Babeş-Bolyai University
Faculty of Mathematics and Computer Science

GRADUATION EXAM
Written Test - September 2026
Mathematics Computer Science Study Programme

SUBJECT I. Algebra

1. **(3 points)** Consider the set $\mathcal{A} = \left\{ \begin{pmatrix} a & b \\ 7b & a \end{pmatrix} \,\middle|\, a, b \in \mathbb{Z} \right\}$.
   (a) Prove that $\mathcal{A}$ is a subring with a multiplicative identity in the ring $(M_2(\mathbb{Q}), +, \cdot)$.
   (b) Is the ring $(\mathcal{A}, +, \cdot)$ a field ($+$ and $\cdot$ are the operations induced on $\mathcal{A}$ by the operations of $M_2(\mathbb{Q})$)? Motivate your answer.
2. **(6 points)** Consider the system with the unknowns $x, y, z \in \mathbb{R}$ and the parameter $a \in \mathbb{R}$:
$$\begin{cases} x + y + z = 1, \\ 2x + 3y + az = 1, \\ x + ay + 3z = 2. \end{cases}$$
   a) Solve the system for $a = 2$.
   b) Determine the values of $a \in \mathbb{R}$ for which the system has a unique solution.
   c) Determine the values of $a \in \mathbb{R}$ for which the system has no solution.
   d) Determine the values of $a \in \mathbb{R}$ for which the system has more than one solution.
   e) Solve the system for $a = 0$.

SUBJECT II. Calculus

1. **(3 points)** Determine the general form of the partial sum of rank n, and then investigate the sum and the nature of the series:
$$\sum_{n \geq 1} \ln\left(\frac{n(n+2)}{(n+1)^2}\right).$$
2. **(3 points)** Write the Taylor's polynomial of random rank n, attached to function f and the point $a = 0$, for $f : \mathbb{R} \to \mathbb{R}$, where $f(x) = \sin(2x)\cos x$, $\forall x \in \mathbb{R}$.
   Note: to determine the n-th order derivative, you can use either Leibniz'z formula or the trigonometric identity $2\sin a \cos b = \sin(a+b) + \sin(a-b)$.
3. **(3 points)** Determine $\int \frac{x}{\sqrt{x^2-4x+5}} dx$, $x \in \mathbb{R}$.

SUBJECT III. Geometry

1. **(5 points)** In the rectangle $ABCD$, the points $A(0,0)$ and $C(6,2)$ are given, and it is known that the side AB is parallel to the line $y = x + 1$.
   a) Write the equation of the diagonal AC and determine the coordinates of the center M of the rectangle.
   b) Determine the equations of the lines containing the sides AB and BC.
   c) Determine the coordinates of the vertices B and D.
   d) Compute the area of the rectangle $ABCD$.
2. **(4 points)** Consider the parabola $\mathcal{P}$ with axis of symmetry Ox, vertex at the origin, and passing through the point $A(4,4)$. Let F denote the focus of the parabola.
   a) Determine the equation of the parabola $\mathcal{P}$ and the coordinates of the point F.
   b) Write the equation of a line that passes through F and is parallel to the line $d : 4x + 3y = 0$.
   c) Determine all points $M \in \mathcal{P}$ at which an incident ray parallel to the axis Ox can be reflected in such a way that the reflected ray is parallel to the line $d : 4x + 3y = 0$.

SUBJECT IV. Computer Science

One of the following programming languages C++, Python, Java or C# can be used to solve problems 1 and 2.

Indicate the programming language used.

Existing libraries (from Python, C++, Java, C#) can be used in the provided solutions.

1. **(2p)** Write a program that:
   a) Implements a class **Employee** with the following protected attributes:
      - ***name*** of type string,
      - ***salary*** of type integer.

