

## COURSE DESCRIPTION

### Complements of Geometry

Academic year 2026-2027

#### 1. Programme-related data

1.1. Higher Education Institution	Babeş-Bolyai University
1.2. Faculty	Mathematics and Computer Science
1.3. Department	Mathematics
1.4. Field	Mathematics
1.5. Level of study	Bachelor
1.6. Degree programme / Qualification	Mathematics and Computer Science (English)
1.7. Form of education	Full-time

#### 2. Course-related data

2.1. Course title	<b>Complements of Geometry</b>			Course code	<b>MLE0041</b>
2.2. Course coordinator	Lect. univ. dr. George-Cătălin Țurcaș				
2.3. Seminar coordinator	Lect. univ. dr. George-Cătălin Țurcaș				
2.4. Year of study	2	2.5. Semester	2	2.6. Type of assessment	Progress check
2.7. Course status	Optional			2.8. Course type	Specialisation subject

#### 3. Total estimated time (hours per semester of teaching activities)

3.1. Number of hours per week	4	of which: 3.2. course	2	3.3. seminar/ laboratory/ project	2
3.4. Total of hours in the curriculum	56	of which: 3.5. course	28	3.6. seminar/ laboratory	28
<b>Time allocation for individual study (IS) and self-taught activities (ST)</b>					<b>hours</b>
Learning from textbooks, course materials, bibliography, and notes (IS)					20
Additional research in the library, on subject-specific electronic platforms, and on-site					20
Preparing seminars/ laboratories/ projects, assignments, reports, portfolios, and essays					30
Tutoring (professional guidance)					14
Examinations					10
Other activities					
<b>3.7. Total hours of individual study (IS) and self-taught activities (ST)</b>				<b>94</b>	
<b>3.8. Total hours per semester</b>				<b>150</b>	
<b>3.9. Number of credits</b>				<b>6</b>	

#### 4. Prerequisites (where applicable)

4.1. curriculum-related	Completion of <b>Analytic Geometry (first year)</b> and an understanding of vector representation of lines and planes in 3D; completion of <b>Analysis II (second semester)</b> and working knowledge of differentiation and integration of vector functions.
4.2. skills-related	Proficiency in differentiating and integrating vector functions; ability to represent lines and planes in three-dimensional space using vector notation.

#### 5. Specific conditions (where applicable)

5.1. course-related	Blackboard, chalk, projector
5.2. seminar/laboratory-related	Blackboard, chalk, projector

#### 6.1. Competencies resulting from the completion of the degree programme (as referred to in the curriculum)<sup>1</sup>

<sup>1</sup> The professional and/or transversal skills targeted by the subject for which the course description is prepared will be copied from the curriculum of the degree programme. For each competency, the complete entry, including

Professional competencies	
Competency code	Competency
CP2	perform analytical mathematical calculations
CP6	think abstractly
CP7	communicate mathematical information
Transversal competencies	
Competency code	Competency
CT4	Solve problems
CT5	Think analytically

## 6.2. Learning outcomes relevant to the degree programme (as referred to in the curriculum)<sup>2</sup>

Learning outcomes targeted by the subject		
Competency code	Knowledge and comprehension	Specific academic skills
CP2	8. The student/graduate defines the concepts from basic computer science and/or applied mathematics disciplines.	8. The student/graduate identifies and applies suitable techniques to solve exercises and problems from the major disciplines of mathematics.
CP6	9. The student/graduate formulates observations and differentiates notions, properties, and assertions from advanced mathematics disciplines through examples and counterexamples.	9. The student/graduate argues the role of elements found in the hypotheses of mathematical assertions, discusses how they articulate within the proof, and independently constructs correct proofs of mathematical assertions from major mathematical disciplines. The student/graduate translates a practical situation into mathematical language, solves the resulting problem, and interprets the obtained results.
CP7	9. The student/graduate formulates observations and differentiates notions, properties, and assertions from advanced mathematics disciplines through examples and counterexamples.	9. The student/graduate argues the role of elements found in the hypotheses of mathematical assertions, discusses how they articulate within the proof, and independently constructs correct proofs of mathematical assertions from major mathematical disciplines. The student/graduate translates a practical situation into mathematical language, solves the resulting problem, and interprets the obtained results.
CT4	9. The student/graduate formulates observations and differentiates notions, properties, and assertions from advanced mathematics disciplines through examples and counterexamples.	8. The student/graduate identifies and applies suitable techniques to solve exercises and problems from the major disciplines of mathematics.

the competency code, will be copied with the exact wording that appears in the curriculum, without any changes. If no competency is copied from either of the two categories, the row corresponding to that category is deleted from the table.

