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Transformation of the traveller wave shape in propagation on a straight and inclined bed

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Abstract. By continuing the papers of the second author based on the relative movements method, we consider Gerstner's potential to study the traveller wave relative movement with respect to a moving dihedron with the translation velocity $c =$ ct., driven on a horizontal bed. In the second case we present the traveller wave relation motion with respect to a moving dihedron, driven on the inclined bed with the angle $\alpha =$ ct.

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In this aim we shall apply the relative movements method, used with special success at the liquid viscous flow through rotor channels of radial turbo machines, through pipelines in vibration state, as well as at aquaplaning phenomenon of a car or airplane tyre.

We shall consider the Ferdinand von Gerstner's potential [1], whose constant C we determined as function of the wave height h, its wave-length λ and water deep in the horizontal channel H (Figure 5) [2], [3]

$$\Phi_1(X_1, Y_1, T_1) = C(h, \lambda, H)\mathrm{ch}k(H - Y_1) \cdot \cos(kX_1 - \omega T_1),$$

using in this particular case the relative movement of the traveller wave with respect to the moving dihedron driven with his propagation velocity c, but taking then into consideration the bed slope, supposed to be constant $\alpha =$ ct.

1. Traveller wave relative movement with respect to the moving dihedron with velocity c

The relations between the absolute, relative and transport variables being (Figure 5)

$$X_1(X, Y, T) = X + X_0(T_1) = X + cT_1, \tag{1.1}$$

$$Y_1(X, Y, T) = Y, \tag{1.2}$$

the time running in the same manner in both fixed and mobile dihedrons, from which by the total differentiation with respect to the time we shall obtain the relations between the absolute, relative and transport velocity components:

$$\begin{gathered} U_1(X_1, Y_1, T_1) = \frac{\partial \Phi_1}{\partial X_1} \\ = -kC\mathrm{ch}k(H - Y_1) \cdot \sin(kX_1 - \omega T_1) = U(X, Y, T) + U_0 = U + c, \end{gathered} \tag{1.3}$$

$$V_1(X_1, Y_1, T_1) = \frac{\partial \Phi_1}{\partial Y_1} = -kC\mathrm{sh}k(H - Y_1) \cdot \cos(kX_1 - \omega T_1) = V(X, Y, T), \tag{1.4}$$

from which one can deduce the expressions of the relative velocity components:

$$U = -kC\mathrm{ch}k(H - Y) \cdot \sin kX - c = \frac{dX}{dT},$$

$$X - X_0 = [-kC\mathrm{ch}k(H - Y) \cdot \sin kX - c](T - T_0) \tag{1.5}$$

$$V = -kC\mathrm{sh}k(H - Y) \cos kX = \frac{dY}{dT} \rightarrow$$

$$Y - Y_0 = -kC\mathrm{sh}k(H - Y) \cos kX(T - T_0), \tag{1.6}$$

from which, eliminating the time and considering that at $X_0 = 0$ we have $Y_0 = H = 1m$, one shall obtain the two relations of iterative calculus, considering in the first relation that $Y_0 = H$

$$Y(Y_0 = H) = H + \frac{kC\mathrm{sh}kH \cdot \cos kX}{kC\mathrm{ch}kH \cdot \sin kX + c}(X - 0), \tag{1.7}$$

and then considering that $Y_0 = Y$, calculated from the anterior relation (1.6)

$$Y(Y_0 = Y) = Y + \frac{kC\mathrm{sh}k(H - Y) \cdot \cos kX}{kC\mathrm{ch}k(H - Y) \cdot \sin kX + c}X, \tag{1.8}$$

the traveller wave trajectory being represented by the two values sequences for

$$H = 1m,\ \lambda = 4m,\ T_0 = 1,671s,\ k = 2\pi/\lambda = 1,57rad/m,\ C = 0,05222$$

and $c = \lambda/T_0 = 2,393776m/s$, presented in the Table 1, as in the Figures 1 and 2.

Table 1

$X(m)$	$Y(Y_0=1)(m)$	$Y(Y)(m)$	$X/Lambda$	$Y(Y_0=1)(m)$	$Y(Y)(m)$
0	1	1	0	1	1
0, 5	0, 9704	0, 9423	0, 125	0, 9979	0, 9958
1	0, 9212	0, 8525	0, 25	0, 9918	0, 9838
1, 5	0, 9212	0, 8520	0, 375	0, 9823	0, 9652
2	0, 9998	0, 9995	0, 5	0, 9404	0, 9423
2, 5	1, 1310	1, 2923	0, 625	0, 9570	0, 9173
3	1, 2362	1, 5865	0, 75	0, 9436	0, 8925
3, 5	1, 2080	1, 5097	0, 875	0, 9313	0, 8703
4	1, 0011	1, 0022	1	0, 9212	0, 8525

