

15.

a, b, c
2 3 4

- C-hamis

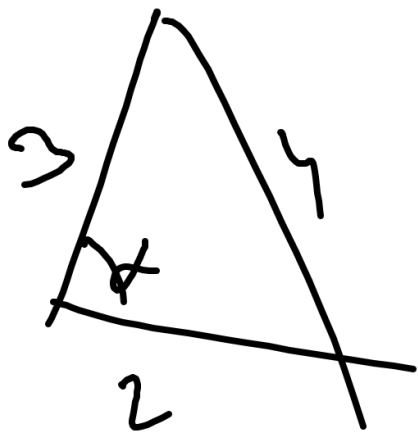
$$2+3>4 \checkmark$$

$$2+4>3 \checkmark$$

$$3+4>2 \checkmark$$

| $\Rightarrow$ szerkeszthető Δ

!!
1D igor



$$4^2 = 2^2 + 3^2 - 2 \cdot 2 \cdot 3 \cdot \cos \alpha$$

$$16 - 4 - 9 = -2 \cdot 2 \cdot 3 \cos \alpha$$

$$-\frac{1}{4} = \cos \alpha \Rightarrow \alpha \text{ kompatibilis Biga A hamis}$$

16.

$$\vec{u} = (m-1)\vec{a} + 2\vec{b}$$

$$\vec{v} = 3\vec{a} + m\vec{b}$$

$$\frac{1}{2}\vec{u} = \frac{1}{2}\vec{v} \quad (m-1)\vec{a} + 2\vec{b} = 3\vec{a} + m\vec{b}$$

$$m-1 = 3$$

$$m = 3 + 1$$

$$2 = 1 + m$$

$$2 = 1 + (3 + 1)$$

$$3\lambda^2 + \lambda - 2 = 0$$

$$\Delta = 1 + 24 = 25$$

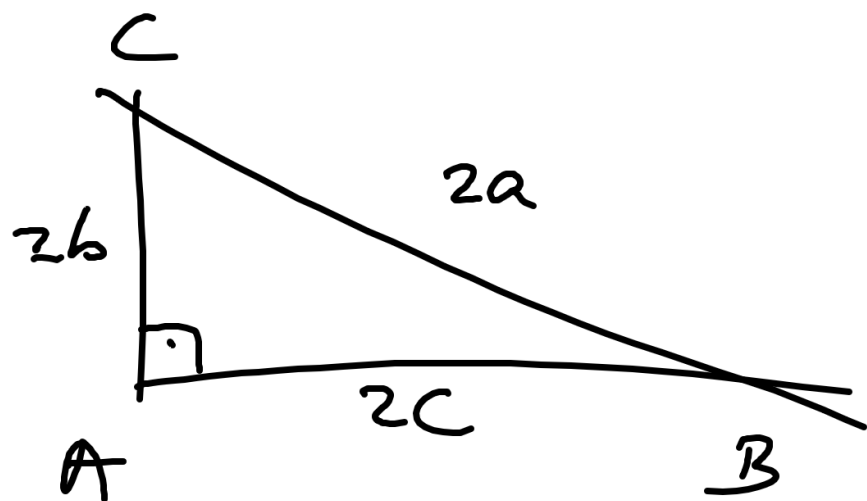
$$\lambda_{1,2} = \frac{-1 \pm 5}{6} \rightarrow \begin{matrix} -1 \\ 2/3 \end{matrix}$$

$$\boxed{m_1 = -2} \quad \boxed{m_2 = 3}$$

C-helyes

A, B, D helyes

(17)



$$b+c-a = \frac{2bc}{b+c+a}$$

(*)

