COURSE DESCRIPTION

Real Analysis

Academic year 2026-2027

1. Programme-related data

1.1. Higher Education Institution	Babeș-Bolyai University
1.2. Faculty	Mathematics and Computer Science
1.3. Department	Mathematics
1.4. Field	Mathematics
1.5. Level of study	Bachelor of Science
1.6. Degree programme / Qualification	Mathematics Computer Science (in English)
1.7. Form of education	Full-time

2. Course-related data

2.1. Course title	**Real Analysis**			Course code	**MLE0074**
2.2. Course coordinator	Conf. dr. Adriana Nicolae				
2.3. Seminar coordinator	Conf. dr. Adriana Nicolae				
2.4. Year of study	**2**	2.5. Semester	**4**	2.6. Type of assessment	Viva voce
2.7. Course status	Compulsory			2.8. Course type	Core subject

3. Total estimated time (hours per semester of teaching activities)

3.1. Number of hours per week	**4**	of which: 3.2. course	**2**	3.3. seminar/ laboratory/ project	**2**
3.4. Total of hours in the curriculum	**56**	of which: 3.5. course	**28**	3.6. seminar/ laboratory	**28**
Time allocation for individual study (IS) and self-taught activities (ST)					**hours**
Learning from textbooks, course materials, bibliography, and notes (IS)					25
Additional research in the library, on subject-specific electronic platforms, and on-site					10
Preparing seminars/ laboratories/ projects, assignments, reports, portfolios, and essays					20
Tutoring (professional guidance)					4
Examinations					10
Other activities					
3.7. Total hours of individual study (IS) and self-taught activities (ST)				**69**	
3.8. Total hours per semester				**125**	
3.9. Number of credits				**5**	

4. Prerequisites (where applicable)

4.1. curriculum-related	Calculus 1, 2; Mathematical Logic and Set Theory
4.2 skills-related	Analytic thinking

5. Specific conditions (where applicable)

5.1. course-related	Lecture hall equipped with blackboard and chalk
5.2. seminar/laboratory-related	Classroom equipped with blackboard and chalk

6.1. Competencies resulting from the completion of the degree programme *(as referred to in the curriculum)*[1]

Professional competencies	
Competency code	**Competency**
PC1	develop problem-solving strategies
PC6	think abstractly
PC8	study relationships between quantities
Transversal competencies	
Competency code	**Competency**
TC4	Solve problems
TC5	Think analytically

6.2. Learning outcomes relevant to the degree programme (as referred to in the curriculum)[2]

Learning outcomes targeted by the subject		
Competency code	**Knowledge and comprehension**	**Specific academic skills**
PC1, PC8	3. The student/graduate formulates observations and differentiates notions, properties, and assertions from the core disciplines of mathematics through examples and counterexamples.	3. The student/graduate identifies and describes the essential elements in the construction of proofs of mathematical assertions (lemmas, propositions, theorems), recognizes errors in reasoning, and corrects them.
PC6	4. The student/graduate defines the basic concepts from advanced mathematics disciplines in the curriculum.	4. The student/graduate answers questions and correctly and rigorously formulates the statements of mathematical assertions (lemmas, propositions, theorems) from the disciplines in the curriculum.
TC4	2. The student/graduate compares and distinguishes related notions and their properties from the core disciplines of mathematics.	2. The student/graduate recognizes and analyzes the necessary and/or sufficient conditions in the statements of mathematical assertions and specifies their role in the proof.
TC5	5. The student/graduate compares and distinguishes related notions and their properties from the advanced mathematics disciplines in the curriculum.	5. The student/graduate reproduces and analyzes the hypotheses and conclusions of mathematical assertions and discusses how these connect within the proof.

7. Subject-specific learning outcomes

Knowledge and comprehension
1. The student has acquired basic knowledge specific to the discipline: σ-algebras, measures, the Lebesque exterior measure, the Lebesgue measure, integration of measurable functions, limit theorems, L^p spaces, Fubini's theorem.
2. The student understands fundamental notions of measure and integration theory, as well as methods for applying them in scientific fields related to mathematics and computer science.

[1] The professional and/or transversal skills targeted by the subject for which the course description is prepared will be copied from the curriculum of the degree programme. For each competency, the complete entry, including the competency code, will be copied with the exact wording that appears in the curriculum, without any changes. If no competency is copied from either of the two categories, the row corresponding to that category is deleted from the table.

[2] The learning outcomes relevant for the degree programme and targeted by the subject for which the course description is prepared will be listed. The entries, copied without any changes from the Curriculum by subject type (Core Subject/Specialisation Subject/Complementary Subject), are listed under the corresponding competency.

Specific academic skills
1. The student is able to construct clear and well-supported mathematical arguments to explain mathematical problems, topics, and ideas both orally and in writing.
2. The student is able to prove theorems using mathematical language in theoretical courses and will be able to present these results both orally and in writing.

