

## COURSE DESCRIPTION

### Calculus 2 (Differential and integral calculus in $\mathbb{R}^n$ )

*Academic year 2026-2027*

#### 1. Programme-related data

1.1. Higher Education Institution	<b>Babeş-Bolyai University</b>
1.2. Faculty	<b>Mathematics and Computer Science</b>
1.3. Department	<b>Mathematics</b>
1.4. Field	<b>Mathematics</b>
1.5. Level of study	<b>Bachelor</b>
1.6. Degree programme / Qualification	<b>Mathematics Computer Science (English)</b>
1.7. Form of education	<b>Full-time</b>

#### 2. Course-related data

2.1. Course title	<b>Calculus 2 (Differential and integral calculus in <math>\mathbb{R}^n</math>)</b>			Course code	<b>MLE0071</b>
2.2. Course coordinator	<i>Conf. dr. Trif Tiberiu</i>				
2.3. Seminar coordinator	<i>Conf. dr. Trif Tiberiu</i>				
2.4. Year of study	1	2.5. Semester	2	2.6. Type of assessment	<b>Exam</b>
2.7. Course status	<b>Compulsory</b>			2.8. Course type	<b>Core subject</b>

#### 3. Total estimated time (hours per semester of teaching activities)

3.1. Number of hours per week	<b>6</b>	of which: 3.2. course	<b>3</b>	3.3. seminar/ laboratory/ project	<b>3</b>
3.4. Total of hours in the curriculum	<b>84</b>	of which: 3.5. course	<b>42</b>	3.6. seminar/ laboratory	<b>42</b>
<b>Time allocation for individual study (IS) and self-taught activities (ST)</b>					<b>hours</b>
<i>Learning from textbooks, course materials, bibliography, and notes (IS)</i>					20
<i>Additional research in the library, on subject-specific electronic platforms, and on-site</i>					10
<i>Preparing seminars/ laboratories/ projects, assignments, reports, portfolios, and essays</i>					20
<i>Tutoring (professional guidance)</i>					6
<i>Examinations</i>					10
<i>Other activities</i>					
<b>3.7. Total hours of individual study (IS) and self-taught activities (ST)</b>				<b>66</b>	
<b>3.8. Total hours per semester</b>				<b>150</b>	
<b>3.9. Number of credits</b>				<b>6</b>	

#### 4. Prerequisites (where applicable)

4.1. curriculum-related	- <i>Calculus 1 (Calculus in <math>\mathbb{R}</math>)</i>
4.2 skills-related	- <i>ability to perform symbolic calculations</i> - <i>ability to operate with abstract concepts</i> - <i>ability to do logical deductions</i> - <i>ability to solve math problems based on acquired notions</i>

#### 5. Specific conditions (where applicable)

5.1. course-related	<i>blackboard, chalk, video projector</i>
5.2. seminar/laboratory-related	<i>blackboard, chalk</i>

#### 6.1. Competencies resulting from the completion of the degree programme (as referred to in the curriculum)<sup>1</sup>

<sup>1</sup> The professional and/or transversal skills targeted by the subject for which the course description is prepared will be copied from the curriculum of the degree programme. For each competency, the complete entry, including the competency code, will be copied with the exact wording that appears in the curriculum, without any changes. If no competency is copied from either of the two categories, the row corresponding to that category is deleted from the table.

Professional competencies	
Competency code	Competency
CP2	<i>perform analytical mathematical calculations</i>
CP6	<i>think abstractly</i>
CP8	<i>study relationships between quantities</i>
Transversal competencies	
Competency code	Competency
CT4	<i>solve problems</i>
CT5	<i>think analytically</i>