$$(b+c)^2 - a^2 = 2bc$$

$$b^2 + c^2 + 2bc - a^2 = 2bc \quad \checkmark$$

A is right

$$4b^2 + 4c^2 = 4a^2$$

Prt. let

C is right

$$T_{ABC} = \frac{2c \cdot 2b}{2} = 2bc \quad \checkmark$$

$$R = \frac{BC}{2} = a \neq \frac{a}{2} \Rightarrow$$

B is not

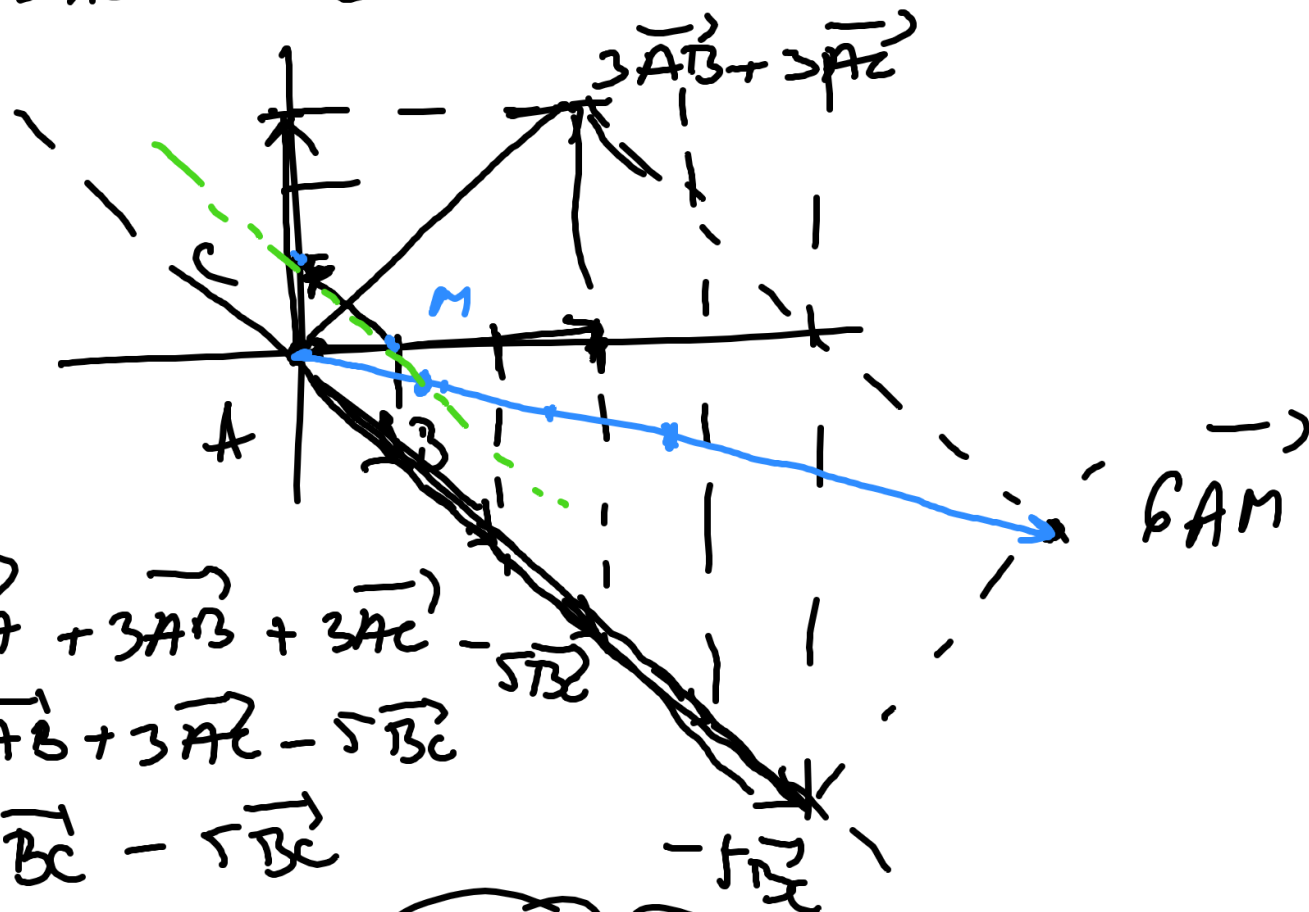
D is not

$$r = \frac{I}{p} = \frac{2bc}{\frac{2a+2b+2c}{2}} = \frac{2bc}{a+b+c}$$

$$2b+2c = 2 \cdot \left(a + \frac{2bc}{a+b+c} \right)$$

(18)

