COURSE DESCRIPTION

Vector Optimization

Academic year 2026-2027

1. Programme-related data

1.1. Higher Education Institution	**Babeş-Bolyai University**
1.2. Faculty	**Mathematics and Computer Science**
1.3. Department	**Mathematics**
1.4. Field	**Mathematics**
1.5. Level of study	**Master**
1.6. Degree programme / Qualification	**Advanced Mathematics**
1.7. Form of education	**Full-time**

2. Course-related data

2.1. Course title	**Vector Optimization**				*Course code*	**MME3403**
2.2. Course coordinator	*Conf. dr. Trif Tiberiu*					
2.3. Seminar coordinator	*Conf. dr. Trif Tiberiu*					
2.4. Year of study	*II*	*2.5. Semester*	*3*	*2.6. Type of assessment*	Progress check	
2.7. Course status		Optional		*2.8. Course type*	Specialisation subject	

3. Total estimated time *(hours per semester of teaching activities)*

3.1. Number of hours per week	**3**	*of which: 3.2. course*	**2**	*3.3. seminar/ laboratory/ project*	**1**
3.4. Total of hours in the curriculum	**42**	*of which: 3.5. course*	**28**	*3.6. seminar/ laboratory*	**14**
Time allocation for individual study (IS) and self-taught activities (ST)					**hours**
Learning from textbooks, course materials, bibliography, and notes (IS)					**ore**
Additional research in the library, on subject-specific electronic platforms, and on-site					**50**
Preparing seminars/ laboratories/ projects, assignments, reports, portfolios, and essays					**30**
Tutoring (professional guidance)					**33**
Examinations					**20**
Other activities					
3.7. Total hours of individual study (IS) and self-taught activities (ST)	**133**				
3.8. Total hours per semester	**175**				
3.9. Number of credits	**7**				

4. Prerequisites *(where applicable)*

4.1. curriculum-related	*Mathematical analysis 1 (Analysis on R);* *Mathematical analysis 2 (Differential Calculus on R^n).*
4.2 skills-related	*Ability to use abstract notions, theoretical results and practical methods of Mathematical Analysis.*

5. Specific conditions *(where applicable)*

5.1. course-related	**blackboard, projector**
5.2. seminar/laboratory-related	**blackboard**

6.1. Competencies resulting from the completion of the degree programme ***(as referred to in the curriculum)***[1]

[1] *The professional and/or transversal skills targeted by the subject for which the course description is prepared will be copied from the curriculum of the degree programme. For each competency, the complete entry, including the competency code, will be copied with the exact wording that appears in the curriculum, without any changes. If no*

Professional competencies	
Competency code	**Competency**
CP3	*perform analytical mathematical calculations*
CP1	*develop problem-solving strategies*
CP6	*disseminate results among the scientific community*
Transversal competencies	
Competency code	**Competency**
CT3	*work independently*
CT6	*think analytically*

6.2. Learning outcomes relevant to the degree programme *(as referred to in the curriculum)*[2]

Learning outcomes targeted by the subject		
Competency code	**Knowledge and comprehension**	**Specific academic skills**
CP3	*5. The graduate formulates observations and differentiates notions, properties and assertions from advanced disciplines of mathematics through examples and counterexamples.*	*5. The graduate verifies, on particular cases or by constructing examples or counterexamples, the validity of mathematical statements. The graduate translates a practical situation into mathematical language, solves the problem obtained and interprets the results obtained.*
CP1	*1. The graduate analyses the hypotheses and conclusions from mathematical assertions and links them within the demonstration.*	*1. The graduate demonstrates the acquisition and use of effective research methods and techniques.*
CP7, CT3	*3. The graduate compares and distinguishes related notions and their properties from advanced mathematics disciplines in the curriculum.*	*3. The graduate is able to identify and formulate significant problems which form the basis for further research.*
CT6	*4. The graduate critically studies the specialized literature, including by using international databases, identifying fundamental concepts.*	*4. The graduate applies appropriate techniques for solving advanced problems.*

7. Subject-specific learning outcomes

Knowledge and comprehension
1. The student/graduate has acquired the knowledge specific to the discipline studied necessary for solving problems.
2. Students should acquire knowledge about vector (multicriteria) optimization. *3. Students will study several classes of practical vector optimization problems.*
Specific academic skills
1. The student/graduate is able to construct clear and well-supported mathematical arguments to explain mathematical problems, topics, and ideas in writing.
2. The student/graduate is able to prove theorems using mathematical language in theoretical courses and will be able to present these results both orally and in writing.

competency is copied from either of the two categories, the row corresponding to that category is deleted from the table.