      Add to the class:
      - a ***constructor*** with parameters,
      - ***get/set*** methods for all attributes,
      - ***toString*** method that returns a string containing the name and the salary of the person separated by space.

   b) Derive the class ***Manager*** from class ***Employee*** having all attributes of class ***Employee*** and add a new private attribute ***department*** of type string. Write the methods ***get/set*** for the new attribute. The method ***toString*** from class ***Manager*** will return the content of the method ***toString*** from class ***Employee*** followed by a space and the ***department*** attribute.

2. **(2p)** Create a vector containing two objects of type ***Employee*** and one object of type ***Manager***. Write a function that receives as parameter the previously created vector, and returns the employee having the minimal salary.

3. **(2p)** Fill in the missing statements from the following function to determine the objects of ***Employee*** type from the vector ***team*** having a salary greater than ***specificSalary***. You can use the function *push_back* which inserts an element at the end of the vector.

```
vector<Employee> filter(const vector<Employee>& team, const int specificSalary){
    vector<Employee> rez;
     ....
    return rez;}
```

4. **(2p)** Write the result of running the following sequence of code. The function *push_back* inserts an element at the end of the vector, the function *pop_back* removes the last element from the vector, and the function *back* returns a reference to the last element of the vector.

```
vector<Employee> staff;
staff.push_back(Employee("John", 5000));
staff.push_back(Manager("Alice", 8500, "IT"));
staff.push_back(Employee("Bob", 4000));
staff.push_back(Manager("Elena", 9000, "HR"));
staff.pop_back();
staff.push_back(Employee("Michael", 4500));
staff.pop_back();
cout<<staff.back().toString()<<endl ;
```

5. **(1p)** Explain the concept of encapsulation.

NOTE.

All subjects are compulsory and full solutions are requested.
An initial score of **1 point** is awarded to each subject. The minimum passing grade is 5,00.
The working time is 3 hours.

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SUBJECT I. Algebra

Default 1p

1. (a) One can check, using the subring characterization theorem, that $\mathcal{A}$ is a subring of $(M_2(\mathbb{Q}), +, \cdot)$

   (i) $O_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \in \mathcal{A}$ 0.5p

   (ii) $\begin{pmatrix} a & b \\ 7b & a \end{pmatrix} - \begin{pmatrix} a' & b' \\ 7b' & a' \end{pmatrix} = \begin{pmatrix} a-a' & b-b' \\ 7(b-b') & a-a' \end{pmatrix} \in \mathcal{A},\ \forall a, b, a', b' \in \mathbb{Z}$ 0.5p

   (iii) $\begin{pmatrix} a & b \\ 7b & a \end{pmatrix} \cdot \begin{pmatrix} a' & b' \\ 7b' & a' \end{pmatrix} = \begin{pmatrix} aa'+7bb' & ab'+ba' \\ 7(ab'+ba') & aa'+7bb' \end{pmatrix} \in \mathcal{A},\ \forall a, b, a', b' \in \mathbb{Z}$ 0.5p

   The multiplicative identity of $M_2(\mathbb{Q})$ is the matrix $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 7\cdot 0 & 1 \end{pmatrix} \in \mathcal{A}$............ 0.5p

   (b) If $\mathcal{A}$ were a field then for any nonzero element from $\mathcal{A}$ the inverse in the monoid $(M_2(\mathbb{Q}), \cdot)$ would also be in $\mathcal{A}$. But, for instance, for the nonzero matrix $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I_2 \in \mathcal{A}$ the inverse in $M_2(\mathbb{Q})$ is $A^{-1} = \frac{1}{2}I_2 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \notin \mathcal{A}$ because $\frac{1}{2} \notin \mathbb{Z}$. Thus $\mathcal{A}$ is not a field. 1p

2. The system matrix is $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & a \\ 1 & a & 3 \end{pmatrix}$ and the augmented matrix is $\overline{A} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & a & 1 \\ 1 & a & 3 & 2 \end{pmatrix}$.