$$6 \vec{AM} = 3\vec{AB} + 3\vec{AC} - 5\vec{BC}$$



$$\begin{aligned} \vec{BM} &= 6\vec{BA} + 6\vec{AM} = 6\vec{BA} + 3\vec{AB} + 3\vec{AC} - 5\vec{BC} \\ &= -3\vec{AB} + 3\vec{AC} - 5\vec{BC} \\ &= 3\vec{BC} - 5\vec{BC} \end{aligned}$$

$$6\vec{BM} = -2\vec{BC}$$

$$3\vec{BM} = -\vec{BC}$$

Aight

1 hour's

$$\vec{BM} \cdot \vec{BC} = -\frac{1}{3}\vec{BC} \cdot \vec{BC} = -\frac{1}{3}BC^2 < 0$$

$$A(1, -1) \quad B(3, -1) \quad A'(-1, -2) \quad B'(0, -2)$$


$$P \in \mathcal{P} \quad C = C' = P$$

$$L = \frac{2 \cdot (y+1)}{2}$$

χ^2_n prüft.

$$\frac{2 \cdot (y+1)}{2} = \frac{4 \cdot (y+2)}{2}$$

$$\text{r.f. } x=2 \Rightarrow \text{Cisat}$$

$$2 + 2y = 4y + 8$$

$y = -3$ is et wenn
beide $y \geq 0$

$$\underline{D} \quad A'C' = 3\sqrt{2}$$

$$(x, x^2)$$

$$(-4, -2)$$

$$(x+4)^2 + (x^2+2)^2 = 18$$

$$x^2 + 8x + 16 + x^4 + 4x^2 + 4 = 18$$

$$\underline{x = -1}$$

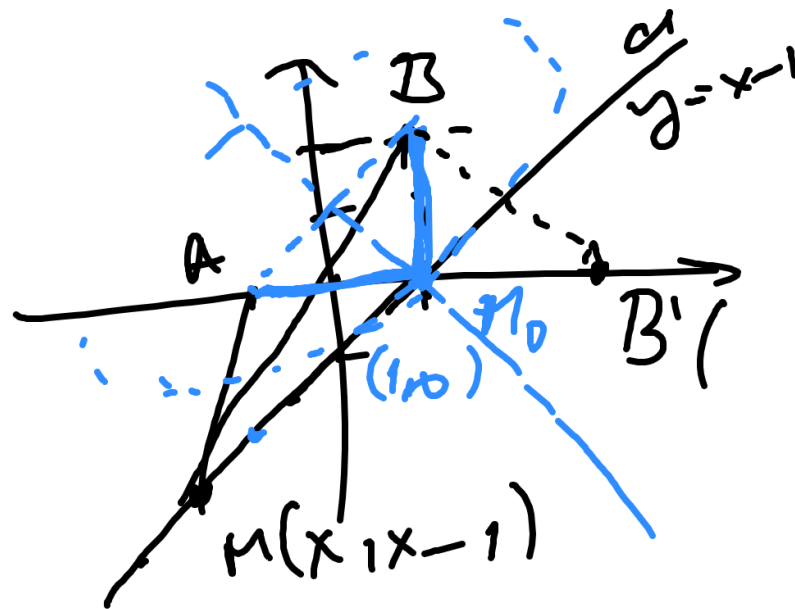
$$3^2 + 3^2 = 18. \checkmark$$

D igar

20. $x - y = 1$

$A(-1, 0)$

$B(1, 2)$



$4 > 2 + \sqrt{2} \Rightarrow$ A'gar

$\sqrt{2} + \sqrt{10} > 4 \Rightarrow$ C hamis

B hamis

D hamis

$AM + MB = AM + MB' \geq$

$\geq AB' = AM_0 + M_0B$
 $= 4$

(21)

$$\lim_{n \rightarrow \infty} n \left(\sqrt[4]{n+1} - \sqrt[4]{n} \right)$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt[4]{x+1} - \sqrt[4]{x}}{x^\alpha} = \lim_{x \rightarrow \infty} \frac{x+1 - x}{x \cdot \left(\sqrt[4]{(x+1)^3} + \dots \right)} =$$

$$n^4 - v^4 = (n-v)(n^3 + n^2v + nv^2 + v^3) \quad \alpha = -\frac{3}{4} \text{ esetén}$$

$$\lim_{n \rightarrow \infty} \underbrace{n^{\frac{1}{4}}}_{\downarrow \infty} \cdot \underbrace{n^{\frac{3}{4}} \left(\sqrt[4]{n+1} - \sqrt[4]{n} \right)}_{\downarrow \frac{1}{4}} = \infty = \frac{1}{4}$$

