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# Exponential growth of solutions to system of nonlinear Klein-Gordon with degenerate damping and source terms

Fares Yazid and Djamel Ouchenane

**Abstract.** In this paper, we consider a system of nonlinear viscoelastic wave equations with degenerate damping and source terms.We proved a global nonexistence result of the solution prove the blow up of solutions in finite by concavity method.

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## 1. Introduction

In this paper, we propose this system of viscoelastic wave equations with degenerate damping and strong nonlinear source terms:

$$\begin{cases} u_{tt} - \Delta u + m_1 u^2 + \int_0^t g(t-s)\Delta u(x,s)\,ds + \left(a|u|^k + b|v|^l\right)|u_t|^{m-1}u_t &= f_1(u,v), \\ v_{tt} - \Delta v + m_2 v^2 + \int_0^t h(t-s)\Delta v(x,s)\,ds + \left(c|v|^\theta + d|u|^\varrho\right)|v_t|^{r-1}v_t &= f_2(u,v), \end{cases} \tag{1.1}$$

where $m, r > 0$, $k, l, \theta, \varrho \geq 1$ and the two functions $f_1(u,v)$ and $f_2(u,v)$ are given by

$$\begin{aligned} f_1(u,v) &= a_1|u+v|^{2(\rho+1)}(u+v) + b_1|u|^\rho u|v|^{(\rho+2)} \\ f_2(u,v) &= a_1|u+v|^{2(\rho+1)}(u+v) + b_1|u|^{(\rho+2)}|v|^\rho v,\ a_1, b_1 > 0, \end{aligned} \tag{1.2}$$

where $\rho > -1$. In (1.1), $u = u(x,t)$, $v = v(x,t)$, where $x \in \Omega$ is a bounded domain of $\mathbb{R}^N$, $(N \geq 1)$ with a smooth boundary $\partial\Omega$ and $t > 0$, $a, b, c, d, m_1, m_2 > 0$.

To above system (1.1) , we add the initial conditions given by

$$(u(0), v(0)) = (u_0, v_0), (u_t(0), v_t(0)) = (u_1, v_1),\ x \in \Omega \tag{1.3}$$

and boundary conditions given by

$$u(x) = v(x) = 0,\ x \in \partial\Omega. \tag{1.4}$$

This kind of problems is generally faced in viscoelasticity and Dafermos was the first to study it in [5], where the general decay was spoken about. In the last decades the problem that was linked to (1.1) draw a lot of attention , and many results appeared on the existence and long time behavior of solutions. See in this directions ([1, 2, 3, 4, 6, 7, 8, 9]) and references therein.

In the absence of viscoelastic term, some special cases of the single wave equations with nonlinear damping and nonlinear source terms in the form

$$u_{tt} - \Delta u + a|u_t|^{m-1}u_t = b|u|^{p-1}u \tag{1.5}$$

With nonlinear damping and source terms are faced in the quantum-field and used to describe the movement of charged electromagnetic fields. Equation (1.5) equiped with initial and limit conditions of Dirichlet type has been extensively studied and many authors studied existence,blow up and asymptotic attitude of smooth and feable solutions. Many authors have examined the single wave equations in the presence of variuos mechanisms of dissipation, damping and non-linear sources and also the results which are related to existence, nonexistence and asymptotic attitude of solutions have been formed.A lot of results can be found in literature; See ([10, 11, 12, 17, 21, 18, 24, 27, 29]) and references therein.

In the work [18], authors considered the nonlinear viscoelastic system:

$$\begin{cases} u_{tt} - \Delta u + \int_0^t g(t-s)\Delta u(x,s)ds + |u_t|^{m-1} u_t = f_1(u,v), \\ v_{tt} - \Delta v + \int_0^t h(t-s)\Delta v(x,s)ds + |v_t|^{r-1} v_t = f_2(u,v), \end{cases} ,\ x \in \Omega, t > 0 \tag{1.6}$$

where

$$\begin{aligned} f_1(u,v) &= a|u+v|^{2(\rho+1)}(u+v) + b|u|^{\rho}u|v|^{(\rho+2)} \\ f_2(u,v) &= a|u+v|^{2(\rho+1)}(u+v) + b|u|^{(\rho+2)}|v|^{\rho}v, \end{aligned} \tag{1.7}$$

The global nonexistence theorem for some solutions with positive energy was proved and they used a metho applied in [24].

The work by Said-Houari, Messaoudi and Guesmia, in [25] wich studied the nonlinear viscoelastic system in (1.6)is related to the study of the decay of solutions equations.And under some restrictions practised on the nonlinearity of damping and source terms, they proved that for some class of relaxation functions and some restrictions which are practised on the initial data, the rate of decay of relaxation functions affects the rate of decay of total energy.

In this paper, we consider system (1.1) and proved a global nonexistence result of the solution. We extended to result in [18] to the more general problem (1.1).

## 2. Preliminaries

In this section, we present some notations and some specialized lemmas to be utilised all throughout this part.

We assume that the relaxation functions $g,\ h:\mathbb{R}^{+}\longrightarrow\mathbb{R}^{+}$ are of class $C^{1}$ and nonincreasing differentiable satisfying:

$$\left\{\begin{array}{ll} 1-\int_{0}^{\infty}g(s)ds=l'>0,\quad g(t)\geq 0,\quad g'(t)\leq 0, & \\ & t\geq 0. \\ 1-\int_{0}^{\infty}h(s)ds=k'>0,\quad h(t)\geq 0,\quad h'(t)\leq 0, & \end{array}\right. \tag{2.1}$$