   (a) For $a = 2$ we have $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix}$

   and $\det A = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{vmatrix} \overset{c_3 - c_1}{=} \begin{vmatrix} 1 & 1 & 0 \\ 2 & 3 & 0 \\ 1 & 2 & 2 \end{vmatrix} = 2(3-2) = 2 \neq 0$, therefore the system is consistent (compatible) with a unique solution given by Cramer theorem: 0.5p

   $x = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 2 & 2 & 3 \end{vmatrix} \overset{c_2 - c_1}{=} \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 1 & 2 & 2 \\ 2 & 0 & 3 \end{vmatrix} = 1\,,$

   $y = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} \overset{c_3 - c_1}{=} \frac{1}{2}\begin{vmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 2 \end{vmatrix} = -1\,,$

   $z = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 2 \end{vmatrix} \overset{c_3 - c_2}{=} \frac{1}{2}\begin{vmatrix} 1 & 1 & 0 \\ 2 & 3 & -2 \\ 1 & 2 & 0 \end{vmatrix} = 1$ 1p

   (b) The given system is consistent (compatible) with a unique solution if and only if $\det A \neq 0$.

   Since $\det A = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & a \\ 1 & a & 3 \end{vmatrix} \underset{c_3 - c_1}{\overset{c_2 - c_1}{=}} \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & a-2 \\ 1 & a-1 & 2 \end{vmatrix} = 2 - a^2 + 3a - 2 = -a(a-3)$, the system is consistent with a unique solution for any $a \in \mathbb{R} \setminus \{0, 3\}$ .... 1p (c),(d) If $a \in \{0,3\}$ then $\operatorname{rank} A = 2$ since the 2-size minor $d = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$ of A is not zero 0.5p

If $a = 0$ then $\overline{A} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 0 & 1 \\ 1 & 0 & 3 & 2 \end{pmatrix}$ and the only 3-size minor of $\overline{A}$ resulted by completing d with constant terms is $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & 0 & 2 \end{vmatrix} \overset{c_3-2c_1}{=} \begin{vmatrix} 1 & 1 & -1 \\ 2 & 3 & -3 \\ 1 & 0 & 0 \end{vmatrix} = 0.$ 0.5p

If $a = 3$ then $\overline{A} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 3 & 1 \\ 1 & 3 & 3 & 2 \end{pmatrix}$ and the only 3-size minor of $\overline{A}$ resulted by completing d with constant terms is $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & 3 & 2 \end{vmatrix} \underset{c_3-c_1}{\overset{c_2-c_1}{=}} \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & -1 \\ 1 & 2 & -1 \end{vmatrix} = -3 \neq 0.$ 0.5p

Using Rouché theorem one deduces that the answer for (c) is $a = 3$, ... 0.5p
and the answer for (d) is $a = 0$. 0.5p

(e) From the above computations it results that, for the case $a = 0$, since the minor $d = \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} \neq 0$, we can consider the first and the last equation as main equations, x, y as main unknowns, and z, being the side unknown, can be treated as a parameter. The system becomes

$$\begin{cases} x + y = 1 - z, \\ x = 2 - 3z, \end{cases}$$

and the solution set is $S = \{(2 - 3z, -1 + 2z, z) \mid z \in \mathbb{R}\}$ 1p

NOTE: Any other correct solution will be scored accordingly.

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SUBJECT II. Calculus

Default 1p

1. We have:

$$s_n = \sum_{k=1}^{n} \ln\left(\frac{k(k+2)}{(k+1)^2}\right) = \ln\left(\prod_{k=1}^{n} \frac{k(k+2)}{(k+1)^2}\right) = \ln\left(\frac{1\cdot 3}{2^2}\cdot\frac{2\cdot 4}{3^2}\cdot\ldots\cdot\frac{n(n+2)}{(n+1)^2}\right)$$
$$= \ln\left(\frac{1}{2}\cdot\frac{n+2}{n+1}\right) = \ln\left(\frac{n+2}{2(n+1)}\right).$$

.... 2p

We calculate the limit:

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty} \ln\left(\frac{n+2}{2(n+1)}\right) = \ln\left(\lim_{n\to\infty}\frac{n+2}{2(n+1)}\right) = \ln\left(\frac{1}{2}\right) = -\ln 2.$$