D igaz
A, B, C hamis

(22.)

$$a_n = \frac{n!}{n^n}$$

$$\frac{(n+1)!}{(n+1)^{n+1}} < \frac{n!}{n^n} \quad ?$$

$(a_n)_{n \geq 1}$ sig. székely $\uparrow$

$$1 < \left(\frac{n+1}{n}\right)^n$$

$\Downarrow$
A hamis

$$1 < \left(1 + \frac{1}{n}\right)^n$$

igaz

$$3 \geq \left(1 + \frac{1}{n}\right)^n \geq 2$$

$\subset e$

$$a_n > 0$$

$$\frac{a_{n+1}}{a_n} = \frac{1}{\left(1 + \frac{1}{n}\right)^n} \rightarrow \frac{1}{e} < 1$$

$\Downarrow$
 $a_n \rightarrow 0$

$$\frac{n!}{n^n} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdots n}{n \cdot n \cdot n \cdots n} \leq \frac{1}{n}$$

$\leq 1 \quad \leq 1 \quad \leq 1 \quad \leq 1$

$\Rightarrow$

$$0 < a_n < \frac{1}{n}$$

$\Rightarrow$

C igaʻ

D igaʻ

B hamis

(23)

$$I = \int_{\frac{\pi}{4}}^{2\pi} \frac{\cos^2 \frac{x}{2}}{x + \sin x} dx$$

$$A = x + \sin x \quad dt = 1 + \cos x = 2 \cos^2 \frac{x}{2}$$

$$2I = \int_{\frac{\pi}{4}}^{2\pi} \frac{dt}{t} = \ln t \Big|_{\frac{\pi}{4}}^{2\pi} = \ln 2\pi - \ln \frac{\pi}{4} = \underline{\underline{\ln 2}}$$

$$I = \frac{\ln 2}{2} \Rightarrow \begin{array}{l} A \text{ iğqa} \\ B, C, D \text{ hamis} \end{array}$$