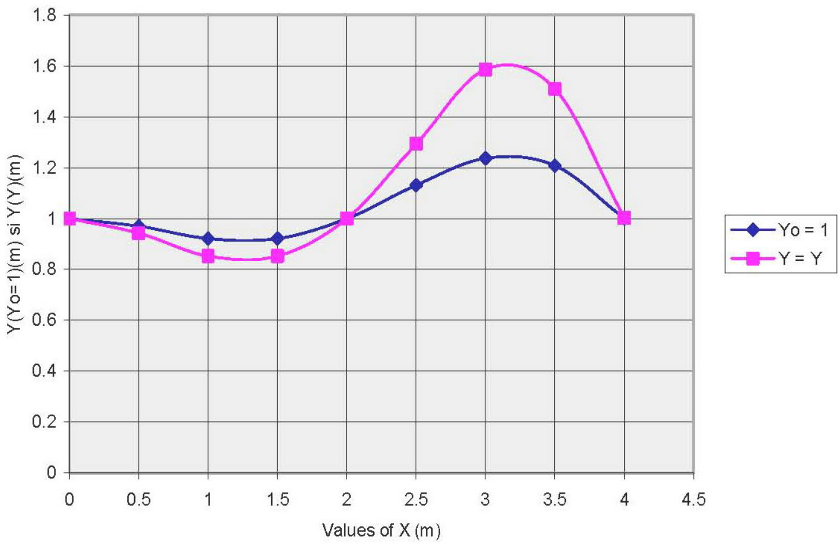

Fig. 1. The variations with abscissa X (m) of the two traveller wave trajectories

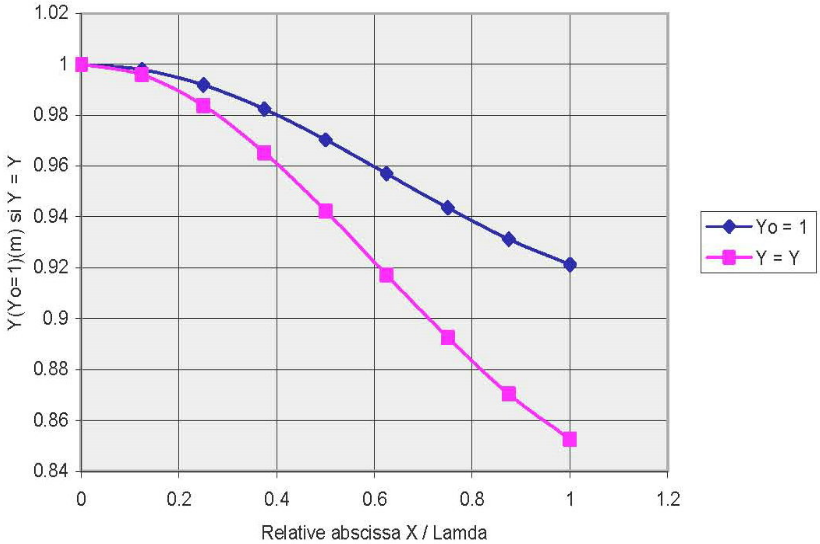

Fig. 2. The variation with relative abscissa X / λ of the two traveller wave trajectories

Concerning the variations (1.5) and (1.8), these are given in the Table 2 and are represented in the Figure 3 in the case when $H = 1 =$ constant and in the Figure 4 in the case when one considers Y variable and one takes anterior value calculated for the anterior X.

Table 2

$X(m)$	$Y(X)\ H = 1$	$X(m)$	$Y(X)\ H - Y$
0	0	0	0
$0,5$	0.026259	$0,5$	0.026259
1	$5.78E-05$	1	$5.54E-05$
$1,5$	-0.07865	$1,5$	-0.07864
2	-0.15748	2	-0.17993
$2,5$	-0.14849	$2,5$	-0.2048
3	-0.00062	3	-0.0009
$3,5$	0.206953	$3,5$	0.20729
4	0.315083	4	0.218097
$4,5$	0.237129	$4,5$	0.163315
5	0.001444	5	0.001101
$5,5$	-0.28739	$5,5$	-0.28688
6	-0.4723	6	-0.76075
$6,5$	-0.38721	$6,5$	-1.54749
7	-0.00336	7	-0.55962
$7,5$	0.44214	$7,5$	1.208477
8	0.63033	8	-0.09131
$8,5$	0.449407		
9	0.004678		
$9,5$	-0.49472		
10	-0.78693		

