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28 11 2020 Lect. dr. Stefan Berinde

# Siruri

1. (*admitere 2016*) Fie functia $f : \mathbb{R} \to \mathbb{R}$, $f(x) = |x|\sqrt[3]{1-x^2}$

   a) Aratati ca $f$ este marginita superior pe $\mathbb{R}$

   b) Calculati $\lim\limits_{n\to\infty} \int_{-1}^{1} x^{2n} f(x)\mathrm{d}x$

2. (*admitere 2019*) Calculati limitele sirurilor

$$x_n = \frac{1}{n}\sum_{k=1}^{n}\frac{k}{\sqrt{n^2+k}}, \quad y_n = \frac{1}{n}\sum_{k=1}^{n}\frac{k}{\sqrt{n^2+k^2}}, \quad n \in \mathbb{N}^{\star}$$

3. (*admitere 2015*) Fie functia $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \mathrm{e}^{x-1}$

   a) Aratati ca $f(x) > x, \forall x \in \mathbb{R} \setminus \{1\}$

   b) Definim sirul $(x_n)_{n\geq 1}$ prin $x_1 = 2$, $x_{n+1} = f(x_n), \forall n \geq 1$. Aratati ca sirul este strict monoton si calculati limita sa. Ce se intampla daca luam $x_1 = 1/2$ ?

4. Calculati urmatoarele limite

   a) $\lim\limits_{n\to\infty} \frac{7^n}{n^7}$

   b) $\lim\limits_{n\to\infty} \sqrt{n}\left(7^{\sqrt{n+1}-\sqrt{n}} - 1\right)$

   c) $\lim\limits_{n\to\infty} n\left(\sqrt[7]{\frac{n+1}{n}} - 1\right)$

   d) $\lim\limits_{n\to\infty} \sqrt{n}\left[(\sqrt{n}+1-\sqrt{n+1})^7 - 1\right]$

   e) $\lim\limits_{n\to\infty} \frac{7^n}{(7n)!}$

   f) $\lim\limits_{n\to\infty} \frac{1\cdot 3\cdot 5\cdot \ldots \cdot (2n-1)}{2\cdot 4\cdot 6\cdot \ldots \cdot (2n)}$