We introduce the "*modified*" energy functional $E$ associated to our system:

$$2E\left(t\right)=\|u_{t}\|_{2}^{2}+\|v_{t}\|_{2}^{2}+2\left(m_{1}^{2}\|u\|_{2}^{2}+m_{2}^{2}\|v\|_{2}^{2}\right)+J\left(u,v\right)-2\int_{\Omega}F\left(u,v\right)dx, \tag{2.2}$$

where $F(u,v)$ is defined

**Lemma 2.1.** *There exists a function $F\left(u,v\right)$ to such an extent that, for all $(u,\ v)\in\mathbb{R}^{2}$,*

$$\begin{aligned} F\left(u,v\right) &= \frac{1}{2\left(\rho+2\right)}\left[uf_{1}\left(u,v\right)+vf_{2}\left(u,v\right)\right], \\ &= \frac{1}{2\left(\rho+2\right)}\left[a_{5}\left|u+v\right|^{2(\rho+2)}+2a_{6}\left|uv\right|^{\rho+2}\right]\geq 0, \end{aligned}$$

*where in Lemma* 2.1

$$\frac{\partial F}{\partial u}=f_{1}\left(u,v\right),\frac{\partial F}{\partial v}=f_{2}\left(u,v\right).$$

*and*

$$\begin{aligned} J\left(u,v\right) &= \left(1-\int_{0}^{t}g\left(s\right)ds\right)\|\nabla u\|_{2}^{2}+\left(1-\int_{0}^{t}h\left(s\right)ds\right)\|\nabla v\|_{2}^{2} \\ &\quad +\left(g\circ\nabla u\right)+\left(h\circ\nabla v\right), \end{aligned} \tag{2.3}$$

*where*

$$\left\{\begin{array}{l} \left(g\circ u\right)\left(t\right)=\int_{0}^{t}g\left(t-\tau\right)\left\|u\left(t\right)-u\left(\tau\right)\right\|_{2}^{2}d\tau, \\ \left(h\circ v\right)\left(t\right)=\int_{0}^{t}h\left(t-\tau\right)\left\|v\left(t\right)-v\left(\tau\right)\right\|_{2}^{2}d\tau. \end{array}\right. \tag{2.4}$$

*As previously, we suppose that $\rho$ satisfies*

$$\left\{\begin{array}{lll} -1<\rho, & \text{if} & N=1,2, \\ -1<\rho\leq\frac{4-N}{N-2} & \text{if} & N\geq 3. \end{array}\right. \tag{2.5}$$

**Lemma 2.2.** [24] *There exist two positive constants $c_0$ and $c_1$ with the end goal that*

$$\frac{c_{0}}{2\left(\rho+2\right)}\left(\left|u\right|^{2(\rho+2)}+\left|v\right|^{2(\rho+2)}\right)\leq F\left(u,v\right)\leq\frac{c_{1}}{2\left(\rho+2\right)}\left(\left|u\right|^{2(\rho+2)}+\left|v\right|^{2(\rho+2)}\right).$$

**Lemma 2.3.** *Assume that (2.5) holds. At that point there exists $\eta > 0$ with the end goal that for any $(u,v) \in H_0^1(\Omega) \times H_0^1(\Omega)$ the inequality*

$$2(\rho+2) \int_\Omega F(u,v)\, dx \leq \eta \left(\|\nabla u\|_2^2 + \|\nabla v\|_2^2\right)^{\rho+2} \tag{2.6}$$

*holds.*

**Lemma 2.4.** *Let $\nu > 0$, be a real positive number and let $L(t)$ be a solution of the ordinary diffenrential inequality*

$$\frac{dL(t)}{dt} \geq \xi L^{1+\nu}(t) \tag{2.7}$$

*defined in $[0,\infty)$.*

*If $L(0) > 0$, then the solution does not exist for $t \geq L(0)^{-\nu} \xi^{-\nu} \nu^{-1}$.*

*Proof.* We beging by simple integration of (2.7), we have

$$L^{-\nu}(0) - L^{-\nu}(t) \geq \xi \nu t.$$

Accordingly, we obtain the following estimate:

$$L^{\nu}(t) \geq \left[L^{-\nu}(0) - \xi \nu t\right]^{-1}. \tag{2.8}$$

Anmistakably the right-hand side of (2.8) is unbounded for

$$\xi \nu t = L^{-\nu}(0).$$

□

The proof is completed

## 3. Blow up result

**Lemma 3.1.** *Assume that (2.5) holds. Let $(u,v)$ be the solution of the system (1.1)–(1.4) then the energy functional is a non-increasing function; that is, for all $t \geq 0$,*

$$\begin{aligned} E'(t) &= -\int_\Omega \left(|u(t)|^k + |v(t)|^l\right) |u_t(t)|^{m+1}\, dx - \int_\Omega \left(|v(t)|^\theta + |u(t)|^\varrho\right) |v_t(t)|^{r+1}\, dx \\ &\quad + \frac{1}{2}(g' \circ \nabla u) + \frac{1}{2}(h' \circ \nabla v) - \frac{1}{2} g(s) \|\nabla u\|_2^2 - \frac{1}{2} h(s) \|\nabla v\|_2^2 . \end{aligned} \tag{3.1}$$

**Lemma 3.2.** *Suppose that (2.5) holds. Let $(u,v)$ be the solution of the system (1.1)-(1.4), then the energy functional is a non-increasing function; that is, for all $t > 0$,*

$$\frac{dE(t)}{dt} = -\int_\Omega \left(|u(t)|^k + |v(t)|^l\right) |u_t(t)|^{m+1}\, dx - \int_\Omega \left(|v(t)|^\theta + |u(t)|^\vartheta\right) |v_t(t)|^{r+1}\, dx \tag{3.2}$$

The proof of Lemma 3.1 can be done as we explained previously after the statement of Lemma 3.2. We omit the details.