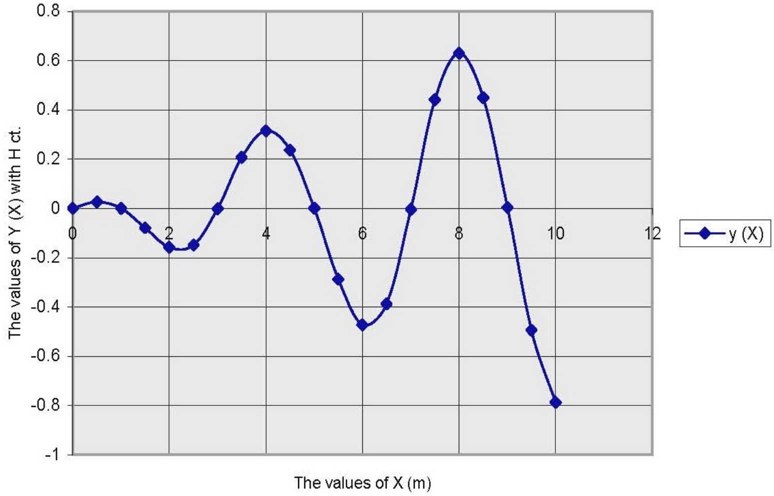

Fig. 3. The wave amplitude variation values Y (X) as function of abscissa X in the consideration case of the position from the proximity of the wave free surface considered at the deep $H = 1$, supposed constant

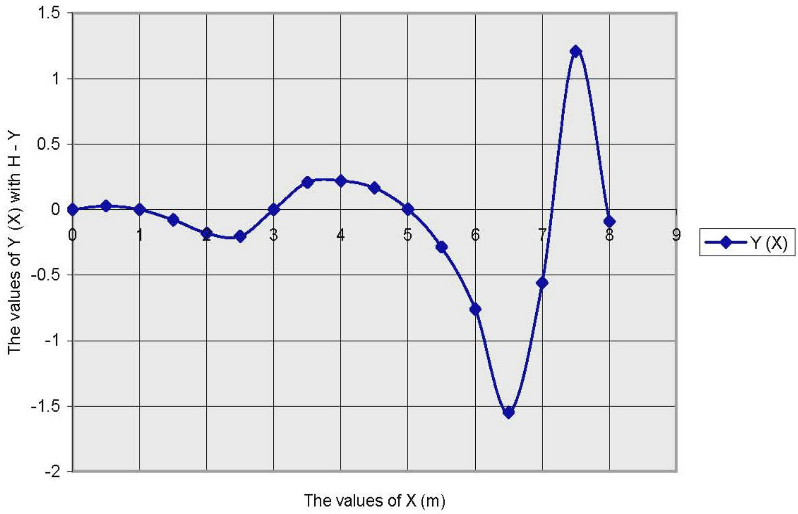

Fig. 4. The wave amplitude variation values Y (X) as function of abscissa X in the consideration case of the position from the proximity of the wave free surface, considering that the deep is variable at the distance H - Y

2. The traveller wave relative motion with respect to the moving dihedron driven on the inclined plane

To study the wave free surface modification, by its propagation on a bed having a constant inclination $\alpha = \text{ct.}$, hypothesis sure in the case of ideal liquid, one shall utilize the relative motion method [3], by that the wave tries to immobilize with respect to the Cartesian dihedron $X0Y$, movable on these two coordinate axes through a translation motion with respect to the fixed absolute dihedron $X_1 0_1 Y_1$ (Figure 5).

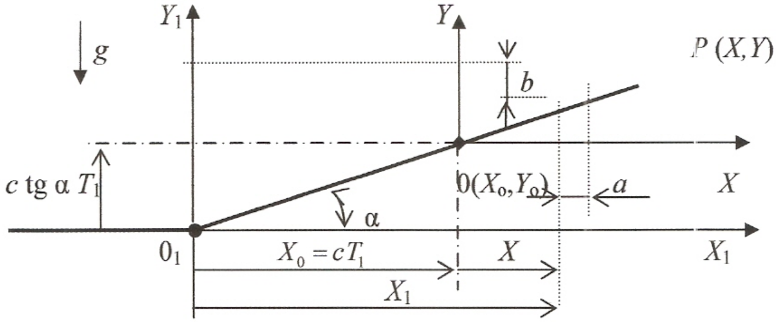

Fig. 5. Coordinate axes of the absolute and relative dihedron

In the hypothesis of wave propagation constant velocity $c = \text{ct.}$, the translation velocity of the mobile dihedron, solidary with the bed bottom, will be:

- velocity c for the axis origin $0X$ on the axis direction 0_1X_1 and consequently,
- velocity $c\ \mathrm{tg}\alpha$ for the axis origin $0Y$ on the axis direction 0_1Y_1, the ratio of the two coordinates of the origin 0 of the moving dihedron being in each moment $Y_0/X_0 = \mathrm{tg}\alpha$.

