COURSE DESCRIPTION

Geometric Function Theory in Several Complex Variables

Academic year 2026-2027

1. Programme-related data

1.1. Higher Education Institution	**Babeş-Bolyai University**
1.2. Faculty	**Mathematics and Computer Science**
1.3. Department	**Mathematics**
1.4. Field	**Mathematics**
1.5. Level of study	**Master**
1.6. Degree programme / Qualification	**Advanced Mathematics**
1.7. Form of education	**Full-time**

2. Course-related data

2.1. Course title	**Geometric Function Theory in Several Complex Variables**				Course code	**MME3115**
2.2. Course coordinator		**Professor PhD Mirela KOHR**				
2.3. Seminar coordinator		**Professor PhD Mirela KOHR**				
2.4. Year of study	**2**	2.5. Semester	**4**	2.6. Type of assessment	Exam	
2.7. Course status		Optional		2.8. Course type	Specialisation subject	

3. Total estimated time (hours per semester of teaching activities)

3.1. Number of hours per week	**3**	of which: 3.2. course	**2**	3.3. seminar/ laboratory/ project	**1 sem**
3.4. Total of hours in the curriculum	**36**	of which: 3.5. course	**24**	3.6. seminar/ laboratory	**12**
Time allocation for individual study (IS) and self-taught activities (ST)					**hours**
Learning from textbooks, course materials, bibliography, and notes (IS)					45
Additional research in the library, on subject-specific electronic platforms, and on-site					45
Preparing seminars/ laboratories/ projects, assignments, reports, portfolios, and essays					45
Tutoring (professional guidance)					34
Examinations					20
Other activities					-
3.7. Total hours of individual study (IS) and self-taught activities (ST)				**189**	
3.8. Total hours per semester				**225**	
3.9. Number of credits				**9**	

4. Prerequisites (where applicable)

4.1. curriculum-related	In-depth knowledge of the following disciplines: • Complex Analysis; • Real Analysis; • Topology; • Partial Differential Equations.
4.2 skills-related	There are useful logical thinking and mathematical notions and results from the above-mentioned fields.

5. Specific conditions (where applicable)

5.1. course-related	Classroom with blackboard, video projector
5.2. seminar/laboratory-related	Classroom with blackboard, video projector

6.1. Competencies resulting from the completion of the degree programme ***(as referred to in the curriculum)***[1]

Professional competencies	
Competency code	**Competency**
PC1	develop problem-solving strategies
PC3	perform analytical mathematical calculations
PC6	disseminate results among the scientific community
PC5	apply the principles of ethics and scientific integrity in research activities
Transversal competencies	
Competency code	**Competency**
TC3	work independently
TC6	think analytically

6.2. Learning outcomes relevant to the degree programme (as referred to in the curriculum)[2]

Learning outcomes targeted by the subject		
Competency code	**Knowledge and comprehension**	**Specific academic skills**
PC1	1. The graduate analyses the hypotheses and conclusions from mathematical assertions and links them within the demonstration.	1. The graduate demonstrates the acquisition and use of effective research methods and techniques.
PC3	5. The graduate formulates observations and differentiates notions, properties and assertions from advanced disciplines of mathematics through examples and counterexamples.	5. The graduate verifies, on particular cases or by constructing examples or counterexamples, the validity of mathematical statements. The graduate translates a practical situation into mathematical language, solves the problem obtained and interprets the results obtained.
PC6	2. The graduate defines the basic concepts from advanced mathematics disciplines in the curriculum.	2. The graduate correctly and rigorously formulates the statements of mathematical assertions (lemmas, propositions, theorems) from the disciplines in the curriculum.
TC3	3. The graduate compares and distinguishes related notions and their properties from advanced mathematics disciplines in the curriculum.	3. The graduate is able to identify and formulate significant problems which form the basis for further research.
PC5, TC6	4. The graduate critically studies the specialized literature, including by using international databases, identifying fundamental concepts.	4. The graduate applies appropriate techniques for solving advanced problems.

7. Subject-specific learning outcomes

Knowledge and comprehension
1. The student/graduate has acquired the knowledge specific to the discipline studied necessary for solving problems, communicating concepts, fundamental and advanced theories in the field of Mathematics.

