game* (submitted).

[3] G. López-Acedo, A. Nicolae, B. Piatek, *"Lion-Man" and the fixed point property*, Geom. Dedicata (in press, https://doi.org/10.1007/s10711-018-0403-9).

[4] J.E. Littlewood, *Littlewood's Miscellany* (ed: B. Bollobás), Cambridge University Press, Cambridge, 1986.

Golden ratio algorithm for variational inequalities

Yura Malitsky

University of Göttingen, Institute for Numerical and Applied Mathematics
E-mail: y.malitskyi@math.uni-goettingen.de

Keywords: Variational inequalities, non-Lipschitz-continuity, first-order methods, linesearch, composite minimization.

Abstract. We present a novel iterative method for solving general (monotone) variational inequalities. In every iteration it defines a stepsize, based on a local information about an operator, without running any linesearch procedure. Thus, the cost of its iteration is the same as, for example, in the forward-backward method, but the method converges without referring to cocoercivity or Lipschitz continuity of the operator. We further discuss possible generalizations of the method when the operator is not monotone.

References

[1] Y. Malitsky, *Golden Ratio Algorithms for Variational Inequalities*, 2018, arXiv: 1803.08832.

Hessian barrier algorithms for linearly constrained optimization problems

Immanuel Bomze[1], **Panayotis Mertikopoulos**[2], **Werner Schachinger**[3] **and Mathias Staudigl**[4]

[1]Universität Wien, ISOR/VCOR & DS:UniVie, Vienna, Austria
E-mail: immanuel.bomze@univie.ac.at

[2]Univ. Grenoble Alpes, CNRS, Inria, Grenoble INP, LIG, 38000, France
E-mail: panayotis.mertikopoulos@imag.fr

[3]Universität Wien, ISOR/VCOR & DS:UniVie, Vienna, Austria
E-mail: werner.schachinger@univie.ac.at

[4]Maastricht University, Department of Quantitative Economics
P.O. Box 616, NL-6200 MD Maastricht, The Netherlands
E-mail: m.staudigl@maastrichtuniversity.nl

Mathematics Subject Classification: 90C51, 90C30, 90C25, 90C26.
Keywords: Hessian Riemannian gradient descent, interior-point methods, mirror descent, non-convex optimization, traffic assignment.

Abstract. In this paper, we propose an interior-point method for linearly constrained optimization problems (possibly nonconvex). The method – which we call the *Hessian barrier algorithm* (HBA) – combines a forward Euler discretization of Hessian Riemannian gradient flows with an Armijo backtracking step-size policy. In this way, HBA can be seen as an explicit alternative to mirror descent (MD), and contains as special cases the affine scaling algorithm, regularized Newton processes, and several other iterative solution methods. Our main result is that, modulo a non-degeneracy condition, the algorithm converges to the problem's set of critical points; hence, in the convex case, the algorithm converges globally to the problem's minimum set. In the case of linearly constrained quadratic programs (not necessarily convex), we also show that the method's convergence rate is $\mathcal{O}(1/k^\rho)$ for some $\rho \in (0, 1]$ that depends only on the choice of kernel function (i.e., not on the problem's primitives). These theoretical results are validated by numerical experiments in standard non-convex test functions and large-scale traffic assignment problems.

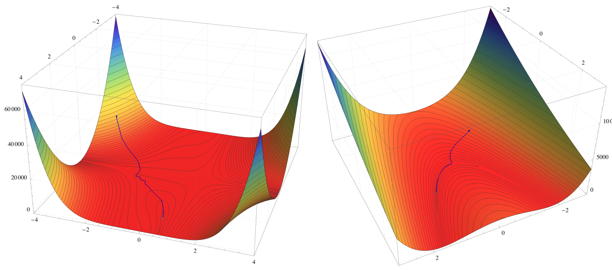

Figure 1: HBA trajectories for Beale and Resenbrock benchmarks

Convergence rates of stochastic first order methods for composite convex optimization