In this case the relations between absolute and relative variables will be:

$$X_1(X, Y, T) = X + X_0(T_1) = X + cT_1, \tag{2.1}$$

$$Y_1(X, Y, T) = Y + Y_0(T_1) = Y + c\ \mathrm{tg}\alpha T_1. \tag{2.2}$$

The relations between the absolute, relative and transport velocities are obtained by total derivation in time of the relations (2.1) and (2.2), partial derivative having not physical sense

$$U_1 = \frac{\partial \Phi_1}{\partial X_1} = -kC\mathrm{ch}k(Y_1)\sin(kX_1 - \omega T_1) = U(X, Y, T) + c, \tag{2.3}$$

$$V_1 = \frac{\partial \Phi_1}{\partial Y_1} = kC\mathrm{sh}k(Y_1)\cos(kX_1 - \omega T_1) = V(X, Y, T) + c\ \mathrm{tg}\alpha, \tag{2.4}$$

the time running in the same manner in any point of the absolute or relative dihedron in which we introduced the two components of the translation velocity of the moving dihedron:

$$\frac{dX_0}{dT} = U_0 = c = \text{ct.} \quad \text{and} \quad \frac{dY_0}{dT} = V_0 = c\ \mathrm{tg}\alpha = \text{ct.} \tag{2.5}$$

and integrating with respect to the time, we shall obtain

$$X - X_0 = \left[-\frac{C}{c\ \mathrm{tg}\alpha} \mathrm{sh}k(Y + c\ \mathrm{tg}\alpha T) \cdot \sin kX - cT \right] (T - T_0) \tag{2.6}$$

$$Y - Y_0 = \left[\frac{C}{c\ \mathrm{tg}\alpha} \mathrm{ch}k(Y + c\ \mathrm{tg}\alpha T) \cdot \cos kX - c\ \mathrm{tg}\alpha T \right] (T - T_0) \tag{2.7}$$

and eliminating the time one shall have

$$Y - Y_0 = \frac{\dfrac{C}{c\ \mathrm{tg}\alpha}\mathrm{ch}k(Y + c\ \mathrm{tg}\alpha T)\cdot\cos kX - c\ \mathrm{tg}\alpha T}{-\dfrac{C}{c\ \mathrm{tg}\alpha}\mathrm{sh}k(Y + c\ \mathrm{tg}\alpha T)\cdot\sin kX - cT}(X - X_0), \tag{2.8}$$

which one shall use for the diverse time moments, for instance $T = 0$, in that case the relation becomes with $\mathrm{tg}\alpha = \mathrm{tg}20^\circ = 0,364$ being represented in the Figure 6

$$Y = H - \frac{\dfrac{C}{c\ \mathrm{tg}\alpha}\mathrm{ch}kY\cdot\cos kX}{\dfrac{C}{c\ \mathrm{tg}\alpha}\mathrm{sh}kY\cdot\sin kX}X \cong H - \frac{\dfrac{C}{c\ \mathrm{tg}\alpha}\mathrm{ch}kY\cdot\cos kX}{\dfrac{C}{c\ \mathrm{tg}\alpha}\mathrm{sh}kY\cdot k}. \tag{2.9}$$

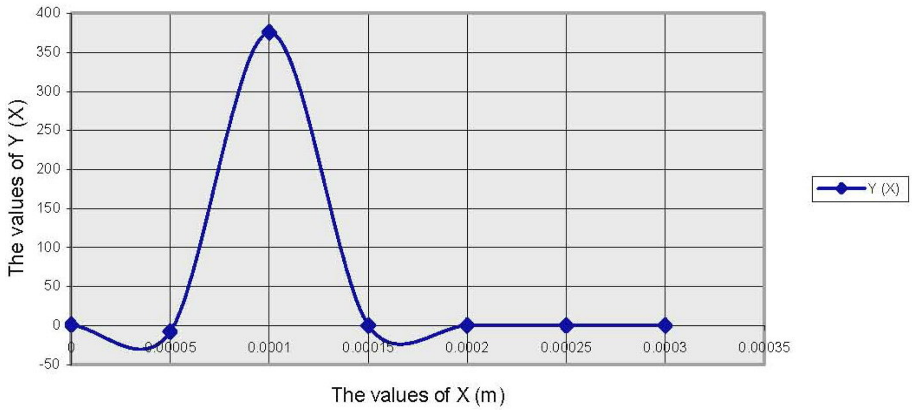

Fig. 6. The values variation Y (X) for the initial moment $T = 0$

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