[1] The professional and/or transversal skills targeted by the subject for which the course description is prepared will be copied from the curriculum of the degree programme. For each competency, the complete entry, including the competency code, will be copied with the exact wording that appears in the curriculum, without any changes. If no competency is copied from either of the two categories, the row corresponding to that category is deleted from the table.

[2] The learning outcomes relevant for the degree programme and targeted by the subject for which the course description is prepared will be listed. The entries, copied without any changes from the Curriculum by subject type (Core Subject/Specialisation Subject/Complementary Subject), are listed under the corresponding competency.

2.	The student/graduate knows, understands and uses fundamental notions of Geometric Function Theory of Several Complex Variables and Complex Analysis in one and higher dimensions necessary for solving specific advanced problems in these fields, as well as special problems in applied mathematics, Fluid Mechanics.
3.	The student/graduate is able to use acquired knowledge in pursuing a doctoral program in Pure Mathematics, Applied Mathematics, or in other fields that use concepts of Complex Analysis.
4.	The student/graduate is able to use advanced skills to develop and manage mathematical projects of research nature, applying a wide range of quantitative and qualitative methods.
	Specific academic skills
1.	The student/graduate is able to identify and state significant problems which can be the basis for subsequent research.
2.	The student/graduate is able to use scientific language and to perform research in Mathematics.
3.	The student/graduate is able to construct clear and well-supported mathematical arguments to explain mathematical problems, topics, and ideas in writing.

8. Contents

8.1. Course	**Teaching and learning methods**	**Remarks**[3]
1. The Carathéodory family *M* of holomorphic mappings in several complex variables. Growth and distortion results, coefficient bounds. Compactness of the family *M*.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
2. Starlike mappings on the unit ball in $\mathbf{C}^n$. Necessary and sufficient conditions for starlikeness. Growth and distortion results and coefficient bounds.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
3. Convex mappings on the unit ball in $\mathbf{C}^n$. Necessary and sufficient conditions for convexity on the Euclidean unit ball and the unit polydisc in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
4. Growth, distortion and coefficient bounds for convex mappings on the unit ball in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
5. Loewner chains and transition mappings (evolution families) in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
6. Loewner chains, Herglotz vector fields and the generalized Loewner differential equation in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
7. Kernel convergence and biholomorphic mappings on the unit ball in $\mathbf{C}^n$. Applications in the theory of Loewner chains.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
8. The solutions of the generalized Loewner differential equation in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
9. The family $S^0(B^n)$ of biholomorphic mappings with parametric representation on the unit ball in $\mathbf{C}^n$. Characterizations in terms of Loewner chains. Compactness of the family $S^0(B^n)$. The Runge property. Open problems.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
10. Extreme points and support points associated with the family $S^0(B^n)$. Approximation properties by automorphisms of the space $\mathbf{C}^n$. Open problems and conjectures.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
11. Univalence criteria on the unit ball in $\mathbf{C}^n$ via the theory of Loewner chains. Parametric representation and asymptotic starlikeness in higher dimensions.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
12. Extension operators that preserve analytic and geometric properties (starlikeness, convexity, Loewner chains,	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

[3] For example, organisational aspects, recommendations for students, specific aspects relating to the course/seminar, such as inviting experts in the field, etc.

parametric representation). Open problems, conjectures, and research directions.		

Bibliography

1. I. Graham, G. Kohr, *Geometric Function Theory in One and Higher Dimensions*, Marcel Dekker Inc., New York, 2003.
2. G. Kohr, *Basic Topics in Holomorphic Functions of Several Complex Variables*, Cluj University Press, Cluj-Napoca, 2003,
3. G. Kohr, *Geometric Function Theory in Several Complex Variables*, Lecture Notes, 2020.