## 6.2. Learning outcomes relevant to the degree programme (as referred to in the curriculum)<sup>2</sup>

Learning outcomes targeted by the subject		
Competency code	Knowledge and comprehension	Specific academic skills
CP2	<i>7. The student/graduates selects, explains, and specifies the mathematical foundations applied in computer science, including formal logic, algebra, probability, and statistics.</i>	<i>7. The student/graduates applies, evaluates, and proposes mathematical methods for modeling, simulating, and solving computer science problems.</i>
CP6	<i>4. The student/graduate defines the basic concepts from advanced mathematics disciplines in the curriculum.</i>	<i>4. The student/graduate answers questions and correctly and rigorously formulates the statements of mathematical assertions (lemmas, propositions, theorems) from the disciplines in the curriculum.</i>
CP8	<i>3. The student/graduate formulates observations and differentiates notions, properties, and assertions from the core disciplines of mathematics through examples and counterexamples.</i>	<i>3. The student/graduate identifies and describes the essential elements in the construction of proofs of mathematical assertions (lemmas, propositions, theorems), recognizes errors in reasoning, and corrects them.</i>
CT4, CT5	<i>2. The student/graduate compares and distinguishes related notions and their properties from the core disciplines of mathematics.</i>	<i>2. The student/graduate recognizes and analyzes the necessary and/or sufficient conditions in the statements of mathematical assertions and specifies their role in the proof.</i>

## 7. Subject-specific learning outcomes

Knowledge and comprehension
<i>1. The student knows the topology of the Euclidean space <math>R^n</math>, the differential calculus of functions of several variables, as well as different types of integrals for functions of several variables (multiple integrals, line and surface integrals).</i>
Specific academic skills
<i>1. The student is able to construct clear and well-supported mathematical arguments to explain mathematical problems, topics, and ideas in writing.</i>
<i>2. The student is able to prove theorems using mathematical language in theoretical courses and will be able to present these results both orally and in writing.</i>

## 8. Contents

8.1. Course	Teaching and learning methods	Remarks <sup>3</sup>
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<sup>2</sup> The learning outcomes relevant for the degree programme and targeted by the subject for which the course description is prepared will be listed. The entries, copied without any changes from the Curriculum by subject type (Core Subject/Specialisation Subject/Complementary Subject), are listed under the corresponding competency.

<i>Week 1. Topology in <math>\mathbf{R}^n</math>: the Euclidean space <math>\mathbf{R}^n</math> (the inner product, the Euclidean norm, the Euclidean distance), the topological structure of <math>\mathbf{R}^n</math> (balls, neighbourhoods, interior points, adherent points, boundary points, and limit points, open and closed sets). Sequences in <math>\mathbf{R}^n</math>: convergent and Cauchy sequences, characterization of adherent points, of limit points, and of closed sets by means of sequences.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 2. Compact sets in <math>\mathbf{R}^n</math>: definition of compact sets, examples of compact sets in <math>\mathbf{R}^n</math>, characterization of compact sets in <math>\mathbf{R}^n</math>. Limits of vector functions of vector variable: definition of the limit, characterization of the limit by means of sequences, operations with functions having a limit.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 3. Continuity of vector functions of vector variable: definition of the continuity at a point, characterization of the continuity by means of sequences, operations with continuous functions, the Weierstrass theorem. Linear mappings and their norm.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 4. Differentiability in <math>\mathbf{R}^n</math>: the derivative of a vector function of a real variable, the mean value theorem for vector functions of a real variable. Differentiability of vector functions of vector variable (definition of the Frechet differential, continuity of Frechet differentiable functions, derivative vs differential for vector functions of a real variable).</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 5. Differentiability in <math>\mathbf{R}^n</math>: the directional derivative of a vector function of vector variable and its relationship with the Frechet differential, partial derivatives and their relationship with the Frechet differential. The chain rule, the differentiability of the inverse function.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 6. Differentiability in <math>\mathbf{R}^n</math>: mean value theorems for functions of several variables. Functions of the class <math>C^1</math>. The inverse function theorem, the implicit function theorem.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 7. Differentiability in <math>\mathbf{R}^n</math>: Lagrange multipliers, second order partial derivatives, the Schwarz and Young theorems concerning the mixed partial derivatives. Necessary and sufficient conditions for extrema.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 8. Riemann integral on a compact interval in <math>\mathbf{R}^n</math>: definition of the Riemann integral on a compact interval in <math>\mathbf{R}^n</math>. Computation of Riemann integrals on compact intervals by means of iterated integrals (Fubini's theorem).</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 9. Riemann integral on bounded sets in <math>\mathbf{R}^n</math>: computation of Riemann integrals on bounded sets in <math>\mathbf{R}^n</math> by means of iterated integrals (Fubini's theorem). Change of variables in multiple integrals. Applications in physics of multiple integrals: centres of gravity and moments of inertia.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 10. Vector functions of bounded variation: definition, examples, properties of the total variation. Additivity of the total variation with respect to the interval, the Jordan representation theorem, computation of the total variation for functions of the class <math>C^1</math>.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 11. Line integrals: paths, examples, equivalent paths, curves and oriented curves. First degree differential forms. Integration of first degree differential forms along a path (the line integral of the second kind), mechanical work.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 12. Line integrals: the Green formula, integration of exact differential forms, the Leibniz-Newton formula, the Poincaré theorem concerning the integration of exact differential forms, mechanical work in the gravitational field.</i>	<i>Explanation, dialogue, examples, proofs</i>	

