

where $f \in C(\mathbb{R}, \mathbb{R})$ and $e \in C([0, R], \mathbb{R})$.

In 2014, Butler et. al, [9] studied the positive radial solutions to the boundary value problem

$$\begin{cases} -\Delta u + u = \lambda a(|x|) f(u), & x \in \Omega, \\ \frac{\partial u}{\partial \eta} + \bar{c}(u) u = 0, & |x| = r_0, \\ u(x) \rightarrow 0, & |x| \rightarrow \infty, \end{cases}$$

where $f \in C([0, \infty), \mathbb{R})$, $\Omega = \{x \in \mathbb{R}^N : N > 2, |x| > r_0 \text{ with } r_0 > 0\}$, λ is a positive parameter, $a \in C([r_0, \infty), \mathbb{R}^+)$ such that $\lim_{r \rightarrow \infty} a(r) = 0$, $\frac{\partial}{\partial \eta}$ is the outward normal derivative and $\bar{c} \in C([0, \infty), (0, \infty))$.

Instead of working directly with (1.1), we note that the change of variable

$$u(x) = u(|x|), \quad t = |x|$$

transforms (1.1) into the following boundary value problem (for details, see [14]:

$$\begin{cases} -u''(t) - \frac{N-1}{t} u'(t) = \lambda f(t, u(t)), & t \in (a, b), \\ u(a) = u(b) = 0, \end{cases}$$

where $\lambda \geq 0$ is a positive parameter and $f \in C([a, b] \times [0, \infty), [0, \infty))$.

Inspired and motivated by the works mentioned above, we deal with existence and nonexistence of radial positive solutions to the BVP (1.1) i.e., an equivalent problem (2.1) by using of the fixed point theorem together with the properties of Green's function and we impose certain conditions on f . The paper is organized as follows. In Section 2, we present that a nontrivial and nonnegative solution of BVP (2.1) is monotone positive solution. In Section 3, we obtain some results of the existence, multiplicity and nonexistence positive solutions for BVP (2.1) depends on the parameter λ and we give an exemple to illustrate our results.

Exemple

2. Preliminaries

We shall consider the Banach space $E = C[a, b]$ equipped with sup norm

$$\|u\| = \max_{a \leq t \leq b} |u(t)|,$$

and $C^+[a, b]$ is the cone of nonnegative functions in $C[a, b]$, where $1 < a < b$.

Definition 2.1. A nonempty closed and convex set $P \subset E$ is called a cone of E if it satisfies

(i) $u \in P, r > 0$ implies $ru \in P$,

(ii) $u \in P, -u \in P$ implies $u = \theta$, where θ denote the zero element of E .

Definition 2.2. A cone P is said to be normal if there exists a positive number N called the normal constant of P , such that $\theta \leq u \leq v$ implies $\|u\| \leq N \|v\|$.