Our main result reads as follows

**Theorem 3.3.** *Suppose that (2.5) holds. Assume further that*

$$\rho > \max\left(\frac{k+m-3}{2}, \frac{l+m-3}{2}, \frac{\theta+r-3}{2}, \frac{\varrho+r-3}{2}\right), \tag{3.3}$$

*and that there exists p such that* $2 < p < 2(\rho+2)$, *for which*

$$\max\left(\int_0^\infty g(s)\,ds, \int_0^\infty h(s)\,ds\right) < \frac{(p/2)-1}{(p/2)-1+1/(2p)}, \tag{3.4}$$

*holds. Then any solution of problem (1.1)–(1.4), with initial data satisfying*

$$\|\nabla u_0\|_2^2+\|\nabla v_0\|_2^2+m_1^2\|u_0\|_2^2+m_2^2\|v_0\|_2^2 > \alpha_1^2, \quad \text{and} \quad E(0) < E_2 \tag{3.5}$$

*blows up in finite time, where the constants* $\alpha_1$ *and* $E_2$ *are defined in (3.6).*

We take $a=b=c=d=1$, $a_1=b_1=1$ for convenience. We introduce the following:

$$\begin{aligned} B &= \eta^{\frac{1}{2(\rho+2)}}, \qquad \alpha_1 = B^{-\frac{\rho+2}{\rho+1}}, \qquad E_1 = \left(\frac{1}{2}-\frac{1}{2(\rho+2)}\right)\alpha_1^2, \\ E_2 &= \left(\frac{1}{p}-\frac{1}{2(\rho+2)}\right)\alpha_1^2, \end{aligned} \tag{3.6}$$

where $\eta$ is the optimal constant in (2.6).

**Lemma 3.4.** [24] *Suppose that (2.5), (3.3) and (3.4) hold. Let* $(u,v)$ *be a solution of (1.1)–(1.4). Assume further that* $E(0) < E_2$ *and*

$$\|\nabla u_0\|_2^2+\|\nabla v_0\|_2^2+m_1^2\|u_0\|_2^2+m_2^2\|v_0\|_2^2 > \alpha_1^2. \tag{3.7}$$

*Then there exists a constant* $\alpha_2 > \alpha_1$ *such that*

$$J(t) > \alpha_2^2, \tag{3.8}$$

*and*

$$2(\rho+2)\int_\Omega F(u,v)\,dx \geq (B\alpha_2)^{2(\rho+2)}, \ \forall t \geq 0. \tag{3.9}$$

*Proof of Theorem* 3.3. The proof of this theorem is similar to the one given in [7] with the necessary modification imposed by the nature of our problem. We assume that the solution exists forever and we get into a contradiction. For this reason, we have set

$$H(t) = E_2 - E(t). \tag{3.10}$$

By utilising the definition of $H(t)$, we obtain

$$\begin{aligned} H'(t) &= -E'(t) \\ &= \int_\Omega \left(|u(t)|^k+|v(t)|^l\right)|u_t(t)|^{m+1}dx + \int_\Omega \left(|v(t)|^\theta+|u(t)|^\varrho\right)|v_t(t)|^{r+1}dx \\ &\quad -\frac{1}{2}\left(g'\circ\nabla u\right)-\frac{1}{2}\left(h'\circ\nabla v\right)+\frac{1}{2}g(s)\|\nabla u\|_2^2+\frac{1}{2}h(s)\|\nabla v\|_2^2 \\ &\geq 0, \forall t \geq 0. \end{aligned} \tag{3.11}$$

Therefore, because $E'$ is absolutely continuous

$$H(0) = E_2 - E(0) > 0. \tag{3.12}$$

Then,

$$\begin{aligned} 0 &< H(0) \leq H(t) \\ &= E_2 - \frac{1}{2}\left(\|u_t\|_2^2 + \|v_t\|_2^2 + m_1^2\|u\|_2^2 + m_2^2\|v\|_2^2\right) - \frac{J(t)}{2} \\ &\quad + \frac{1}{2(\rho+2)}\left[\|u+v\|_{2(\rho+2)}^{2(\rho+2)} + 2\|uv\|_{\rho+2}^{\rho+2}\right]. \end{aligned} \tag{3.13}$$

Note that from (2.1) and (3.8), we get

$$\begin{aligned} E_2 - \frac{1}{2}\left(\|u_t\|_2^2 + \|v_t\|_2^2\right) - \frac{J(t)}{2} &< E_2 - \frac{1}{2}\alpha_2^2 \\ &< E_2 - \frac{1}{2}\alpha_1^2 \\ &< E_1 - \frac{1}{2}\alpha_1^2 \\ &= -\frac{1}{2(\rho+2)}\alpha_1^2 < 0, \quad \forall t \geq 0. \end{aligned} \tag{3.14}$$

Thus, by using (3.14) and Lemma 2.2, we get

$$\begin{aligned} 0 &< H(0) \leq H(t) \leq \frac{1}{2(\rho+2)}\left[\|u+v\|_{2(\rho+2)}^{2(\rho+2)} + 2\|uv\|_{\rho+2}^{\rho+2}\right] \\ &\leq \frac{c_1}{2(\rho+2)}\left(\|u\|_{2(\rho+2)}^{2(\rho+2)} + \|v\|_{2(\rho+2)}^{2(\rho+2)}\right), \forall t \geq 0. \end{aligned} \tag{3.15}$$

At that point the function $M$ is defined by

$$M(t) = \frac{1}{2}\int_\Omega \left(u^2 + v^2\right)(x,t)\,dx. \tag{3.16}$$

We present

$$L(t) = H^{1-\sigma}(t) + \varepsilon M'(t), \tag{3.17}$$

for $\varepsilon$ small to be chosen later and

$$\begin{aligned} 0 &< \sigma \leq \min\left\{\frac{1}{2}, \frac{2\rho+3-(k+m)}{2(m+1)(\rho+2)}, \frac{2\rho+3-(l+m)}{2(m+1)(\rho+2)},\right. \\ &\quad \left.\frac{2\rho+3-(\varrho+r)}{2(r+1)(\rho+2)}, \frac{2\rho+3-(\theta+r)}{2(r+1)(\rho+2)}, \frac{2\rho+2}{4(\rho+2)}\right\}. \end{aligned} \tag{3.18}$$

