COURSE DESCRIPTION

Introduction to Functional Analysis

Academic year 2026-2027

1. Programme-related data

1.1. Higher Education Institution	Babeș-Bolyai University
1.2. Faculty	Mathematics and Computer Science
1.3. Department	Mathematics
1.4. Field	Mathematics
1.5. Level of study	Bachelor
1.6. Degree programme / Qualification	Mathematics Computer Science (English)
1.7. Form of education	Full-time

2. Course-related data

2.1. Course title	**Introduction to Functional Analysis**			Course code	**MLE0101**
2.2. Course coordinator	Assoc. Prof. dr. Brigitte Breckner				
2.3. Seminar coordinator	Assoc. Prof. dr. Brigitte Breckner				
2.4. Year of study	3	2.5. Semester	5	2.6. Type of assessment	Exam
2.7. Course status	Optional			2.8. Course type	Specialisation subject

3. Total estimated time (hours per semester of teaching activities)

3.1. Number of hours per week	**4**	of which: 3.2. course	**2**	3.3. seminar/ laboratory/ project	**2**
3.4. Total of hours in the curriculum	56	of which: 3.5. course	28	3.6. seminar/ laboratory	**28**
Time allocation for individual study (IS) and self-taught activities (ST)					**hours**
Learning from textbooks, course materials, bibliography, and notes (IS)					45
Additional research in the library, on subject-specific electronic platforms, and on-site					15
Preparing seminars/ laboratories/ projects, assignments, reports, portfolios, and essays					4
Tutoring (professional guidance)					1
Examinations					4
Other activities					-
3.7. Total hours of individual study (IS) and self-taught activities (ST)				**69**	
3.8. Total hours per semester				**125**	
3.9. Number of credits				**5**	

4. Prerequisites (where applicable)

4.1. curriculum-related	linear algebra; general topology; mathematical analysis
4.2 skills-related	abstract and logical thinking

5. Specific conditions (where applicable)

5.1. course-related	blackboard, chalk, video projector
5.2. seminar/laboratory-related	blackboard, chalk

6.1. Competencies resulting from the completion of the degree programme *(as referred to in the curriculum)*[1]

Professional competencies	
Competency code	**Competency**
PC2	perform analytical mathematical calculations
PC6	think abstractly
PC8	study relationships between quantities
Transversal competencies	
Competency code	**Competency**
TC4	Solve problems
TC5	Think analytically

6.2. Learning outcomes relevant to the degree programme (as referred to in the curriculum)[2]

Learning outcomes targeted by the subject		
Competency code	**Knowledge and comprehension**	**Specific academic skills**
PC5, PC7	9. The student/graduate formulates observations and differentiates notions, properties, and assertions from advanced mathematics disciplines through examples and counterexamples.	9. The student/graduate argues the role of elements found in the hypotheses of mathematical assertions, discusses how they articulate within the proof, and independently constructs correct proofs of mathematical assertions from major mathematical disciplines. The student/graduate translates a practical situation into mathematical language, solves the resulting problem, and interprets the obtained results.

7. Subject-specific learning outcomes

Knowledge and comprehension
1. The student acquires the basic notions of the discipline: linear topological space, seminorm, norm, normed space, Banach space, inner product space, Hilbert space, linear continuous operator, the dual of a normed space
2. The student knows the fundamental concepts of functional analysis as well as methods of applying them in areas of science related to mathematics and computer science.
Specific academic skills
1. The student can construct clear and well-supported mathematical arguments to explain mathematical problems, topics and ideas in writing.
2. The student can prove theorems, using mathematical language, and is able to present these results both orally and in writing.

[1] The professional and/or transversal skills targeted by the subject for which the course description is prepared will be copied from the curriculum of the degree programme. For each competency, the complete entry, including the competency code, will be copied with the exact wording that appears in the curriculum, without any changes. If no competency is copied from either of the two categories, the row corresponding to that category is deleted from the table.

[2] The learning outcomes relevant for the degree programme and targeted by the subject for which the course description is prepared will be listed. The entries, copied without any changes from the Curriculum by subject type (Core Subject/Specialisation Subject/Complementary Subject), are listed under the corresponding competency.