   G. Kohr, M. Kohr, *Geometric Function Theory in Several Complex Variables*, Lecture Notes, 2025.
4. P. Duren, I. Graham, H. Hamada, G. Kohr, *Solutions for the generalized Loewner differential equation in several complex variables*, Mathematische Annalen, **347** (2010), 411-435.
5. I. Graham, H. Hamada, G. Kohr, M. Kohr, *Extremal properties associated with univalent subordination chains in* $\mathbf{C}^n$, Mathematische Annalen, **359** (2014), 61-99.
6. H. Hamada, G. Kohr, M. Kohr, Subordination chains and solutions to the Loewner PDE infinite dimensions, The Journal of Geometric Analysis, **36** (2026), 29.
7. S. Gong, *Convex and Starlike Mappings in Several Complex Variables*, Kluwer Acad. Publ., Dordrecht, 1998.
8. P. Duren, *Univalent Functions*, Springer-Verlag, New York, 1983.
9. M. Elin, S. Reich. D. Shoikhet, *Numerical Range of Holomorphic Mappings and Applications*, Birkhäuser, Springer, Cham, 2019.
10. S.G. Krantz, *Function Theory of Several Complex Variables*, Reprint of the 1992 Edition, AMS Chelsea Publishing, Providence, Rhode Island, 2001.
11. Ch. Pommerenke, *Univalent Functions*, Vandenhoeck & Ruprecht, Göttingen, 1975.
12. T. Poreda, *On generalized differential equations in Banach spaces*, Dissertationes Mathematicae, **310** (1991), 1-50.
13. M. Range, *Holomorphic Functions and Integral Representations in Several Complex Variables* Springer-Verlag, New York, 1986.
14. W. Rudin, *Function Theory in the Unit Ball of* $\mathbf{C}^n$, Springer-Verlag, New York, 1980.

8.2. Seminar	**Teaching and learning methods**	**Remarks**
1. Examples of mappings in the Carathéodory family *M*. Special subclasses of *M*. Distortion and coefficient bounds.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
2. Sufficient conditions of starlikeness on the unit ball in $\mathbf{C}^n$. Examples of starlike mappings.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
3. Sufficient conditions of convexity on the unit ball in $\mathbf{C}^n$. Examples of convex mappings.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
4. Starlike mappings of order α on the Euclidean unit ball in $\mathbf{C}^n$, $0 \le \alpha < 1$. Growth and coefficient bounds. Examples.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
5. Loewner chains and transition mappings (evolution families) in several complex variables. Examples.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
6. Loewner chains and the associated Loewer PDE in higher dimensions. Applications.	Applications of course concepts. Description of arguments and proofs for	1 hour/week

	solving problems. Homework assignments. Direct answers to students.	
7. The analytical characterizations of starlikeness and spirallikeness of type α on the unit ball in $\mathbf{C}^n$ in terms of Loewner chains.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
8. Variation of Loewner chains in $\mathbf{C}^n$. Applications to extremal problems for univalent mappings with parametric representation on the unit ball in $\mathbf{C}^n$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
9. Bounded mappings with parametric representation on the unit ball in $\mathbf{C}^n$. Growth and coefficient bounds. Applications to extremal problems.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
10. Univalence criteria on the unit ball in $\mathbf{C}^n$ via the theory of Loewner chains.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
11. Kernel convergence and Loewner chains in $\mathbf{C}^n$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
12. Extension operators that preserve analytic and geometric properties. Open problems, conjectures, and research directions.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week