(21.) $f: [0,1] \rightarrow \mathbb{R} \quad f(x) = x^2(x+a)$

$$f'(x) = 3x^2 + 2ax$$

$$f'(x) = 0 \Leftrightarrow x(3x + 2a) = 0$$

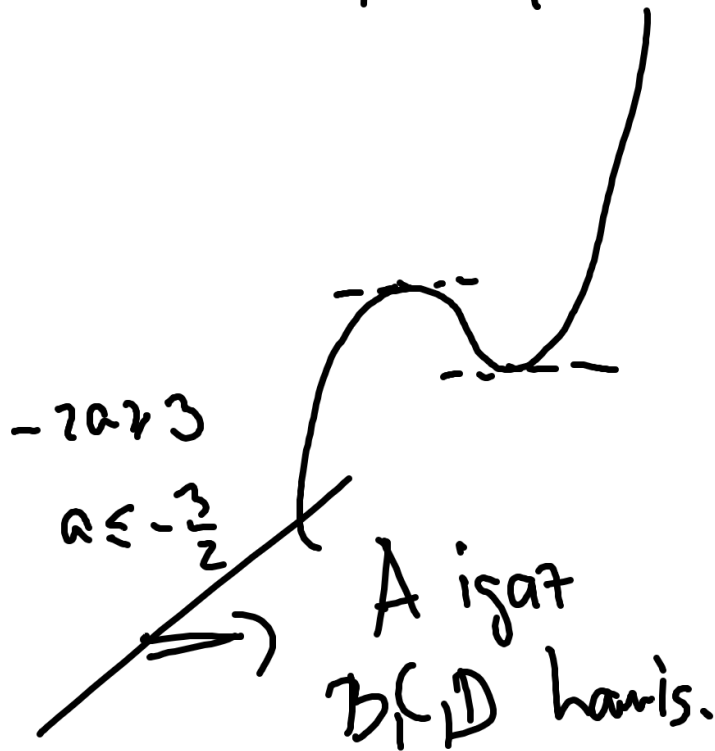
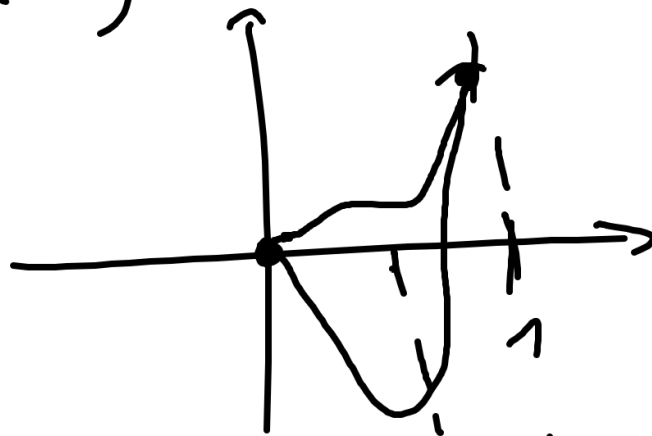
$$x \in \{0, -\frac{2a}{3}\}$$

$$-\frac{2a}{3} \leq 0$$

$$\Rightarrow a \geq 0$$

$$-\frac{2a}{3} \geq 1$$

$$a \leq -\frac{3}{2}$$



25

$$f(x) = \begin{cases} \cos x - 2 & -\frac{\pi}{2} \leq x \leq 0 \\ \frac{\sin x}{x} & 0 < x \leq \frac{\pi}{2} \end{cases}$$

$$\lim_{\substack{x \rightarrow 0 \\ x < 0}} f(x) = -1$$

$$\lim_{\substack{x \rightarrow 0 \\ x > 0}} f(x) = 1$$

$$\lim_{x \rightarrow 0} |f(x)| = 1 = |f(0)|$$

$\Downarrow$
 $|f|$ folytonos
0-ban
 $\Rightarrow$ A igaz

D igaz

B hamis

C igaz

$$\begin{cases} \cos x = 2 & \text{mindig nagyobb} \\ \frac{\sin x}{x} < 0 & \text{mind nagyobb } (0, \frac{\pi}{2}] \end{cases}$$

26.

$$f(x) = 2^{x^2}$$

$$\lim_{x \rightarrow \infty} f(-x) = \lim_{x \rightarrow -\infty} 2^{x^2} = +\infty \Rightarrow \text{A hant}$$

$$1 \leq x_1 < x_2 \Rightarrow x_1^2 < x_2^2 \Rightarrow 2^{x_1^2} < 2^{x_2^2}$$

tehát f emög. növ. $[1, \infty)$ -n.

B igaz

~~$f'(x) = 2^x \cdot \ln 2 \cdot 2^x \geq 2^x$~~

$x > 0$!

$$2 \ln 2 \geq 1$$
$$\ln 4 \geq 1$$

igaz $\forall x \geq 0$

C hamis

$x < 0$ -ra nem igaz

$$\lim_{x \rightarrow \infty} \frac{f'(x)}{(2x+1)f(x)} = \lim_{x \rightarrow \infty} \frac{2x \cdot \ln 2 \cdot 2^x}{(2x+1)2^x} = \ln 2 \neq 1$$