8. Contents

8.1. Course	Teaching and learning methods	Remarks[3]
1. Complements of linear algebra (linear spaces, linear subspaces, linear hull, base, linear operators, linear functionals, sublinear functionals)	Lecture with mathematical proofs, problematization, discussion	
2. Complements of linear algebra (Helly's Lemma; the Hahn-Banach Theorem for real linear spaces)	Lecture with mathematical proofs, problematization, discussion	
3. Complements of linear algebra (the Hahn-Banach Theorem for complex linear spaces; seminorms; the Bohnenblust-Sobczyk-Suhomlinov Theorem and its consequence). Normed spaces (definition of the norm and of the metric induced by a norm)	Lecture with mathematical proofs, problematization, discussion	
4. Normed spaces (definition of the topology compatible with the algebraic structure of a linear space; definition of the topological linear space; proof of the fact that every normed space is a topological linear space; topological properties of the balls in a normed space)	Lecture with mathematical proofs, problematization, discussion	
5. Normed spaces (series and absolutely convergent series in a normed space; the notion of Cauchy sequence and of complete metric space; the notion of Banach space; the characterization of the completeness of a normed space with the aid of series). Finite dimensional normed spaces (equivalent norms; the characterization of equivalent norms)	Lecture with mathematical proofs, problematization, discussion	
6. Finite dimensional normed spaces (the equivalence of norms in finite dimensional linear spaces; the completeness of finite dimensional normed spaces; the Riesz Lemma)	Lecture with mathematical proofs, problematization, discussion	
7. Finite dimensional normed spaces (characterizations of finite dimensional normed spaces). Inner product spaces (the definition of the inner product; properties of inner products; the notion of inner product space)	Lecture with mathematical proofs, problematization, discussion	
8. Inner product spaces (the continuity of the inner product; the characterization of the norms induced by an inner product; the notion of orthogonality of two vectors; Pythagora's equality)	Lecture with mathematical proofs, problematization, discussion	
9. Inner product spaces (orthonormal families; properties of orthonormal families; orthonormal bases; characterizations of orthonormal bases; the notions of Fourier coefficients and of Fourier expansion)	Lecture with mathematical proofs, problematization, discussion	
10. Inner product spaces (proof of the fact that nonempty, complete and convex subsets of inner product spaces are	Lecture with mathematical proofs, problematization, discussion	

[3] For example, organisational aspects, recommendations for students, specific aspects relating to the course/seminar, such as inviting experts in the field, etc.

Chebyshev sets; the notion of the orthogonal complement of a subset of an inner product space; characterizations of the points of best approximation in a linear subspace; the orthogonal decomposition of a Hilbert space; chracterizations of orthonormal bases in Hilbert spaces)		
11. Linear continuous operators between normed spaces (characterizations of the continuity of linear operators between normed spaces; the normed space of linear continuous operators between normed spaces; linear continuous functionals on normed spaces; the dual of a normed space)	Lecture with mathematical proofs, problematization, discussion	
12. Linear continuous operators between normed spaces (isomorphisms and isometric isomorphisms between normed spaces; the Neumann series associated to a linear continuous operator)	Lecture with mathematical proofs, problematization, discussion	
13. Fundamental results in functional analysis (the uniform boundedness and the pointwise boundedness of a family of linear continuous operators between two normed spaces; the definition of Baire space; the uniform boundedness principle; the open mapping theorem; the bounded inverse theorem)	Lecture with mathematical proofs, problematization, discussion	
14. The extension of linear continuous functionals (the two theorems of Hahn and their consequences). Recap	Lecture with mathematical proofs, problematization, discussion	

Bibliography

1. BRECKNER W. W.: Analiză funcţională, Presa Universitară Clujeană, Cluj-Napoca, 2009.

2. BREZIS H.: Analiză funcţională. Teorie şi aplicaţii, Ed. Academiei Române, Bucureşti, 2002.

3. CONWAY J. B.: A Course in Functional Analysis. Second Edition, Springer-Verlag, New-York –Berlin – Heidelberg, 1999.

4. HEUSER H.: Funktionalanalysis. Theorie und Anwendung, 3. Auflage, B. G. Teubner, Stuttgart, 1992.

5. KANTOROVICI L.V., AKILOV G. P.: Analiză funcţională. Editura Ştiinţifică şi Enciclopedică, Bucureşti, 1986.

6. MUNTEAN I.: Analiză funcţională, Universitatea "Babeş-Bolyai", Cluj-Napoca, 1993.

7. PRECUPANU T.: Analiză funcţională pe spaţii liniare normate, Editura Universităţii "Alexandru Ioan Cuza", Iaşi, 2005.

8. WERNER D.: Funktionalanalysis, Vierte, überarbeitete Auflage., Springer-Verlag, Berlin - Heidelberg - New York, 2002.

8.2. Seminar/ laboratory	**Teaching and learning methods**	**Remarks**
1. Complements of linear algebra	Problematization, discussion, team work	
2. Complements of linear algebra (the relationship between complex linear functionals and real linear functionals; applications of the Hahn-Banach Theorem for real linear spaces)	Problematization, discussion, team work	
3. Complements of linear algebra (seminorms; an application of the Hahn-Banach Theorem); Minkowski's	Problematization, discussion, team work	