By differentiation of (3.17) with respect to time and using (1.1), we get

$$\begin{aligned} L'(t) &= (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon \left(\|u_t\|_2^2 + \|v_t\|_2^2\right) \\ &-\varepsilon \left(\|\nabla u\|_2^2 + \|\nabla v\|_2^2\right) \\ &-\varepsilon \int_\Omega u \left(|u(t)|^k + |v(t)|^l\right) |u_t|^{m-1} u_t dx \\ &-\varepsilon \int_\Omega v \left(|v(t)|^\theta + |u(t)|^\varrho\right) |v_t|^{r-1} v_t dx \\ &+\varepsilon \int_\Omega \left(u f_1(u, v) + v f_2(u, v)\right) dx \\ &+\varepsilon \int_\Omega \nabla u(t) \int_0^t g(t-s) \nabla u(\tau) dx ds \\ &+\varepsilon \int_\Omega \nabla v(t) \int_0^t h(t-s) \nabla v(\tau) dx ds. \end{aligned} \tag{3.19}$$

Then,

$$\begin{aligned} L'(t) &= (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon \left(\|u_t\|_2^2 + \|v_t\|_2^2 + m_1^2 \|u\|_2^2 + m_2^2 \|v\|_2^2\right) \\ &-\varepsilon \left(\|\nabla u\|_2^2 + \|\nabla v\|_2^2\right) \\ &-\varepsilon \int_\Omega u \left(|u(t)|^k + |v(t)|^l\right) |u_t|^{m-1} u_t dx \\ &-\varepsilon \int_\Omega v \left(|v(t)|^\theta + |u(t)|^\varrho\right) |v_t|^{r-1} v_t dx \\ &+\varepsilon \left(\|u+v\|_{2(\rho+2)}^{2(\rho+2)} + 2\|uv\|_{\rho+2}^{\rho+2}\right) \\ &+\varepsilon \left(\int_0^t g(s) ds\right) \|\nabla u\|_2^2 + \left(\int_0^t h(s) ds\right) \|\nabla v\|_2^2 \\ &+\varepsilon \int_0^t g(t-s) \int_\Omega \nabla u(t) . [\nabla u(\tau) - \nabla u(t)] dx ds \\ &+\varepsilon \int_0^t h(t-s) \int_\Omega \nabla v(t) . [\nabla v(\tau) - \nabla v(t)] dx ds. \end{aligned} \tag{3.20}$$

By using Cauchy-Schwarz and Young's inequalities, we obtain the following estimate

$$\begin{aligned} & \int_0^t g(t-s) \int_\Omega \nabla u(t) . [\nabla u(\tau) - \nabla u(t)] dx ds \\ \leq & \int_0^t g(t-s) \|\nabla u\|_2 \|\nabla u(\tau) - \nabla u(t)\|_2 d\tau \\ \leq & \lambda (g \circ \nabla u) + \frac{1}{4\lambda} \left(\int_0^t g(s) ds\right) \|\nabla u\|_2^2, \qquad \lambda > 0 \end{aligned} \tag{3.21}$$

and

$$\begin{aligned} & \int_0^t h(t-s) \int_\Omega \nabla v(t) \,.\, [\nabla v(\tau) - \nabla v(t)]\, dxds \\ \leq \; & \lambda (h \circ \nabla v) + \frac{1}{4\lambda} \left( \int_0^t h(s)\, ds \right) \|\nabla v\|_2^2, \qquad \lambda > 0. \end{aligned} \tag{3.22}$$

Adding and inserting $pE(t)$ and using the definition of $H(t)$ and $E_2$ lead to

$$\begin{aligned} L'(t) \geq \; & (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon \left(1 + \frac{p}{2}\right) \left( \|u_t\|_2^2 + \|v_t\|_2^2 + m_1^2 \|u\|_2^2 + m_2^2 \|v\|_2^2 \right) \\ & + \varepsilon \left( \frac{p}{2} - \lambda \right) [(g \circ \nabla u) + (h \circ \nabla v)] + p\varepsilon H(t) - p\varepsilon E_2 \\ & - \varepsilon \int_\Omega u \left( |u(t)|^k + |v(t)|^l \right) |u_t|^{m-1} u_t dx \\ & - \varepsilon \int_\Omega v \left( |v(t)|^\theta + |u(t)|^\varrho \right) |v_t|^{r-1} v_t dx \\ & + \varepsilon \left( 1 - \frac{p}{2(\rho+2)} \right) \left( \|u+v\|_{2(\rho+2)}^{2(\rho+2)} + 2\|uv\|_{\rho+2}^{\rho+2} \right) \\ & + \varepsilon \left[ \left( \frac{p}{2} - 1 \right) - \left( \frac{p}{2} - 1 + \frac{1}{4\lambda} \right) \int_0^\infty g(s)\, ds \right] \|\nabla u\|_2^2 \\ & + \varepsilon \left[ \left( \frac{p}{2} - 1 \right) - \left( \frac{p}{2} - 1 + \frac{1}{4\lambda} \right) \int_0^\infty h(s)\, ds \right] \|\nabla v\|_2^2, \end{aligned} \tag{3.23}$$

for some $\lambda$ such that

$$a_1 = \frac{p}{2} - \lambda > 0,$$

and

$$a_2 = \left[ \left( \frac{p}{2} - 1 \right) - \left( \frac{p}{2} - 1 + \frac{1}{4\lambda} \right) \max \left( \int_0^\infty g(s)\, ds, \; \int_0^\infty h(s)\, ds \right) \right] > 0.$$