$\Rightarrow$ D harris

(27)

$$\lim_{x \rightarrow \pm \infty} \frac{f(x)}{x} = 0 \Rightarrow \text{A igaz}$$



f -polyt $\Rightarrow$ B igaz

C harris

$$f(x) = x^{2021}$$

$$\lim_{x \rightarrow \infty} (f(x) - x^{2021}) = \infty$$

$\Rightarrow$ D igaz

28

$$I = \int_0^1 x \ln(1+x) dx =$$

$$= \int_0^1 \left(\frac{x^2}{2}\right)' \ln(1+x) dx = \frac{x^2}{2} \ln(1+x) \Big|_0^1 - \int_0^1 \frac{x^2}{2(1+x)} dx$$

$$= \frac{\ln 2}{2} - \frac{1}{2} \int_0^1 \frac{x^2 - 1}{x+1} + \frac{1}{x+1} dx$$

$$= \frac{\ln 2}{2} - \frac{1}{2} \left(\frac{x^2}{2} - x \right) \Big|_0^1 = \frac{1}{2} \ln(x+1) \Big|_0^1$$

$$= \frac{\ln 2}{2} + \frac{1}{4} - \frac{\ln 2}{2} = \frac{1}{4} \in \mathbb{Q} \Rightarrow \text{Bigat}$$

$$\frac{1}{2} - 1 = -\frac{1}{2}$$

A hamis

$$0 < \frac{1}{4} < \ln 2$$

$$1 < \ln 2^4 \text{ igaz.} \Rightarrow \text{C igaz}$$

$$\frac{1}{2} \ln \frac{3}{2} < \frac{1}{4} < \frac{1}{2} \ln 2$$

$$\ln \frac{3}{2} < \frac{1}{2} < \ln 2 \quad / \cdot 2$$

$$\ln \frac{9}{4} < 1 < \ln 4$$

$\ln e$

$$\frac{9}{4} < e < 4$$

$$2,25 < e \text{ igaz.}$$

D helyes

(29)

$$\sqrt{3-x} - x = a$$

$$\frac{x \leq 3}{3-x \geq 0}$$

$$f: (-\infty, 3] \rightarrow \mathbb{R} \quad f(x) = \sqrt{3-x} - x$$

$$f'(x) = \frac{-1}{2\sqrt{3-x}} - 1 < 0 \Rightarrow f \text{ nig cötkenő}$$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \sqrt{3-x} - x = +\infty$$

$$f(3) = -3$$

$$\Rightarrow \text{Im} f = [-3, \infty)$$

$$f(x) = a \text{ - nat } \exists$$

* havis

$$\text{meg. } (=) \text{ a } \in [-3, \infty)$$

31 am is

$$C \text{ igaz } D \text{ igaz.}$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin x - x \cos x}{x^2 + \sin^2 x} dx = - \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{x \cos x - \sin x}{x^2 \left(1 + \left(\frac{\sin x}{x}\right)^2\right)} dt$$

$$\left(\frac{\sin x}{x}\right)' = \frac{\cos x \cdot x - \sin x}{x^2}$$

$$\frac{\sin x}{x} = t \quad \frac{\cos x \cdot x - \sin x}{x^2} dx = dt$$

$$\left(\arctan\left(\frac{\sin x}{x}\right)\right)' = \frac{1}{1 + \left(\frac{\sin x}{x}\right)^2} \cdot \left(\frac{\sin x}{x}\right)'$$

$$= - \arctan \frac{\sin x}{x} \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} = - \arctan \frac{\frac{3\sqrt{3}}{2\pi}}{\frac{3}{2\pi}} + \arctan$$

$$\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{3\sqrt{3}}{2\pi}$$

$$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{1 + \left(\frac{x}{\sin x}\right)^2} \cdot \left(\frac{x}{\sin x}\right)' dx$$

$$= \operatorname{arctg} \frac{x}{\sin x} \Bigg|_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \operatorname{arctg} \frac{\frac{\pi}{3}}{\frac{\sqrt{3}}{2}} - \operatorname{arctg} \frac{\frac{\pi}{4}}{\frac{\sqrt{2}}{2}}$$

$$= \operatorname{arctg} \frac{2\pi\sqrt{3}}{3} - \operatorname{arctg} \frac{\pi\sqrt{2}}{2}$$

B helyes
A, C, D
hamis

$$\left(\frac{x}{\sin x}\right)' = \frac{\sin x - x \cdot \cos x}{\sin^2 x}$$

$$\frac{\frac{\pi}{4} \cdot \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

$$\frac{2\pi}{4\sqrt{2}}$$

$$\frac{\pi}{2\sqrt{2}}$$

$$\frac{2\pi\sqrt{2}}{4}$$