[2] *The learning outcomes relevant for the degree programme and targeted by the subject for which the course description is prepared will be listed. The entries, copied without any changes from the Curriculum by subject type (Core Subject/Specialisation Subject/Complementary Subject), are listed under the corresponding competency.*

8. Contents

8.1. Course	Teaching and learning methods	Remarks[3]
1. Preorder relations; maximal elements of a set with respect to a preference relation; formulation of general optimization problems. Linear prorder relations (compatible with the vector addition and multiplication of vectors by scalars).	*Explanation, dialogue, examples, proofs*	
2. Cones; characterizations of (convex, pointed, generating, totally-generating) cones; the relationship between linear preorder relations and convex cones. Topological properties of convex cones: (relative) solid and closed convex cones; the polar cone of a set; polyhedral cones.	*Explanation, dialogue, examples, proofs*	
3. Concepts of efficiency in vector optimization; efficient points and weakly efficient points w.r.t. a convex cone; efficient solutions and weakly efficient solutions of vector optimization problems.	*Explanation, dialogue, examples, proofs*	
4. Monotone and strictly monotone scalar functions (w.r.t. a preorder relation) and the their extremum points; examples of linear/nonlinear monotone functions; conical sections of a set; the existence of efficient/weakly efficient points.	*Explanation, dialogue, examples, proofs*	
5. Sufficient conditions for efficiency and weak efficiency. Cone-convex sets; necessary conditions for weak-efficiency. Proper efficient points.	*Explanation, dialogue, examples, proofs*	
6. Cone-convex vector-valued functions, their characterizations by means of the epigraph and the polar cone; the cone-convexity of the images of convex sets by cone-convex functions.	*Explanation, dialogue, examples, proofs*	
7. Explicitly cone-quasiconvex functions and lexicographic quasiconvex vector-valued functions, their characterization and some of important properties; the relationship between explicit cone- convexity and lexicographic quasiconvexity.	*Explanation, dialogue, examples, proofs*	
8. Scalarization methods for vector optimization problems: the weighting method (for convex objective functions); the parametric method (for quasiconvex/, explicitly quasiconvex/ explicitly quasiaffine objective functions).	*Explanation, dialogue, examples, proofs*	
9. The geometric and topological structure of the boundary of a closed radiant set (the homeomorphism of Bonnisseau-Cornet).	*Explanation, dialogue, examples, proofs*	
10. Simply shaded and completely shaded sets (w.r.t. a convex cone) and their characterizations. The connectedness /contractibility of the sets of efficient points.	*Explanation, dialogue, examples, proofs*	
11. The role of Helly's Theorem in reducing the number of criteria involved in vector optimization with convex/quasiconvex objective functions.	*Explanation, dialogue, examples, proofs*	
12. Pareto reducible vector optimization problems involving explicitly / lexicographic quasiconvex objective functions.	*Explanation, dialogue, examples, proofs*	
13. Approximate efficient / weakly efficient solutions and their role in numerical methods.	*Explanation, dialogue, examples, proofs*	
14. Efficient sequences and their relationship with the minimizing sequences of certain scalarization functions.	*Explanation, dialogue, examples, proofs*	

[3] *For example, organisational aspects, recommendations for students, specific aspects relating to the course/seminar, such as inviting experts in the field, etc.*

Bibliography

1. *BRECKNER, B.E., POPOVICI, N.: Convexity and Optimization. An Introduction, EFES, Cluj-Napoca, 2006.*
2. *EHRGOT, M.: Multicriteria Optimization. Springer, Berlin Heidelberg New York, 2005.*
3. *GOEPFERT, A., RIAHI, H., TAMMER, C., ZALINESCU, C.: Variational Methods in Partially Ordered Spaces. Springer-Verlag, New York, 2003.*
4. *JAHN, J.: Vector Optimization. Theory, Applications, and Extensions. Springer, Berlin, 2004.*
5. *LUC, D.T.: Theory of Vector Optimization. Springer Verlag, Berlin, 1989.*
6. *POPOVICI, N.: Optimizare vectoriala, Casa Cartii de Stiinta, Cluj-Napoca, 2005.*