Thus, the sum of the series is $-\ln 2$ and the series is convergent. 1p

2. The Taylor polynomial of rank n associated with a function f and the point $a = 0$ is the function

$$T_{n;0}f : \mathbb{R} \to \mathbb{R}$$

having the expression

$$T_{n;0}f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^k, \quad \forall x \in \mathbb{R}$$

The function $f(x) = \sin(2x)\cos x$ is infinitely differentiable on $\mathbb{R}$, being a composition and product of elementary functions. 0.5p

To determine the n-th order derivative, we first transform the product into a sum using the trigonometric identity:

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

We thus obtain:

$$f(x) = \frac{1}{2}\left[\sin(2x+x) + \sin(2x-x)\right] = \frac{1}{2}\sin(3x) + \frac{1}{2}\sin x$$

By induction, it can be proven for any $k \in \mathbb{R}^*$ that:

$$(\sin(kx))^{(n)} = k^n \sin\left(kx + \frac{n\pi}{2}\right)$$

Applying the linearity of the derivative, we obtain the expression of the n-th order derivative:

$$f^{(n)}(x) = \frac{1}{2}\cdot 3^n \sin\left(3x + \frac{n\pi}{2}\right) + \frac{1}{2}\sin\left(x + \frac{n\pi}{2}\right)$$

.... 1.5p

To construct the Taylor polynomial centered at $a = 0$, we evaluate the k-th order derivative at the point $x = 0$:

$$f^{(k)}(0) = \frac{3^k + 1}{2}\sin\left(\frac{k\pi}{2}\right)$$

We observe that the value depends on the parity of k:

- If $k = 2m$ (even), then $\sin(m\pi) = 0 \implies f^{(2m)}(0) = 0$
- If $k = 2m+1$ (odd), then $\sin\left(m\pi + \frac{\pi}{2}\right) = (-1)^m \implies f^{(2m+1)}(0) = (-1)^m \frac{3^{2m+1}+1}{2}$

Therefore, all terms with even powers vanish (including $f(0) = 0$), and the Taylor polynomial contains only odd powers:

$$T_n(x) = 2x - \frac{14}{3!}x^3 + \frac{122}{5!}x^5 - \cdots + \frac{(-1)^m(3^{2m+1}+1)}{2 \cdot (2m+1)!}x^{2m+1}$$

where $2m+1 \leq n$. 1p

3. The integral

$$I = \int \frac{x}{\sqrt{x^2-4x+5}}\,dx, \quad \forall x \in \mathbb{R}$$

We observe that $(x^2-4x+5)' = 2x-4$. 0.5p

Then

$$I = \frac{1}{2}\int \frac{2x}{\sqrt{x^2-4x+5}}\,dx = \frac{1}{2}\int \frac{(2x-4)+4}{\sqrt{x^2-4x+5}}\,dx$$

We split the fraction into two distinct integrals:

$$I = \frac{1}{2}\int \frac{2x-4}{\sqrt{x^2-4x+5}}\,dx + 2\int \frac{1}{\sqrt{x^2-4x+5}}\,dx$$

.... 0.5p

Using $\int \frac{u'}{\sqrt{u}}\,dx = 2\sqrt{u}$ and $\int \frac{1}{\sqrt{y^2+a^2}}\,dy = \ln\left|y + \sqrt{y^2+a^2}\right|$, we obtain:

$$I = \sqrt{x^2-4x+5} + 2\ln\left|x-2+\sqrt{x^2-4x+5}\right| + \mathcal{C},$$

.... 2p

NOTE: Any other correct solution will be scored accordingly.