<sup>2</sup> The learning outcomes relevant for the degree programme and targeted by the subject for which the course description is prepared will be listed. The entries, copied without any changes from the Curriculum by subject type (Core Subject/Specialisation Subject/Complementary Subject), are listed under the corresponding competency.

<b>CT5</b>	9. The student/graduate formulates observations and differentiates notions, properties, and assertions from advanced mathematics disciplines through examples and counterexamples.	9. The student/graduate argues the role of elements found in the hypotheses of mathematical assertions, discusses how they articulate within the proof, and independently constructs correct proofs of mathematical assertions from major mathematical disciplines. The student/graduate translates a practical situation into mathematical language, solves the resulting problem, and interprets the obtained results.
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## 7. Subject-specific learning outcomes

<b>Knowledge and comprehension</b>
1. Understand the fundamental concepts of differential geometry of curves and surfaces.
2. Understand the role of parametrizations, tangent objects, and local geometric invariants.
3. Understand the geometric meaning of curvature and its relation to the local behavior of curves and surfaces.
4. Understand the interplay between analytic methods, geometric intuition, and rigorous mathematical reasoning in differential geometry.
<b>Specific academic skills</b>
1. Compute and use parametrizations, tangent vectors, normal vectors, arc length, and curvature in the study of curves and surfaces.
2. Solve problems involving the local geometry of curves and surfaces using appropriate analytical and geometric methods.
3. Formulate, justify, and communicate rigorous arguments in differential geometry using correct mathematical language and notation.

## 8. Contents

8.1. Course	Teaching and learning methods	Remarks <sup>3</sup>
Week 1. Definition of curves; Arc-length	Lecture, demonstration, examples	
Week 2. Reparameterization; Closed curves	Lecture, demonstration, examples	
Week 3. Level curves vs parameterized curves; Definition of curvature; Curvature for plane curves	Lecture, demonstration, examples	
Week 4. Signed curvature determines the curve; Space curves	Lecture, demonstration, examples	
Week 5. Global properties of curves; The isoperimetric inequality; The four vertex theorem	Lecture, demonstration, examples	
Week 6. First partial exam	Partial exam (assessment)	
Week 7. Definition of surfaces; Smooth surfaces; Smooth maps	Lecture, demonstration, examples	
Week 8. Tangents and derivatives; Normals and orientability	Lecture, demonstration, examples	
Week 9. The first fundamental form	Lecture, demonstration, examples	
Week 10. Curvature of surfaces; Second fundamental form	Lecture, demonstration, examples	
Week 11. Gaussian, mean and principal curvatures	Lecture, demonstration, examples	

<sup>3</sup> For example, organisational aspects, recommendations for students, specific aspects relating to the course/seminar, such as inviting experts in the field, etc.