Ion Necoară

Abstract. Many problems from engineering, statistics and machine learning can be formulated as stochastic composite optimization problems. We propose a general framework for the analysis of stochastic composite convex optimization that includes the most well-known classes of objective functions analyzed in the literature: non-smooth functions, and composition of a (potentially) non-smooth function and a smooth function, with or without a quadratic functional growth property. Based on this framework we derive a complete convergence analysis for the most known classes of stochastic first order methods, that is the stochastic proximal gradient and proximal point algorithms. These schemes are typically the method of choice in practice for applications due to their cheap iteration and superior empirical performance. Usually, the convergence theory of these methods have been derived for simple stochastic optimization models, the rates are in general sublinear and hold only for decreasing stepsizes.

In this paper we analyze the convergence rates of stochastic first order methods with constant or variable stepsize for composite convex optimization problems, expressed in terms of expectation operator. We show that these methods can achieve linear convergence rate up to a constant proportional the stepsize and under some strong stochastic bounded gradient condition even pure linear convergence. When the strong stochastic bounded gradient condition does not hold we show that restarted variants of these methods can achieve linear convergence. Moreover, when variable stepsize is chosen we derive sublinear convergence rates for these stochastic first order methods under the same assumptions on the optimization model. Finally, convergence rates for minibatch variants of these methods are also derived. Moreover, the stochastic gradient mapping and the Moreau smoothing mapping used in the present paper lead to more elegant and intuitive proofs.

"Lion and man" Revisited II

Adriana Nicolae

Department of Mathematics, Babeş-Bolyai University
Kogălniceanu 1, 400084 Cluj-Napoca, Romania
E-mail: anicolae@math.ubbcluj.ro

Abstract. This talk continues the analysis of the pursuit-evasion game discussed in "Lion and man" Revisited I. In this second part we relate the existence of different types of rays with the success of the lion. This allows to establish a link between the solution of the game and the seemingly unconnected fixed point property for continuous and nonexpansive mappings.

References

[1] S. Alexander, R. Bishop, R. Ghrist, *Total curvature and simple pursuit on domains of curvature bounded above*, Geom. Dedicata, **149**(2010), 275-290.

[2] G. López-Acedo, A. Nicolae, B. Piatek, *"Lion-Man" and the fixed point property*, Geom. Dedicata (in press, https://doi.org/10.1007/s10711-018-0403-9).

[3] G. López-Acedo, A. Nicolae, B. Piatek, *"Lion-Man" and geodesic rays* (in progress).

[4] J.E. Littlewood, *Littlewood's Miscellany* (ed: B. Bollobás), Cambridge University Press, Cambridge, 1986.

Data Science problems and complex networks optimization... in a nutshell

Justo Puerto

Instituto Universitario de Investigación Matemática (IMUS)
Universidad de Sevilla
E-mail: puerto@us.es

Keywords: Data Science, continuous and combinatorial optimization, network design.

Abstract. In today's world, "Big data" is a very popular word. Recent technological advances offer a great diversity of tools for collecting and storing huge volumes of data, the goal of data science is to extract information from these large data sets. However, the technology required to advantageously process these data sets, often endowed with complex structures, needs a deep analysis which is currently far from the capacity of nowadays technology to collect and store them efficiently. Indeed, when high-volume streams of data arrive, traditional methods for managing the information flow are inadequate and a further research, to provide efficient tools, is needed.

This talk addresses theoretical results, methodology and resolution procedures that allow one to make use of the tools and models of complex networks analysis in Data Science. Our aim is to develop tools that allow us to extract additional information from datasets beyond what is reported by traditional methods of Data Science. Specifically, we will revisit models of supervised and unsupervised classification and regression analysis under the lens of continuous and combinatorial optimization giving partial answer to some of those questions.