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SUBJECT III. Geometry

Default 1p

1. a) The line AC passes through $A(0,0)$ and $C(6,2)$, hence it has slope $m_{AC} = \frac{2}{6} = \frac{1}{3}$. Therefore

$$AC: \ y = \frac{1}{3}x \Longleftrightarrow x - 3y = 0.$$

.... 0,5p

The diagonals of a rectangle bisect each other, hence the center of the rectangle is the midpoint of the segment $[AC]$:

$$M\left(\frac{0+6}{2}, \frac{0+2}{2}\right) = M(3,1).$$

.... 0,5p

b) Since AB is parallel to $y = x + 1$ and $A(0,0) \in AB$, we obtain

$$AB: \ y = x.$$

.... 0,75p

In a rectangle, $BC \perp AB$. Since the slope of AB is 1, the slope of BC is -1, and $C(6,2) \in BC$. Thus

$$BC: \ y - 2 = -(x - 6) \Longleftrightarrow x + y - 8 = 0.$$

.... 0,75p

c) The vertex B is at the intersection of the lines AB and BC:

$$\begin{cases} y = x, \\ x + y - 8 = 0, \end{cases}$$

hence $x = 4$, $y = 4$, so $B(4,4)$. 0,75p

Since M is the midpoint of the segment $[BD]$, we have

$$D = (2, -2).$$

.... 0,75p

d) We have

$$AB = \sqrt{4^2 + 4^2} = 4\sqrt{2}, \qquad BC = \sqrt{(6-4)^2 + (2-4)^2} = 2\sqrt{2}.$$

Hence

$$\text{Aria}(ABCD) = AB \cdot BC = 4\sqrt{2} \cdot 2\sqrt{2} = 16.$$

.... 1p

2. a) Since the parabola has vertex at the origin and axis of symmetry Ox, its equation is of the form

$$P: \ y^2 = 2px.$$

.... 0,75p

Since $A(4,4) \in P$, we obtain

$$16 = 2p \cdot 4 \Longleftrightarrow p = 2,$$

hence the equation of the parabola is

$$P: \ y^2 = 4x.$$

.... 0,75p

For the parabola $y^2 = 2px$, the focus has coordinates $\left(\frac{p}{2}, 0\right)$. Therefore $F(1,0)$. 0,5p

b) The required line passes through $F(1,0)$ and is parallel to d; therefore it has equation

$$r : 4x + 3y - 4 = 0$$

. . . . 0,5p

c) The reflected ray must be parallel to the line $4x + 3y = 0$ and, by the optical property, must pass through the focus $F(1,0)$. Therefore it lies on the line determined above,

$$r : \; 4x + 3y - 4 = 0.$$

. . . . 0,5p

The points of reflection are the intersections of the line r with the parabola P:

$$\begin{cases} y^2 = 4x, \\ 4x + 3y - 4 = 0. \end{cases}$$

Substituting $4x = y^2$ into the second equation, we obtain

$$y^2 + 3y - 4 = 0 \Longleftrightarrow y \in \{1, -4\}.$$

. . . . 0,75p

For $y = 1$ we have $x = \dfrac{1}{4}$, and for $y = -4$ we have $x = 4$. Thus the required points are

$$M_1\left(\frac{1}{4}, 1\right), \qquad M_2(4, -4).$$

. . . . 0,25p

NOTE: Any other correct solution will be scored accordingly.

BABEŞ -BOLYAI UNIVERSITY CLUJ-NAPOCA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Rubric for Computer Science Subject
Exam session 1: Assessment of basic and specialist knowledge, license exam September 2026
Specialization Mathematics and Informatics

Computer Science

1. a) Definition of the class Employee (constructor, methods, data access)......... b) Definition of the derived class Manager (inheritance, constructor, method)	**2p** 1p 1p
2. Vector creation.. Computing minimal salary... Returning the employee with the minimal salary…………………………..	**2p** 0.5p 1p 0.5p
3. Elements iteration…..…………………………………………………… Elements comparison ……………………………………………………… Results update…. ………………………………………………………..	**2p** 0.5p 1p 0.5p
4. Correct identification of displayed object…………………………………. Correct specification of string representation of the displayed object…….	**2p** 1p 1p
5. Theoretical explanation	**1p** 1p

Remark:
(1p) Default