<sup>3</sup> For example, organisational aspects, recommendations for students, specific aspects relating to the course/seminar, such as inviting experts in the field, etc.

<i>Week 13. Surface integrals: parametrized surfaces, examples. Differential forms of the second degree and their integrals over parametrized surfaces (surface integrals of the second kind).</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 14. Stokes and Gauss-Ostrogradski formulas.</i>	<i>Explanation, dialogue, examples, proofs</i>	

#### **Bibliography**

1. BALÁZS M., KOLUMBÁN I.: *Matematikai analízis*, Dacia Könyvkiado, Kolozsvár-Napoca, 1978.
2. BOBOC N.: *Analiză matematică. Vol. 2*, Editura Universităţii din Bucureşti, 1998.
3. BRECKNER W. W.: *Analiza matematica. Topologia spatiului  $R^n$* . Universitatea din Cluj-Napoca, 1985.
4. BROWDER A.: *Mathematical Analysis. An Introduction*, Springer-Verlag, New York, 1996.
5. COBZAS ST.: *Analiză matematică (Calcul diferential)*, Presa Universitară Clujeană, Cluj-Napoca, 1997.
6. *Colectiv al catedrei de analiză matematică a Universităţii Bucureşti: Analiză matematică. Vol. 2, Editura didactică și pedagogică, Bucureşti, 1980.*
7. FINTA Z.: *Matematikai Analízis I, II*, Kolozsvári Egyetemi Kiadó, Kolozsvár, 2007
8. FITZPATRICK P.M.: *Advanced Calculus: Second Edition*, AMS, 2006.
9. HEUSER H.: *Lehrbuch der Analysis, Teil 1, 11. Auflage*, B. G. Teubner, Stuttgart, 1994; *Teil 2, 9. Auflage*, B. G. Teubner, Stuttgart, 1995.
10. MEGAN M.: *Bazele analizei matematice, Vol. I + Vol. II*, Editura EUROBIT, Timisoara, 1997. *Vol. III, Editura EUROBIT, Timisoara, 1998.*
11. NICULESCU C. P.: *Calculul integral al funcțiilor de mai multe variabile. Teorie și aplicații*. Editura Universitaria, Craiova, 2002.
12. RUDIN W.: *Principles of Mathematical Analysis, 2nd Edition*, McGraw-Hill, New York, 1964.
13. WALTER W.: *Analysis, I, II*, Springer-Verlag, Berlin, 1990.