8. Contents

8.1. Course	**Teaching and learning methods**	**Remarks[3]**
1. Introduction: the problem of measure. Measurable spaces and measure spaces	Lecture, discussion, didactical demonstration, problematisation	
2. The Lebesgue exterior measure	Lecture, discussion, didactical demonstration, problematisation	
3. The Lebesgue measure	Lecture, discussion, didactical demonstration, problematisation	
4. Properties of the Lebesgue measure	Lecture, discussion, didactical demonstration, problematisation	
5. Measurable functions	Lecture, discussion, didactical demonstration, problematisation	
6. Operations with measurable functions. Approximation of measurable functions	Lecture, discussion, didactical demonstration, problematisation	
7. Properties that hold almost everywhere. Littlewood's three principles	Lecture, discussion, didactical demonstration, problematisation	
8. Integration of measurable functions. Limit theorems and applications (I)	Lecture, discussion, didactical demonstration, problematisation	
9. Integration of measurable functions. Limit theorems and applications (II)	Lecture, discussion, didactical demonstration, problematisation	
10. The relation between the Riemann and Lebesgue integrals.	Lecture, discussion, didactical demonstration, problematisation	
11. L^p spaces (I)	Lecture, discussion, didactical demonstration, problematisation	
12. L^p spaces (II)	Lecture, discussion, didactical demonstration, problematisation	
13. Types of convergence	Lecture, discussion, didactical demonstration, problematisation	
14. Measure and integration on product spaces	Lecture, discussion, didactical demonstration, problematisation	

Bibliography

1. V. Anisiu, Topologie şi teoria măsurii, Universitatea "Babeş-Bolyai", Cluj-Napoca, 1993.
2. J.J. Benedetto, W. Czaja, Integration and modern analysis, Birkhäuser, Boston, MA, 2009.
3. D.L. Cohn, Measure theory, 2nd ed., Birkhäuser/Springer, New York, 2013.
4. G.B. Folland, Real analysis. Modern techniques and their applications, 2nd ed., John Wiley & Sons, Inc., New York, 1999.
5. F. Jones, Lebesgue integration on Euclidean space, Jones and Bartlett Publishers, Boston, MA, 1993.
6. H.L. Royden, P.M. Fitzpatrick, Real analysis, 4th ed., Pearson, 2010.
7. W. Rudin, Real and complex analysis, 3rd ed., McGraw-Hill Book Co., New York, 1987.
8. E. Stein, R. Shakarchi, Real analysis. Measure theory, integration, and Hilbert spaces, Princeton University Press, Princeton, NJ, 2005.
9. D.W. Stroock, A concise introduction to the theory of integration, 2nd ed., Birkhäuser Boston, Inc., Boston, MA, *1994.*
10. T. Tao, An introduction to measure theory, American Mathematical Society, Providence, RI, 2011.

8.2. Seminar/ laboratory	**Teaching and learning methods**	**Remarks**
1. A brief review of some definitions and properties	Discussion, problem solving, didactical demonstration	
2. Measurable spaces	Discussion, problem solving, didactical demonstration	

[3] For example, organisational aspects, recommendations for students, specific aspects relating to the course/seminar, such as inviting experts in the field, etc.

3. Measure spaces	Discussion, problem solving, didactical demonstration	
4. The Lebesgue exterior measure	Discussion, problem solving, didactical demonstration	
5. Lebesgue measurability and the Lebesgue measure (I)	Discussion, problem solving, didactical demonstration	
6. Lebesgue measurability and the Lebesgue measure (II)	Discussion, problem solving, didactical demonstration.	
7. Measurable functions	Discussion, problem solving, didactical demonstration	
8. Properties that hold almost everywhere. Integration of nonnegative measurable functions	Discussion, problem solving, didactical demonstration	
9. Integration of arbitrary measurable functions	Discussion, problem solving, didactical demonstration	
10. Limit theorems and applications (I)	Discussion, problem solving, didactical demonstration	
11. Limit theorems and applications (II). Differentiation under the integral sign	Discussion, problem solving, didactical demonstration	
12. The relation between the Riemann and Lebesgue integrals.	Discussion, problem solving, didactical demonstration	
13. L^p spaces	Discussion, problem solving, didactical demonstration	
14. Measure and integration on product spaces	Discussion, problem solving, didactical demonstration	
Bibliography (in addition to the books mentioned before which also contain exercises) 1. R.L. Schilling, Measures, integrals and martingales, Cambridge University Press, New York, 2005. 2. W.J. Kaczor, M.T. Nowak, Problems in Mathematical Analysis III. Integration, American Mathematical Society, Providence, RI, 2003. 3. A. Torchinsky, Problems in real and functional analysis, American Mathematical Society, Providence, RI, 2015.		

9. Evaluation

Type of activity	9.1 Evaluation criteria[4]	9.2 Evaluation methods[5]	9.3 Percentage in the final grade
9.4. Course	- Knowledge of basic notions, examples and results - Ability to prove theoretical results	- Test, viva voce - Lecture and seminar activity	- Test: 35% - Viva voce: 65% - Lecture and seminar activity: bonus max. 10%
9.5. Seminar/ laboratory	- Problem solving using concepts and results acquired during the lecture classes		
9.6 Minimum standard for passing			
- The accumulation of at least 10 attendances at the seminar. - Both the test grade at the end of the semester and the final grade should be at least 5. The bonus points are only awarded in this case.			

10. SDG labels (Sustainable Development Goals)[6]

[4] The evaluation criteria must directly reflect the learning outcomes targeted at the level of the degree programme respectively at the level of the subject. More specifically, the learning outcomes set out in the expected learning outcomes are assessed.

[5] Both final evaluation methods and ongoing evaluation strategies should be established.

[6] Select a single label which, according to the *Implementation of SDG labels in the academic process*, best matches the subject. If the subject addresses sustainable development in a generic manner (i.e. by presenting/introducing the general framework of sustainable development, etc.), then the Sustainable Development generic label may be applied. If none of the labels describe the subject, select the last option: *"No label applies."*

		Sustainable Development Generic Label

								
								X
								No label applies

Date of entry:
10.04.2026

Signature of course coordinator

Conf. dr. Adriana Nicolae

Signature of seminar coordinator

Conf. dr. Adriana Nicolae

Date of approval in the department:
24.04.2026

Signature of the head of department

Prof. dr. Andrei Mărcuş