An alternating semi-proximal method for nonconvex problems

Shoham Sabach

Faculty of Industrial Engineering and Management, Technion, Haifa, Israel
E-mail: ssabach@ie.technion.ac.il

Abstract. We consider a broad class of regularized structured total-least squares problems (RSTLS) encompassing many scenarios in image processing. This class of problems results in a nonconvex and often nonsmooth model in large dimension. To tackle this difficult class of problems we introduce a novel algorithm that blends proximal and alternating minimization methods by beneficially exploiting data information and structures inherently present in RSTLS. The proposed algorithm, which can also be applied to more general problems is proven to globally converge to critical points and is amenable to efficient and simple computational steps.

Existence of monotone solutions with respect to a preorder and applications

Wissam Latreche and Oana-Silvia Serea

Department of Computing, Mathematics, and Physics
Western Norway University of Applied Sciences, Bergen, Norway
E-mail: oserea@yahoo.fr

Abstract. In this paper, we study the existence of $P(\cdot) \times Q(\cdot)$-monotone solutions of the differential inclusions with discontinuous right-hand side of the form

$$\begin{cases} x'(t) \in \operatorname{proj}_{T_{P(x(t))}(x(t))}(-\partial_x \Gamma(x(t), y(t))), \ \text{a.e. } t \in [0, +\infty), \\ y'(t) \in \operatorname{proj}_{T_{Q(y(t))}(y(t))}(\partial_y^+ \Gamma(x(t), y(t))), \ \text{a.e. } t \in [0, +\infty), \\ x(0) = x_0, \ y(0) = y_0. \end{cases}$$

The monotonicity that we consider is with respect to a preorder. Moreover, we show that the limit points $(\widetilde{x}, \widetilde{y})$ of a solution is a saddle-point for the convex-concave function $\Gamma(\cdot, \cdot)$ in $P(\widetilde{x}) \times Q(\widetilde{y})$. The obtained results can be used for studying a game involving two players with a collective pay-off. Another application is in a production problem of a company which wishes to maximize its profit. Finally, some particular cases are also considered i.e. with a function $w(\cdot)$ which is supposed to be convex, concave, then prox-regular.

No regret criteria in learning, games and convex optimization

Sylvain Sorin

Institut de Mathématiques Jussieu-PRG, Sorbonne Université
UPMC - PARIS 6, CNRS UMR 7586
E-mail: sylvain.sorin@imj-prg.fr

Abstract. The purpose of this work is to underline links between no-regret algorithms used in learning, games and convex optimization. In particular we will study the continuous and discrete time versions and their connections.

Sparse non-negative super-resolution: simplified and stabilized

Jared Tanner

University of Oxford, Mathematical Institute
E-mail: tanner@maths.ox.ac.uk

Abstract. Super-resolution is a technique by which one seeks to overcome the inherent accuracy of a measurement device by exploiting further information. Applications are very broad, but in particular these methods have been used to great effect in modern microscopy methods and underpin recent Nobel prizes in chemistry. This topic has received a renewed theoretical interest starting in approximately 2013 where notions from compressed sensing were extended to this continuous setting. We extend recent results by by Schiebinger, Robava, and Recht to show uniqueness and consistency of solutions to the measurement process.

A dual moving balls algorithm for a class of constrained nonsmooth convex minimization

Marc Teboulle

School of Mathematical Sciences, Tel Aviv University, Israel
E-mail: teboulle@post.tau.ac.il

Abstract. We consider the problem of minimizing a nonsmooth convex objective over a smooth convex inequality constraint. This particular model arises in a number of interesting applications e.g., in sparse recovery and machine learning problems. Exploiting the smoothness of the feasible set, and using duality, we introduce a novel simple algorithm, free of any exogenous parameters, (e.g., penalty or smoothing parameters), which is proven to globally converge to an optimal solution with a $O(1/\varepsilon)$ efficiency estimate. The performance and efficiency of the algorithm is demonstrated by solving large scale instances of the convex sparse recovery problem.