Then, (3.23) can be estimated as follows

$$\begin{aligned} L'(t) \geq \; & (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon \left(1 + \frac{p}{2}\right) \left( \|u_t\|_2^2 + \|v_t\|_2^2 \right) \\ & + \varepsilon a_1 [(g \circ \nabla u) + (h \circ \nabla v)] + p\varepsilon H(t) - p\varepsilon E_2 \\ & - \varepsilon \int_\Omega u \left( |u(t)|^k + |v(t)|^l \right) |u_t|^{m-1} u_t dx \\ & - \varepsilon \int_\Omega v \left( |v(t)|^\theta + |u(t)|^\varrho \right) |v_t|^{r-1} v_t dx \\ & + \varepsilon \left( 1 - \frac{p}{2(\rho+2)} \right) \left( \|u+v\|_{2(\rho+2)}^{2(\rho+2)} + 2\|uv\|_{\rho+2}^{\rho+2} \right) \\ & + \varepsilon a_2 \left( \|\nabla u\|_2^2 + \|\nabla v\|_2^2 \right). \end{aligned} \tag{3.24}$$

By taking $c_3 = 1 - \frac{p}{\rho+2} - 2E_2 (B\alpha_2)^{-2(\rho+2)} > 0$, since $\alpha_2 > B^{-\frac{2(\rho+2)}{\rho+1}}$.
Consequently, (3.24) takes the form

$$\begin{aligned} L'(t) \geq\ & (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon\left(1+\frac{p}{2}\right)\left(\|u_t\|_2^2 + \|v_t\|_2^2 + m_1^2\|u\|_2^2 + m_2^2\|v\|_2^2\right) \\ & + \varepsilon a_1 [(g\circ\nabla u) + (h\circ\nabla v)] \\ & + \varepsilon a_2 \left(\|\nabla u\|_2^2 + \|\nabla v\|_2^2\right) + p\varepsilon H(t) \\ & + \varepsilon c_3 \left(\|u+v\|_{2(\rho+2)}^{2(\rho+2)} + 2\|uv\|_{\rho+2}^{\rho+2}\right) \\ & - \varepsilon \int_\Omega u\left(|u(t)|^k + |v(t)|^l\right)|u_t|^{m-1} u_t dx \\ & - \varepsilon \int_\Omega v\left(|v(t)|^\theta + |u(t)|^\varrho\right)|v_t|^{r-1} v_t dx. \end{aligned} \tag{3.25}$$

By using Young inequality, we have

$$XY \leq \frac{\delta^\alpha X^\alpha}{\alpha} + \frac{\delta^{-\beta} Y^\beta}{\beta}, \tag{3.26}$$

where $X, Y \geq 0$, $\delta > 0$, and $\alpha, \beta > 0$ such that $1/\alpha + 1/\beta = 1$, we obtain

$$\left|u |u_t|^{m-1} u_t\right| \leq \frac{\delta_1^{m+1}}{m+1}|u|^{m+1} + \frac{m}{m+1}\delta_1^{-(m+1)/m}|u_t|^{m+1}, \forall delta_1 \geq 0 \tag{3.27}$$

and

$$\begin{aligned} \int_\Omega \left(|u(t)|^k + |v(t)|^l\right)\left|u|u_t|^{m-1}u_t\right| dx \leq \frac{\delta_1^{m+1}}{m+1}\int_\Omega \left(|u(t)|^k + |v(t)|^l\right)|u|^{m+1} dx \\ + \frac{m}{m+1}\delta_1^{-(m+1)/m}\int_\Omega \left(|u(t)|^k + |v(t)|^l\right)|u_t|^{m+1} dx. \end{aligned} \tag{3.28}$$

Similarly, for any $\delta_2 > 0$,

$$\left|v|v_t|^{r-1} v_t\right| \leq \frac{\delta_2^{r+1}}{r+1}|v|^{r+1} + \frac{r}{r+1}\delta_2^{-(r+1)/r}|v_t|^{r+1}, \tag{3.29}$$

which gives

$$\begin{aligned} \int_\Omega \left(|v(t)|^\theta + |u(t)|^\varrho\right)\left|v|v_t|^{r-1}v_t\right| dx \leq \frac{\delta_2^{r+1}}{r+1}\int_\Omega \left(|v(t)|^\theta + |u(t)|^\varrho\right)|v|^{r+1} dx \\ + \frac{r}{r+1}\delta_2^{-(r+1)/r}\int_\Omega \left(|v(t)|^\theta + |u(t)|^\varrho\right)|v_t|^{r+1} dx. \end{aligned} \tag{3.30}$$

Then, we obtain

$$\begin{aligned} L'(t) \geq\ & (1-\sigma) H^{-\sigma}(t) H^{'}(t)+\varepsilon\left(1+\frac{p}{2}\right)\left(\|u_t\|_2^2+\|v_t\|_2^2+m_1^2\|u\|_2^2+m_2^2\|v\|_2^2\right) \\ & +\varepsilon a_1\left[(g\circ\nabla u)+(h\circ\nabla v)\right] \\ & +\varepsilon a_2\left(\|\nabla u\|_2^2+\|\nabla v\|_2^2\right)+p\varepsilon H(t) \\ & +\varepsilon c_3\left(\|u+v\|_{2(\rho+2)}^{2(\rho+2)}+2\|uv\|_{\rho+2}^{\rho+2}\right) \\ & -\varepsilon\frac{\delta_1^{m+1}}{m+1}\int_\Omega\left(|u(t)|^k+|v(t)|^l\right)|u|^{m+1}dx \\ & -\varepsilon\frac{m}{m+1}\delta_1^{-\frac{(m+1)}{m}}\int_\Omega\left(|u(t)|^k+|v(t)|^l\right)|u_t|^{m+1}dx \\ & -\varepsilon\frac{\delta_2^{r+1}}{r+1}\int_\Omega\left(|v(t)|^\theta+|u(t)|^\varrho\right)|v|^{r+1}dx \\ & -\varepsilon\frac{r}{r+1}\delta_2^{-\frac{(r+1)}{r}}\int_\Omega\left(|v(t)|^\theta+|u(t)|^\varrho\right)|v_t|^{r+1}dx. \end{aligned} \tag{3.31}$$