<b>8.2. Seminar/ laboratory</b>	<b>Teaching and learning methods</b>	<b>Remarks</b>
<i>Week 1. The Euclidean space <math>R^n</math>: problems concerning the Euclidean space <math>R^n</math>.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 2. Compact sets in <math>R^n</math>: problems concerning compact sets in <math>R^n</math>.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 3. Limits of vector functions of vector variable, continuity of vector functions of vector variable. Linear mappings and their norm: computation of the norm for some concrete linear mappings.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 4. Computation of directional derivatives, partial derivatives, and differentials for concrete functions.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 5. Differentials: study of the Frechet differentiability for concrete functions. Applications to the chain rule.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 6. Mean value theorems for functions of several variables. Diffeomorphisms and implicit functions.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 7. Extrema for functions of several variables.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 8. Calculation of double integrals over rectangles. Calculation of triple integrals over parallelepipeds. Double and triple integrals over simple sets with respect to an axis.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 9. Calculation of double integrals by means of change of variables (polar coordinates).</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 10. Calculation of triple integrals by means of change of variables (spherical coordinates, cylindrical coordinates).</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 11. Line integrals of the first kind: definition, main theoretical results, calculation of line integrals of the first kind along concrete paths.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 12. Line integrals of the second kind: calculation of the integrals of certain first degree differential forms along concrete paths. Integration of some exact differential forms. Applications to the Green formula.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Week 13. Calculation of surface integrals of the first and of</i>	<i>Explanation, dialogue, examples,</i>	

<i>the second kind.</i>	<i>proofs</i>	
<i>Week 14. Problems concerning the Stokes and the Gauss-Ostrogradski formulas.</i>	<i>Explanation, dialogue, examples, proofs</i>	
<i>Bibliography</i> <ol style="list-style-type: none"> <li><i>BUCUR G., CÂMPU E., GAINA S.: Culegere de probleme de calcul diferential si integral, Vol. II, Editura Tehnica Bucuresti 1966. Vol. III, Editura Tehnica, Bucuresti, 1967.</i></li> <li><i>CĂȚINAȘ D. et al.: Calcul integral. Culegere de probleme pentru seminarii, examene și concursuri. Editura U. T. Pres, Cluj-Napoca, 2000.</i></li> <li><i>DE SOUZA P. N., SILVA J.-N.: Berkeley Problems in Mathematics. Springer, 1998.</i></li> <li><i>DONCIU N., FLONDOR D.: Analiză matematică. Culegere de problema. Vol. 2, Editura All, București, 1998.</i></li> <li><i>KACZOR W. J., NOWAK M. T.: Problems in Mathematical Analysis III: Integration. American Mathematical Society, 2003.</i></li> <li><i>KEDLAYA K. S., POONEN B., VAKIL R.: The William Lowell Putnam Mathematical Competition 1985 – 2000. Problems, Solutions, and Commentary. The Mathematical Association of America, 2002.</i></li> <li><i>RĂDULESCU S., RĂDULESCU M.: Teoreme și probleme de analiză matematică. Editura Didactică și Pedagogică, București, 1982.</i></li> <li><i>TRIF T.: Probleme de calcul diferential si integral în <math>R^n</math>, Universitatea Babes-Bolyai, Cluj-Napoca, 2003.</i></li> </ol>		









## 9. Evaluation

<i>Activity type</i>	<i>9.1 Evaluation criteria</i>	<i>9.2 Evaluation methods</i>	<i>9.3 Percentage of final grade</i>
<i>9.4 Course</i>	<i>Knowledge of fundamental notions and results</i>	<i>Written paper plus discussion on it</i>	<i>90%</i>
<i>9.5 Seminar/laboratory</i>	<i>Solving problems based on learned notions and theorems</i>	<i>Solving exercises on the blackboard</i>	<i>10%</i>
<i>9.6 Minimum standard of performance</i>			
<ul style="list-style-type: none"> <li><i>to acquire minimum 5 (out of 10) points to pass the exam</i></li> </ul>			

## 10. SDG labels (Sustainable Development Goals)<sup>4</sup>

		<i>Sustainable Development Generic Label</i>						
								

<sup>4</sup> Select a single label which, according to the [Implementation of SDG labels in the academic process](#), best matches the subject. If the subject addresses sustainable development in a generic manner (i.e. by presenting/introducing the general framework of sustainable development, etc.), then the Sustainable Development generic label may be applied. If none of the labels describe the subject, select the last option: "No label applies."

<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	X
								No label applies
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Date of entry:  
10.04.2026

Signature of course coordinator

Conf. dr. Trif Tiberiu

Signature of seminar coordinator

Conf. dr. Trif Tiberiu

Date of approval in the department:  
24.04.2026

Signature of the head of department

Prof. dr. Andrei Mărcuș