Metric regularity and directional metric regularity of multifunctions

Michel Théra

Departement Maths-Info, Université de Limoges, France
E-mail: michel.thera@icloud.com

Abstract. My aim in this talk is to make an overview on metric regularity, a concept which plays an important role in optimization and related topics and has attracted over the recent years a large number of contributions.

I intend to give some new advances I obtained with my co-authors Huynh Van Ngai and Nguyen Huu Tron on directional metric regularity.

Solving quadratic multi-leader-follower games by smoothing the follower's best response

Anna Thünen

RWTH Aachen University
E-mail: thuenen@igpm.rwth-aachen.de

Mathematics Subject Classification: 91A06, 91A10, 90C33, 91A65, 49J52.
Keywords: Multi-leader-follower games, Nash equilibria, game theory, equilibrium problems with equilibrium constraints.

Abstract. The multi-leader-follower game is a particular subset of classical game theory. These models serve as an analytical tool to study the strategic behavior of individuals in a noncooperative manner. In particular, the individuals (players) are divided into two groups, namely the leaders and the followers, according to their position in the game. Mathematically, this leads optimization problems with optimization problems as constraints.

We derive Nash-s-stationary equilibria for a class of quadratic multi-leader-follower games using nonsmooth best response function. To overcome the challenge of nonsmoothness, we pursue a smoothing approach resulting in a reformulation as smooth Nash equilibrium problem. We prove existence and uniqueness for all smoothing parameter. For a decreasing sequence of this smoothing parameters accumulation points of Nash equilibria exists and we show that they fulfill the conditions of s-stationarity. Finally, we propose an update on the leader variables for efficient computation and numerically compare nonsmooth Newton and subgradient methods.

Nonconvex second-order damped gradient systems and metastability

Adrian Viorel

Department of Mathematics, Technical University of Cluj-Napoca

Abstract. A couple of decades ago, Samson et. al. [4] (cf. also [2]) have proposed a Mumford-Shah-type nonconvex functional that can achieve both image classification and restoration simultaneously. Their variational, Γ-convergence based, approach relies on ideas form phase transition mechanics [1], albeit that the functional that they consider is nonconvex both in terms of the classification energy as well as in the regularization energy

$$E_\varepsilon(u) = \int_\Omega (u(x) - u_0(x))^2 dx + \varepsilon \int_\Omega \varphi(|\nabla u(x)|) dx + \frac{1}{\varepsilon} \int_\Omega W(u(x)) dx,$$

where u_0 is the image to be restored and classified, $\varepsilon > 0$ is a small parameter and $\varphi : \mathbb{R} \to \mathbb{R}$, $\varphi(w) = \frac{w^2}{w^2+1}$ while W is a double-well potential, e.g., $W : \mathbb{R} \to \mathbb{R}$, $W(u) = \frac{1}{4}(u^2 - 1)^2$.

Having this particular optimization problem in mind, we analyze the asymptotic behavior of damped second-order gradient systems

$$\ddot{u} + \gamma B \dot{u} = -\nabla E_\varepsilon(u), \quad \text{where} \quad E_\varepsilon(u) = E_{\varepsilon 1}(Au) + E_{\varepsilon 2}(u)$$

where B is allowed to be an unbounded linear operator, especially in connection with the concept of metastability and recent developments in that field (cf. [3]).

References

[1] S. Allen, J. Cahn, *A microscopic theory for antiphase boundary motion and its application to antiphase domain coarsening*, Acta Metall., **27**(1979), no. 6, 1085-1095.

[2] G. Aubert, B. Kornprobst, *Mathematical Problems in Image Processing: Partial Differential Equations and the Calculus of Variations*, 2nd ed. Applied Mathematical Sciences, vol. 147, Springer, New York, 2006.