Let us choose $\delta_1$ and $\delta_2$ such that

$$\delta_1^{-\frac{(m+1)}{m}}=M_1H(t)^{-\sigma},\quad \delta_2^{-\frac{(r+1)}{r}}=M_2H(t)^{-\sigma}, \tag{3.32}$$

for $M_1$ and $M_2$ large constants to be fixed later. Thus, by using (3.32), we obtain

$$\begin{aligned} L'(t) \geq\ & ((1-\sigma)-M\varepsilon) H^{-\sigma}(t) H^{'}(t)+\varepsilon\left(1+\frac{p}{2}\right)\left(\|u_t\|_2^2+\|v_t\|_2^2+m_1^2\|u\|_2^2+m_2^2\|v \right. \\ & +\varepsilon a_1\left[(g\circ\nabla u)+(h\circ\nabla v)\right] \\ & +\varepsilon a_2\left(\|\nabla u\|_2^2+\|\nabla v\|_2^2\right)+p\varepsilon H(t) \\ & +\varepsilon c_3\left(\|u+v\|_{2(\rho+2)}^{2(\rho+2)}+2\|uv\|_{\rho+2}^{\rho+2}\right) \\ & -\varepsilon M_1^{-m}H^{\sigma m}(t)\int_\Omega\left(|u(t)|^k+|v(t)|^l\right)|u|^{m+1}dx \\ & -\varepsilon\frac{m}{m+1}\delta_1^{-\frac{(m+1)}{m}}\int_\Omega\left(|u(t)|^k+|v(t)|^l\right)|u_t|^{m+1}dx \\ & -\varepsilon M_2^{-r}H^{\sigma r}(t)\int_\Omega\left(|v(t)|^\theta+|u(t)|^\varrho\right)|v|^{r+1}dx \\ & -\varepsilon\frac{r}{r+1}\delta_2^{-\frac{(r+1)}{r}}\int_\Omega\left(|v(t)|^\theta+|u(t)|^\varrho\right)|v_t|^{r+1}dx, \end{aligned} \quad (3$$

where $M=m/(m+1)M_1+r/(r+1)M_2$. Therefore, we have

$$\int_\Omega\left(|u(t)|^k+|v(t)|^l\right)|u|^{m+1}dx=\|u\|_{k+m+1}^{k+m+1}+\int_\Omega|v|^l|u|^{m+1}dx \tag{3.34}$$

and

$$\int_\Omega\left(|v(t)|^\theta+|u(t)|^\varrho\right)|v|^{r+1}dx=\|v\|_{\theta+r+1}^{\theta+r+1}+\int_\Omega|u|^\varrho|v|^{r+1}dx. \tag{3.35}$$

Also by using Young's inequality, we obtain

$$\begin{aligned}\int_{\Omega}|v|^{l}|u|^{m+1} &\leq \frac{l}{l+m+1}\delta_1^{(l+m+1)/l}\|v\|_{l+m+1}^{l+m+1}+\frac{m+1}{l+m+1}\delta_1^{-(l+m+1)/(m+1)}\|u\|_{l+m+1}^{l+m+1}\\ \int_{\Omega}|u|^{\varrho}|v|^{r+1} &\leq \frac{\varrho}{\varrho+r+1}\delta_2^{(\varrho+r+1)/\varrho}\|u\|_{\varrho+r+1}^{\varrho+r+1}+\frac{r+1}{\varrho+r+1}\delta_2^{-(\varrho+r+1)/(r+1)}\|v\|_{\varrho+r+1}^{\varrho+r+1}.\end{aligned}$$

Therefore,

$$\begin{aligned}&H^{\sigma m}(t)\int_{\Omega}\left(|u(t)|^{k}+|v(t)|^{l}\right)|u|^{m+1}dx\\ =\;& H^{\sigma m}(t)\|u\|_{k+m+1}^{k+m+1}+\frac{l}{l+m+1}\delta_1^{(l+m+1)/l}H^{\sigma m}(t)\|v\|_{l+m+1}^{l+m+1}\\ &+\frac{m+1}{l+m+1}\delta_1^{-(l+m+1)/(m+1)}H^{\sigma m}(t)\|u\|_{l+m+1}^{l+m+1} \qquad (3.36)\end{aligned}$$

and

$$\begin{aligned}&H^{\sigma r}(t)\int_{\Omega}\left(|v(t)|^{\theta}+|u(t)|^{\varrho}\right)|v|^{r+1}dx\\ =\;& H^{\sigma r}(t)\|v\|_{\theta+r+1}^{\theta+r+1}+\frac{\varrho}{\varrho+r+1}\delta_2^{\frac{\varrho+r+1}{\varrho}}H^{\sigma r}(t)\|u\|_{\varrho+r+1}^{\varrho+r+1}\\ &+\frac{r+1}{\varrho+r+1}\delta_2^{-\frac{(\varrho+r+1)}{r+1}}H^{\sigma r}(t)\|v\|_{\varrho+r+1}^{\varrho+r+1}. \qquad (3.37)\end{aligned}$$

Since (3.3) holds, we get by using (3.18)

$$\begin{cases} H^{\sigma m}(t)\|u\|_{k+m+1}^{k+m+1}\leq c_5\left(\|u\|_{2(\rho+2)}^{2\sigma m(\rho+2)+k+m+1}+\|v\|_{2(\rho+2)}^{2\sigma m(\rho+2)}\|u\|_{k+m+1}^{k+m+1}\right),\\ H^{\sigma r}(t)\|v\|_{\theta+r+1}^{\theta+r+1}\leq c_6\left(\|v\|_{2(\rho+2)}^{2\sigma r(\rho+2)+\theta+r+1}+\|u\|_{2(\rho+2)}^{2\sigma r(\rho+2)}\|v\|_{\theta+r+1}^{\theta+r+1}\right).\end{cases} \qquad (3.38)$$