Week 12. Gauss' Theorema Eggregium	Lecture, demonstration, examples	
Week 13. The Gauss-Bonnet Theorem	Lecture, demonstration, examples	
Week 14. Second partial exam	Partial exam (assessment)	
Bibliography		
1. A. Pressley, <i>Elementary Differential Geometry</i> , 2nd ed., Springer, London, 2010. 2. M. P. do Carmo, <i>Differential Geometry of Curves and Surfaces</i> , revised and updated 2nd ed., Dover, New York, 2016. 3. B. O'Neill, <i>Elementary Differential Geometry</i> , 2nd ed., Academic Press, 2006. 4. T. F. Banchoff and S. T. Lovett, <i>Differential Geometry of Curves and Surfaces</i> , 2nd ed., CRC Press, 2016.		
<b>8.2. Seminar/ laboratory</b>	<b>Teaching and learning methods</b>	<b>Remarks</b>
Week 1. Problems with the definition of curves; Arc-length	Examples, dialogue, explanation, demonstration, problem-solving	
Week 2. Reparameterization; Closed curves	Examples, dialogue, explanation, demonstration, problem-solving	
Week 3. Level curves vs parameterized curves; Definition of curvature; Curvature for plane curves	Examples, dialogue, explanation, demonstration, problem-solving	
Week 4. Signed curvature determines the curve; Space curves	Examples, dialogue, explanation, demonstration, problem-solving	
Week 5. Global properties of curves; The isoperimetric inequality; The four vertex theorem	Examples, dialogue, explanation, demonstration, problem-solving	
Week 6. Discussion of the partial exam. More problems with curves	Examples, dialogue, explanation, demonstration, problem-solving	
Week 7. Definition of surfaces; Smooth surfaces; Smooth maps	Examples, dialogue, explanation, demonstration, problem-solving	
Week 8. Tangents and derivatives; Normals and orientability	Examples, dialogue, explanation, demonstration, problem-solving	
Week 9. The first fundamental form	Examples, dialogue, explanation, demonstration, problem-solving	
Week 10. Curvature of surfaces; Second fundamental form	Examples, dialogue, explanation, demonstration, problem-solving	
Week 11. Gaussian, mean and principal curvatures	Examples, dialogue, explanation, demonstration, problem-solving	
Week 12. Gauss' Theorema Eggregium	Examples, dialogue, explanation, demonstration, problem-solving	
Week 13. The Gauss-Bonnet Theorem	Examples, dialogue, explanation, demonstration, problem-solving	
Week 14. Discussion of the final partial exam. More problems with curves	Examples, dialogue, explanation, demonstration, problem-solving	
Bibliography		
1. A. Pressley, <i>Elementary Differential Geometry</i> , 2nd ed., Springer, London, 2010. 2. M. P. do Carmo, <i>Differential Geometry of Curves and Surfaces</i> , revised and updated 2nd ed., Dover, New York, 2016. 3. B. O'Neill, <i>Elementary Differential Geometry</i> , 2nd ed., Academic Press, 2006. 4. T. F. Banchoff and S. T. Lovett, <i>Differential Geometry of Curves and Surfaces</i> , 2nd ed., CRC Press, 2016.		

## 9. Evaluation















Type of activity	9.1 Evaluation criteria <sup>4</sup>	9.2 Evaluation methods <sup>5</sup>	9.3 Percentage in the final grade
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<sup>4</sup> The evaluation criteria must directly reflect the learning outcomes targeted at the level of the degree programme respectively at the level of the subject. More specifically, the learning outcomes set out in the expected learning outcomes are assessed.

<sup>5</sup> Both final evaluation methods and ongoing evaluation strategies should be established.

9.4. Course	Knowledge of fundamental concepts and results; problem-solving ability	Two written tests	80%
9.5. Seminar/ laboratory	Problem-solving ability based on learned concepts and theorems; Ability to synthesize mathematical content and effectively communicate it through clear presentations	Homework; solving problems at the board; individual projects and presentations	20%
9.6 Minimum standard for passing			
Obtaining a minimum weighted average of 5 (on a scale from 1 to 10), calculated according to the evaluation percentages specified above (80% course assessments and 20% seminar activities).			

## 10. SDG labels (Sustainable Development Goals)<sup>6</sup>

	<input type="radio"/>	Sustainable Development Generic Label						
								
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	X
								No label applies
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Date of entry:  
15.04.2026

Signature of course coordinator

Lector univ. dr. George Țurcaș

Signature of seminar coordinator

Lector univ. dr. George Țurcaș

Date of approval in the department:  
24.04.2026

Signature of the head of department

Prof. dr. Andrei Mărcuș

<sup>6</sup> Select a single label which, according to the [Implementation of SDG labels in the academic process](#), best matches the subject. If the subject addresses sustainable development in a generic manner (i.e. by presenting/introducing the general framework of sustainable development, etc.), then the Sustainable Development generic label may be applied. If none of the labels describe the subject, select the last option: "No label applies."