[3] R. Folino, C. Lattanzio, C. Mascia, *Metastable dynamics for hyperbolic variations of the Allen-Cahn equation*, Commun. Math. Sci., **15**(2017), no. 7, 2055-2085.

[4] C. Samson, L. Blanc-Féraud, G. Aubert, J. Zerubia, *A variational model for image classification and restoration*, IEEE Trans. Pattern Anal. Mach. Intell., **22**(2000), no. 5, 460-472.

The Boosted DC Algorithm for nonsmooth functions

Francisco J. Aragón Artacho[1], **Phan T. Vuong**[2]

[1]Department of Mathematics, University of Alicante
Alicante, Spain
E-mail: francisco.aragon@ua.es

[2]Faculty of Mathematics, University of Vienna
Oskar-Morgenstern-Platz 1, 1090 Vienna, Austria
E-mail: vuong.phan@univie.ac.at

Keywords: Difference of convex functions, Boosted difference of convex functions algorithm, Kurdyka-Lojasiewicz property, clustering problem, multidimensional scaling problem.

Abstract. The Boosted Difference of Convex functions Algorithm (BDCA) was recently proposed for minimizing smooth difference of convex (DC) functions. BDCA accelerates the convergence of the classical Difference of Convex functions Algorithm (DCA) thanks to an additional line search step. The purpose of this paper is twofold. Firstly, to show that this scheme can be generalized and successfully applied to certain types of nonsmooth DC functions, namely, those that can be expressed as the difference of a smooth function and a possibly nonsmooth one. Secondly, to show that there is complete freedom in the choice of the trial step size for the line search, which is something that can further improve its performance. We prove that any limit point of the BDCA iterative sequence is a critical point of the problem under consideration, and that the corresponding objective value is monotonically decreasing and convergent. The global convergence and convergent rate of the iterations are obtained under the Kurdyka-Lojasiewicz property. Applications and numerical experiments for two problems in data science are presented, demonstrating that BDCA outperforms DCA. Specifically, for the Minimum Sum-of-Squares Clustering problem, BDCA was on average sixteen times faster than DCA, and for the Multidimensional Scaling problem, BDCA was three times faster than DCA.

The proximal alternating minimization algorithm

Sandy Bitterlich, Radu Ioan Boţ, Ernö Robert Csetnek and Gert Wanka

Abstract. The Alternating Minimization Algorithm (AMA) introduced by Tseng (1991) is an algorithm to solve separable convex programming problems with two-block separable linear constraints and objectives, whereby one of the components of the latter is assumed to be strongly convex. The fact that one of the subproblems to be solved within the iteration process of AMA does not usually correspond to the calculation of a proximal operator through a closed formula, affects the implementability of the algorithm. We allow in each block of the objective a further smooth convex function and propose a proximal version of AMA, called Proximal AMA, which is achieved by equipping the algorithm with proximal terms induced by variable metrics. For suitable choices of the latter, the solving of the two subproblems in the iterative scheme can be reduced to the computation of proximal operators. We investigate the convergence of the proposed algorithm in a real Hilbert space setting and illustrate its numerical performances on two applications in image processing and machine learning.

On unconstrained optimization problems solved using CDT and triality theory

Constantin Zălinescu

Faculty of Mathematics, University ”Al. I. Cuza” Iaşi, Romania
E-mail: zalinesc@uaic.ro

Abstract. D.Y. Gao solely or in collaboration applied his Canonical Duality Theory for solving a class of unconstrained optimization problems, getting the so-called ”triality theorems”. Unfortunately, the ”double-min duality” from these results published before 2010 revealed to be false, even if in 2003 D.Y. Gao announced that ”certain additional conditions” are needed for getting it. After 2010, D.Y. Gao together with some of his collaborators published several papers in which they added additional conditions for getting ”double-min” and ”double-max” dualities in the triality theorems. Our aim is to treat rigorously this kind of problems and to discuss several results concerning the ”triality theory” obtained up to now.

Index