This implies

$$\begin{aligned}&\tfrac{l}{l+m+1}\delta_1^{\frac{l+m+1}{l}}H^{\sigma m}(t)\|v\|_{l+m+1}^{l+m+1}\\ &\leq c_7\tfrac{l}{l+m+1}\delta_1^{\frac{l+m+1}{l}}\left(\|v\|_{2(\rho+2)}^{2\sigma m(\rho+2)+l+m+1}+\|u\|_{2(\rho+2)}^{2\sigma m(\rho+2)}\|v\|_{l+m+1}^{l+m+1}\right)\end{aligned} \qquad (3.39)$$

and

$$\begin{aligned}&\tfrac{\varrho}{\varrho+r+1}\delta_2^{\frac{\varrho+r+1}{\varrho}}H^{\sigma r}(t)\|u\|_{\varrho+r+1}^{\varrho+r+1}\\ &\leq c_8\tfrac{\varrho}{\varrho+r+1}\delta_2^{\frac{\varrho+r+1}{\varrho}}\left(\|u\|_{2(\rho+2)}^{2\sigma r(\rho+2)+\varrho+r+1}+\|v\|_{2(\rho+2)}^{2\sigma r(\rho+2)}\|u\|_{\varrho+r+1}^{\varrho+r+1}\right).\end{aligned} \qquad (3.40)$$

Using (3.18) and the algebraic inequality, we get

$$z^{\nu}\leq(z+1)\leq\left(1+\frac{1}{a}\right)(z+a),\quad \forall z\geq0,\ 0<\nu\leq1,\ a>0, \qquad (3.41)$$

we have, for all $t \geq 0$,

$$\begin{cases} \|u\|_{2(\rho+2)}^{2\sigma m(\rho+2)+k+m+1} \leq d\left(\|u\|_{2(\rho+2)}^{2(\rho+2)} + H(0)\right) \leq d\left(\|u\|_{2(\rho+2)}^{2(\rho+2)} + H(t)\right), \\ \|v\|_{2(\rho+2)}^{2\sigma r(\rho+2)+\theta+r+1} \leq d\left(\|v\|_{2(\rho+2)}^{2(\rho+2)} + H(t)\right),\ \forall t \geq 0, \end{cases} \tag{3.42}$$

where $d = 1 + 1/H(0)$. Similarly

$$\begin{cases} \|v\|_{2(\rho+2)}^{2\sigma m(\rho+2)+l(m+1)} \leq d\left(\|v\|_{2(\rho+2)}^{2(\rho+2)} + H(0)\right) \leq d\left(\|v\|_{2(\rho+2)}^{2(\rho+2)} + H(t)\right), \\ \|u\|_{2(\rho+2)}^{2\sigma r(\rho+2)+\varrho(r+1)} \leq d\left(\|u\|_{2(\rho+2)}^{2(\rho+2)} + H(t)\right),\ \forall t \geq 0. \end{cases} \tag{3.43}$$

Also, since

$$(X+Y)^s \leq C\left(X^s + Y^s\right), \qquad X, Y \geq 0,\ s > 0, \tag{3.44}$$

by utilising (3.18) and (3.41) we infer

$$\begin{aligned} \|v\|_{2(\rho+2)}^{2\sigma m(\rho+2)} \|u\|_{k+m+1}^{k+m+1} &\leq c_9\left(\|v\|_{2(\rho+2)}^{2(\rho+2)} + \|u\|_{k+m+1}^{2(\rho+2)}\right) \\ &\leq c_{10}\left(\|v\|_{2(\rho+2)}^{2(\rho+2)} + \|u\|_{2(\rho+2)}^{2(\rho+2)}\right), \end{aligned} \tag{3.45}$$

similarly

$$\|u\|_{2(\rho+2)}^{2\sigma r(\rho+2)} \|v\|_{\theta+r+1}^{\theta+r+1} \leq c_{11}\left(\|u\|_{2(\rho+2)}^{2(\rho+2)} + \|v\|_{2(\rho+2)}^{2(\rho+2)}\right), \tag{3.46}$$

$$\|u\|_{2(\rho+2)}^{2\sigma m(\rho+2)} \|v\|_{l+m+1}^{l+m+1} \leq c_{12}\left(\|u\|_{2(\rho+2)}^{2(\rho+2)} + \|v\|_{2(\rho+2)}^{2(\rho+2)}\right) \tag{3.47}$$

and

$$\|v\|_{2(\rho+2)}^{2\sigma r(\rho+2)} \|u\|_{\varrho+r+1}^{\varrho+r+1} \leq c_{13}\left(\|v\|_{2(\rho+2)}^{2(\rho+2)} + \|u\|_{2(\rho+2)}^{2(\rho+2)}\right). \tag{3.48}$$

Taking into account (3.36)-(3.48), then ( 3.33) writes on the form

$$\begin{aligned} L'(t) \geq\ & ((1-\sigma) - M\varepsilon) H^{-\sigma}(t) H'(t) + 2\varepsilon\left(\|u_t\|_2^2 + \|v_t\|_2^2 + m_1^2\|u\|_2^2 + m_2^2\|v\|_2^2\right) \\ & + \varepsilon\left[2 - CM_1^{-m}\left(1 + \frac{l}{l+m+1}\delta_1^{\frac{l+m+1}{l}} + \frac{m+1}{l+m+1}\delta_1^{-\frac{(l+m+1)}{m+1}}\right)\right. \\ & \left. - CM_2^{-r}\left(1 + \frac{\varrho}{\varrho+r+1}\delta_2^{\frac{\varrho+r+1}{\varrho}} + \frac{r+1}{\varrho+r+1}\delta_2^{-\frac{(\varrho+r+1)}{r+1}}\right)\right] H(t) \\ & + \varepsilon\left[c_4 - CM_1^{-m}\left(1 + \frac{l}{l+m+1}\delta_1^{\frac{l+m+1}{l}} + \frac{m+1}{l+m+1}\delta_1^{-\frac{(l+m+1)}{m+1}}\right)\right. \\ & \left. - CM_2^{-r}\left(1 + \frac{\varrho}{\varrho+r+1}\delta_2^{\frac{\varrho+r+1}{\varrho}} + \frac{r+1}{\varrho+r+1}\delta_2^{-\frac{(\varrho+r+1)}{r+1}}\right)\right] \\ & \times\left(\|u\|_{2(\rho+2)}^{2(\rho+2)} + \|v\|_{2(\rho+2)}^{2(\rho+2)}\right). \end{aligned} \tag{3.49}$$