Bibliography

1. I. Graham, G. Kohr, *Geometric Function Theory in One and Higher Dimensions*, Marcel Dekker Inc., New York, 2003.
2. G. Kohr, *Basic Topics in Holomorphic Functions of Several Complex Variables*, Cluj University Press, Cluj-Napoca, 2003.
3. G. Kohr, *Geometric Function Theory in Several Complex Variables*, Seminar Notes, 2020.
4. F. Bracci, I. Graham, H. Hamada, G. Kohr, *Variation of Loewner chains, extreme and support points in the class* S^0 *in higher dimensions*, Constructive Approximation, **43** (2016), 231-251.
5. P. Duren, I. Graham, H. Hamada, G. Kohr, *Solutions for the generalized Loewner differential equation in several complex variables*, Mathematische Annalen, **347** (2010), 411-435.
6. I. Graham, H. Hamada, G. Kohr, M. Kohr, *Extremal properties associated with univalent subordination chains in* $\mathbf{C}^n$, Mathematische Annalen, **359** (2014), 61-99.
7. I. Graham, H. Hamada, G Kohr, M Kohr, *Loewner PDE in infinite dimensions*, Computational Methods and Function Theory, **25** (2025), 151–171.
8. G. Kohr, P. Liczberski, *Univalent Mappings of Several Complex Variables*, Cluj University Press, Cluj-Napoca, 1998.
9. P. Curt, *Special Chapters in Geometric Function Theory of Several Complex Variables*, Editura Albastră, Cluj-Napoca, 2001 (in Romanian).
10. S. Gong, *Convex and Starlike Mappings in Several Complex Variables*, Kluwer Acad. Publ., Dordrecht, 1998.
11. S. Gong, The Bieberbach Conjecture, Amer. Math. Soc. Intern. Press, Providence, R.I., 1999.
12. S.G. Krantz, *Function Theory of Several Complex Variables*, Reprint of the 1992 Edition, AMS Chelsea Publishing, Providence, Rhode Island, 2001.
13. Ch. Pommerenke, *Univalent Functions*, Vandenhoeck & Ruprecht, Göttingen, 1975.
14. F. Bracci (Ed.), *Geometric Function Theory in Higher Dimension*, Springer INdAM Series, vol. **26** (2017), Springer International Publishing AG, Cham, Switzerland.

9. Evaluation

Type of activity	9.1 Evaluation criteria[4]	9.2 Evaluation methods[5]	9.3 Percentage in the final grade
9.4. Course	Knowledge of concepts and basic results.	Written exam.	60%
	Ability to justify by proofs theoretical results.		
9.5. Seminar	Ability to apply concepts and results acquired in the course in the study of advanced topics of geometric function theory in $\mathbf{C}^n$ and related area.	Evaluation of reports and homework during the semester, and active participation in the seminar activity.	15%
		A midterm written test.	25%
There are valid the official rules of the faculty concerning the attendance of students at teaching activities.			
9.6 Minimum standard for passing			
• The final grade should be at least 5 (from a scale of 1 to 10).			

10. SDG labels (Sustainable Development Goals)[6]

	○	Sustainable Development Generic Label						
1 FĂRĂ SĂRĂCIE	2 FOAMETE „ZERO”	3 SĂNĂTATE ȘI BUNĂSTARE	4 EDUCAȚIE DE CALITATE	5 EGALITATE DE GEN	6 APĂ CURATĂ ȘI SANITAȚIE	7 ENERGIE CURATĂ ȘI LA PREȚURI ACCESIBILE	8 MUNCĂ DECENTĂ ȘI CREȘTERE ECONOMICĂ	9 INDUSTRIE, INOVAȚIE ȘI INFRASTRUCTURĂ
○	○	○	○	○	○	○	○	X
10 INEGALITĂȚI REDUSE	11 ORAȘE ȘI COMUNITĂȚI DURABILE	12 CONSUM ȘI PRODUCȚIE RESPONSABILE	13 ACȚIUNE CLIMATICĂ	14 VIAȚA ACVATICĂ	15 VIAȚA TERESTRĂ	16 PACE, JUSTIȚIE ȘI INSTITUȚII EFICIENTE	17 PARTENERIATE PENTRU REALIZAREA OBIECTIVELOR	No label applies
○	○	○	○	○	○	○	○	○

Date of entry:
10.04.2026

Signature of course coordinator
Prof.PhD. Mirela KOHR

Signature of seminar coordinator
Prof.PhD. Mirela KOHR

Date of approval in the department:
24.04.2026

Signature of the head of department
Prof.PhD. Andrei MĂRCUȘ

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[4] The evaluation criteria must directly reflect the learning outcomes targeted at the level of the degree programme respectively at the level of the subject. More specifically, the learning outcomes set out in the expected learning outcomes are assessed.

[5] Both final evaluation methods and ongoing evaluation strategies should be established.

[6] Select a single label which, according to the *Implementation of SDG labels in the academic process*, best matches the subject. If the subject addresses sustainable development in a generic manner (i.e. by presenting/introducing the general framework of sustainable development, etc.), then the Sustainable Development generic label may be applied. If none of the labels describe the subject, select the last option: *"No label applies."*