If we let

$$P = \left\{ u \in C^{+}[a,b] : \min_{a_1 \le t \le b_1} u(t) \ge c \|u\| \right\}, \tag{2.5}$$

then it is easy to see that P is a cone in $C[a,b]$. It is evident that BVP (2.1) has an integral formulation given by

$$u(t) = \lambda \int_a^b s^{N-1} G(t,s) f(s, u(s))\, ds,$$

where G defined in (2.2).
Now, we define an integral operator $T_\lambda : P \to C[a,b]$ by

$$(T_\lambda u)(t) = \lambda \int_a^b s^{N-1} G(t,s) f(s, u(s))\, ds.$$

Lemma 2.6. *Let $y \in C^{+}[a,b]$. If $u \in C^2[a,b]$ satisfies*

$$\begin{cases} u''(t) = y(t), & a \le t \le b, \\ u(a) = 0, \ u(b) = 0, \end{cases} \to \ -u''(t) - \frac{N-1}{t} u(t) = y(t), \ t \in [a,b$$

then
(i) $u(t) \ge 0$ for $t \in [a,b]$,
(ii) $u'(t) \ge 0$ for $t \in [a,b]$.

Proof. From Lemma 2.4, we obtain $u(t) \ge 0$ and $u'(t) \ge 0$ for $t \in [a,b]$. □

Lemma 2.7. $T_\lambda(P) \subset P$.

Proof. For any $u \in P$, we have

$$\begin{aligned}
\min_{a_1 \le t \le b_1} T_\lambda u(t) &= \frac{\lambda}{(N-2)(b^{N-2} - a^{N-2})} \min_{a_1 \le t \le b_1} \left\{ \int_a^t \left(1 - \left(\frac{a}{s}\right)^{N-2}\right) s^{N-1} f(s, u(s)) \right. \\
&\quad \times \left(\left(\frac{b}{t}\right)^{N-2} - 1\right) ds + \left. \int_t^b \left(1 - \left(\frac{a}{t}\right)^{N-2}\right) \left(\left(\frac{b}{s}\right)^{N-2} - 1\right) s^{N-1} f(s, u(s))\, ds \right\} \\
&\ge \frac{\lambda}{(N-2)(b^{N-2} - a^{N-2})} \min_{a_1 \le t \le b_1} \left\{ \int_a^t \left(1 - \left(\frac{a}{s}\right)^{N-2}\right) \left(\left(\frac{b}{b_1}\right)^{N-2} - 1\right) \right. \\
&\quad \times s^{N-1} f(s, u(s))\, ds + \left. \int_t^b \left(1 - \left(\frac{a}{a_1}\right)^{N-2}\right) \left(\left(\frac{b}{s}\right)^{N-2} - 1\right) s^{N-1} f(s, u(s))\, ds \right\} \\
&\ge \frac{\lambda \min\left\{ \left(\frac{b}{b_1}\right)^{N-2} - 1, 1 - \left(\frac{a}{a_1}\right)^{N-2} \right\}}{(N-2)(b^{N-2} - a^{N-2})} \min_{a_1 \le t \le b_1} \left\{ \int_a^t s^{N-1} f(s, u(s)) \right.
\end{aligned}$$