8.2. Seminar/ laboratory	**Teaching and learning methods**	**Remarks**
1. Geometric interpretation of the preference relations induced by the objective functions of some practical optimization problems (Fermat-Weber-type location problems, resource allocation problems, etc.)	*dialogue, examples, proofs*	
2. Particular classes of convex cones in the n-dimensional Euclidean space (polyhedral cones, the lexicographic cone, Phelps-type cones).	*dialogue, examples, proofs*	
3. Exercises involving the concepts of: polar cone, basis of a convex cone, the (relative) interior, and the facial structure of a convex cone.	*dialogue, examples, proofs*	
4. Finding the efficient / weakly efficient solutions of certain vector optimization problems by a geometric approach.	*dialogue, examples, proofs*	
5. Exercises concerning the (strict) monotony of certain scalar functions.	*dialogue, examples, proofs*	
6. Identifying the (weakly) efficient solutions of some concrete vector optimization problems in R^2 by means of the necessary and sufficient conditions of (weakly) efficiency.	*dialogue, examples, proofs*	
7. Geometric representations of the direct images of convex/polyhedral sets by certain cone-convex functions and their (weakly) efifcient points.	*dialogue, examples, proofs*	
8. Geometric representation of the level sets of certain cone-quasiconvex vector-valued functions.	*dialogue, examples, proofs*	
9. Exercises concerning explicitly quasiconvex functions (in particular, lexicographic convex functions and linear-fractional functions).	*dialogue, examples, proofs*	
10. Bicriteria optimization problems solved by a geometrical approach.	*dialogue, examples, proofs*	
11. Linear vector optimization problems solved by the weighting scalarization method.	*dialogue, examples, proofs*	
12. Nonlinear vector optimization problems solved by the weighting scalarization method. Integration of some exact differential forms. Applications to the Green formula.	*dialogue, examples, proofs*	
13. Linear vector optimization problems solved by the parametric method.	*dialogue, examples, proofs*	
14. Nonlinear vector optimization problems solved by the parametric method.	*dialogue, examples, proofs*	

Bibliography

1. *ALZORBA, S., GUNTHER, C., POPOVICI, N., TAMMER, C.: A new algorithm for solving planar multiobjective location problems involving the Manhattan norm, European Journal of Operational Research, Vol. 258 (1) 2017, pp. 35-46.*
2. *EHRGOT, M.: Multicriteria Optimization. Springer, Berlin Heidelberg New York, 2005.*
3. *POPOVICI, N.: Pareto reducible multicriteria optimization problems, Optimization, Vol. 54 (2005), pp. 253-263.*
4. *SAWARAGI, Y., NAKAYAMA, H., TANINO, T.: Theory of Multiobjective Optimization. Academic Press, New York, 1985.*
5. *YU, P.L.: Multiple criteria decision making: concepts, techniques and extensions. Plenum Press, New York - London, 1985.*

9. Evaluation

Activity type	9.1 Evaluation criteria	9.2 Evaluation methods	9.3 Percentage of final grade
9.4 Course	*Knowledge of fundamental notions and results*	*Written paper*	*90%*
9.5 Seminar/laboratory	*Solving problems based on learned notions and theorems*	*Solving the exercises on the board*	*10%*
9.6 Minimum standard of performance			
• *to aquire 5 points to pass the exam.*			

10. SDG labels (Sustainable Development Goals)[4]

	○	*Sustainable Development Generic Label*						
1 FĂRĂ SĂRĂCIE	2 FOAMETE „ZERO"	3 SĂNĂTATE ȘI BUNĂSTARE	4 EDUCAȚIE DE CALITATE	5 EGALITATE DE GEN	6 APĂ CURATĂ ȘI SANITAȚIE	7 ENERGIE CURATĂ ȘI LA PREȚURI ACCESIBILE	8 MUNCĂ DECENTĂ ȘI CREȘTERE ECONOMICĂ	9 INDUSTRIE, INOVAȚIE ȘI INFRASTRUCTURĂ
○	○	○	○	○	○	○	○	*X*
10 INEGALITĂȚI REDUSE	11 ORAȘE ȘI COMUNITĂȚI DURABILE	12 CONSUM ȘI PRODUCȚIE RESPONSABILE	13 ACȚIUNE CLIMATICĂ	14 VIAȚA ACVATICĂ	15 VIAȚA TERESTRĂ	16 PACE, JUSTIȚIE ȘI INSTITUȚII EFICIENTE	17 PARTENERIATE PENTRU REALIZAREA OBIECTIVELOR	*No label applies*
○	○	○	○	○	○	○	○	○

Date of entry: 11.04.2026	*Signature of course coordinator* *Conf. dr. Trif Tiberiu*	*Signature of seminar coordinator* *Conf. dr. Trif Tiberiu*

[4] *Select a single label which, according to the Implementation of SDG labels in the academic process, best matches the subject. If the subject addresses sustainable development in a generic manner (i.e. by presenting/introducing the general framework of sustainable development, etc.), then the Sustainable Development generic label may be applied. If none of the labels describe the subject, select the last option: "No label applies."*

Date of approval in the department:
25.04.2026

Signature of the head of department

Prof. dr. Andrei Mărcuș