inequality. Examples of norms (the $\|\cdot\|_p$ norms on $\mathbf{K}^m$; the supremum norm on B(T, **K**); the norm on l^p, where p is a real number greater or equal than 1)		
4. Normed spaces (the continuity of the norm; the characterization of the distances on a linear space that are induced by norms; examples of distances that are not induced by norms; properties of the spaces l^p)	Problematization, discussion, team work	
5. Normed spaces (the notion of the sum of a family of points and notions related to this concept; unconditionally convergent series; bounded sets and sequences; examples of equivalent norms on $\mathbf{K}^m$)	Problematization, discussion, team work	
6. Examples of Banach spaces (the spaces B(T, **K**), CB(T, **K**) and C(T, **K**)). Examples of equivalent norms. An example of a non-complete normed space	Problematization, discussion, team work	
7. Examples of Banach spaces (the spaces C^1([a,b]), l^∞, c, c_0 and l^p). An example of a bounded sequence without any convergent subsequence. An exemple of a non-complete normed space	Problematization, discussion, team work	
8. Inner product spaces (examples and properties). Hilbert spaces (definition andexamples). Orthogonality in inner product spaces	Problematization, discussion, team work	
9. The Chebyshev approximation problem. The notions of best approximation point, Chebyshev set, proximinal set, and best approximation problem in a normed space. Determination of the best approximation points in concrete cases. The Bessel inequality in inner product spaces	Problematization, discussion, team work	
10. Examples of orthonormal bases. The orthogonal decomposition of Hilbert spaces	Problematization, discussion, team work	
11. The determination of the norm of linear continuous operators/functionals	Problematization, discussion, team work	
12. The pointwise convergence of a sequence of linear continuos operators between two normed spaces. The Frechet-Riesz Theorem concerning the general form of linear continuous functionals on Hilbert spaces. Applications of the Frechet-Riesz Theorem	Problematization, discussion, team work	
13. The general form of linear continuous functionals on the l^p normed spaces, where p is a real number greater or equal than 1. Applications of the uniform boundedness principle and of the open mapping theorem	Problematization, discussion, team work	
14. Applications of the theorems of Hahn. Applications of the bounded inverse theorem. An example of a linear operator which is not continuous	Problematization, discussion, team work	

Bibliography
1. BREZIS H.: Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, 2011.

2. HEUSER H.: Funktionalanalysis. Theorie und Anwendung, 3. Auflage. B. G. Teubner, Stuttgart, 1992.

3. POPA E.: Culegere de probleme de analiză funcţională, Editura Didactică şi Pedagogică, Bucureşti, 1981.

4. WERNER D.: Funktionalanalysis. Vierte, überarbeitete Auflage, Springer-Verlag, Berlin - Heidelberg - New York, 2002 .

9. Evaluation

Type of activity	9.1 Evaluation criteria[4]	9.2 Evaluation methods[5]	9.3 Percentage in the final grade
9.4. Course	Knowledge of concepts and basic results	There will be a midterm test concerning the contents of the first 7 lectures and seminars. Those who are pleased with their grade, will perform the written exam at the end of the semester concerning only the contents of the last 7 lectures and seminars. In this case, the final grade will be the arithmetic mean of the two grades. Those who are not pleased with the grade obtained in the midterm test, will perform the exam concerning the whole contents of the lectures and seminars. In this case, the final grade will be the grade obtained in the exam. The retaken exam will be about the whole contents of the lectures and seminars.	
9.5. Seminar/ laboratory	Ability to apply theoretical results to solve exercises and problems		
9.6 Minimum standard for passing			
1. The ability to prove that a functional is a norm/seminorm 2. The ability to prove that a linear operator/functional is continuous and to determine its norm 3. Basic knowledge on the topics from the lectures and seminars			

[4] The evaluation criteria must directly reflect the learning outcomes targeted at the level of the degree programme respectively at the level of the subject. More specifically, the learning outcomes set out in the expected learning outcomes are assessed.

[5] Both final evaluation methods and ongoing evaluation strategies should be established.

10. SDG labels (Sustainable Development Goals)[6]

	○	Sustainable Development Generic Label

								
○	○	○	○	○	○	○	○	X
								No label applies
○	○	○	○	○	○	○	○	○

Date of entry:
15.04.2026

Signature of course coordinator

Assoc. Prof. dr. Brigitte E. Breckner

Signature of seminar coordinator

Assoc. Prof. dr. Brigitte E. Breckner

Date of approval in the department:
24.04.2026

Signature of the head of department

Prof. dr. Andrei Mărcuş

[6] Select a single label which, according to the *Implementation of SDG labels in the academic process*, best matches the subject. If the subject addresses sustainable development in a generic manner (i.e. by presenting/introducing the general framework of sustainable development, etc.), then the Sustainable Development generic label may be applied. If none of the labels describe the subject, select the last option: *"No label applies."*