At this point, and for large values of $M_1$ and $M_2$, we can find positive constants $\Lambda_1$ and $\Lambda_2$ such that (3.49) becomes

$$\begin{aligned} L^{'}(t) \geq & ((1-\sigma)-M\varepsilon) H^{-\sigma}(t) H^{'}(t)+2\varepsilon\left(\|u_t\|_2^2+\|v_t\|_2^2+m_1^2\|u\|_2^2+m_2^2\|v\|_2^2\right) \\ & +\varepsilon\Lambda_1\left(\|u(t)\|_{2(\rho+2)}^{2(\rho+2)}+\|v(t)\|_{2(\rho+2)}^{2(\rho+2)}\right)+\varepsilon\Lambda_2 H(t). \end{aligned} \tag{3.50}$$

Once $M_1$ and $M_2$ are fixed (hence, $\Lambda_1$ and $\Lambda_2$), we choose $\varepsilon$ small enough so that $((1-\sigma)-M\varepsilon)\geq 0$ and

$$L(0)=H^{1-\sigma}(0)+\varepsilon\int_\Omega [u_0.u_1+v_0.v_1]\,dx>0. \tag{3.51}$$

Therefore, there exists $\Gamma>0$ such that (3.50) can be write

$$L^{'}(t)\geq\varepsilon\Gamma\left(H(t)+\|u_t\|_2^2+\|v_t\|_2^2+\|u\|_{2(\rho+2)}^{2(\rho+2)}+\|v\|_{2(\rho+2)}^{2(\rho+2)}\right). \tag{3.52}$$

In this way, we have $L(t)\geq L(0)>0$, for all $t\geq 0$. Next, by using Holder's and Young's inequalities, we have the estimate

$$\begin{aligned} & \left(\int_\Omega u.u_t(x,t)\,dx+\int_\Omega v.v_t(x,t)\,dx\right)^{\frac{1}{1-\sigma}} \\ \leq\ & C\left(\|u\|_{2(\rho+2)}^{\frac{\tau}{1-\sigma}}+\|u_t\|_2^{\frac{s}{1-\sigma}}+\|v\|_{2(\rho+2)}^{\frac{\tau}{1-\sigma}}+\|v_t\|_2^{\frac{s}{1-\sigma}}\right), \end{aligned} \tag{3.53}$$

for $1/\tau+1/s=1$. We takes $s=2(1-\sigma)$, to get $\dfrac{\tau}{1-\sigma}=\dfrac{2}{1-2\sigma}$. From (3.10) and (3.41) we see that

$$\|u\|_{2(\rho+2)}^{\frac{2}{1-2\sigma}}\leq d\left(\|u\|_{2(\rho+2)}^{2(\rho+2)}+H(t)\right) \tag{3.54}$$

and

$$\|v\|_{2(\rho+2)}^{\frac{2}{1-2\sigma}}\leq d\left(\|v\|_{2(\rho+2)}^{2(\rho+2)}+H(t)\right),\ \forall t\geq 0. \tag{3.55}$$

Consequently, (3.53) can be writed on the form

$$\begin{aligned} & \left(\int_\Omega uu_t(x,t)\,dx+\int_\Omega vv_t(x,t)\,dx\right)^{\frac{1}{1-\sigma}} \\ \leq\ & c_{14}\left(\|u\|_{2(\rho+2)}^{2(\rho+2)}+\|v\|_{2(\rho+2)}^{2(\rho+2)}+\|u_t\|_2^2+\|v_t\|_2^2+m_1^2\|u\|_2^2+m_2^2\|v\|_2^2+H(t)\right),\ \forall t\geq 0. \end{aligned}$$

Also, we can write

$$\begin{aligned} L^{\frac{1}{1-\sigma}}(t) &= \left(H^{1-\sigma}(t)+\varepsilon\int_\Omega (u.u_t+v.v_t)(x,t)\,dx\right)^{\frac{1}{(1-\sigma)}} \\ &\leq c_{15}\left(H(t)+\left|\int_\Omega (u.u_t(x,t)+v.v_t(x,t))\,dx\right|^{\frac{1}{(1-\sigma)}}\right) \\ &\leq c_{16}\left[H(t)+\|u\|_{2(\rho+2)}^{2(\rho+2)}+\|v\|_{2(\rho+2)}^{2(\rho+2)}+\|u_t\|_2^2+\|v_t\|_2^2+m_1^2\|u\|_2^2+m_2^2\|v\|_2^2\right], \end{aligned}$$

from (3.56) and (3.52), we get

$$L^{'}(t) \geq a_0 L^{\frac{1}{1-\sigma}}(t), \ \forall t \geq 0. \tag{3.57}$$

Finally, a simple integration of (3.57) gives the desired result. □

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Fares Yazid
University of Setif 1,
Faculty of sciences,
Algeria
e-mail: fsmmath@yahoo.com

Djamel Ouchenane
Amar Telidji University,
Faculty of Sciences,
Algeria
e-mail: ouchenanedjamel@gmail.com