COURSE DESCRIPTION

Integral Equations with Applications

Academic year 2026-2027

1. Programme-related data

1.1. Higher Education Institution	**Babeş-Bolyai University**
1.2. Faculty	**Faculty of Mathematics and Computer Science**
1.3. Department	**Department of Mathematics**
1.4. Field	**Mathematics**
1.5. Level of study	**Master**
1.6. Degree programme / Qualification	**Advanced Mathematics**
1.7. Form of education	**Full-time**

2. Course-related data

2.1. Course title	**Integral Equations with Applications**			Course code	**MME3160**
2.2. Course coordinator	**Prof. Sanda Micula, PhD. Habil.**				
2.3. Seminar coordinator	**Prof. Sanda Micula, PhD. Habil.**				
2.4. Year of study	2	2.5. Semester	3	2.6. Type of assessment	Exam
2.7 Course status	Optional			2.8. Course type	Specialization subject

3. Total estimated time (hours per semester of teaching activities)

3.1. Number of hours per week	**3**	of which: 3.2. course	**2**	3.3. seminar/ laboratory/ project	**1**
3.4. Total of hours in the curriculum	**42**	of which: 3.5. course	**28**	3.6. seminar/ laboratory	**14**

Time allocation for individual study (IS) and self-taught activities (ST)	**hours**
Learning from textbooks, course materials, bibliography, and notes (IS)	40
Additional research in the library, on subject-specific electronic platforms, and on-site	30
Preparing seminars/ laboratories/ projects, assignments, reports, portfolios, and essays	40
Tutoring (professional guidance)	14
Examinations	9
Other activities	
3.7. Total hours of individual study (IS) and self-taught activities (ST)	**133**
3.8. Total hours per semester	**175**
3.9. Number of credits	**7**

4. Prerequisites (where applicable)

4.1. curriculum-related	• Mathematical Analysis, Numerical Analysis
4.2 skills-related	• Knowledge of basic notions of operator theory • Average programming skills

5. Specific conditions (where applicable)

5.1. course-related	• Classroom with large blackboard and video projector
5.2. seminar/laboratory-related	• Classroom with large blackboard and video projector/computers with Matlab

6.1. Competencies resulting from the completion of the degree programme *(as referred to in the curriculum)*[1]

Professional competencies	
Competency code	**Competency**
PC3	perform analytical mathematical calculations
PC5	apply the principles of ethics and scientific integrity in research activities
Transversal competencies	
Competency code	**Competency**
TC5	solve problems
TC6	think analytically

6.2. Learning outcomes relevant to the degree programme (as referred to in the curriculum)[2]

Learning outcomes targeted by the subject		
Competency code	**Knowledge and comprehension**	**Specific academic skills**
PC5, TC6	4. The graduate critically studies the specialized literature, including by using international databases, identifying fundamental concepts.	4. The graduate applies appropriate techniques for solving advanced problems.
PC3, TC5	5. The graduate formulates observations and differentiates notions, properties and assertions from advanced disciplines of mathematics through examples and counterexamples.	5. The graduate verifies, on particular cases or by constructing examples or counterexamples, the validity of mathematical statements. The graduate translates a practical situation into mathematical language, solves the problem obtained and interprets the results obtained.

7. Subject-specific learning outcomes

Knowledge and comprehension
1. Acquire knowledge of the general theory of integral equations, with focus on applications.
2. Gain the ability to apply concepts and results from integral equations theory to specific problems.
Specific academic skills
1. Understand and be able to use main concepts and results from general integral equations theory.
2. Understand, use and be able to derive numerical methods for the approximate solution of integral equations arising in applications from various fields.

[1] The professional and/or transversal skills targeted by the subject for which the course description is prepared will be copied from the curriculum of the degree programme. For each competency, the complete entry, including the competency code, will be copied with the exact wording that appears in the curriculum, without any changes. If no competency is copied from either of the two categories, the row corresponding to that category is deleted from the table.

[2] The learning outcomes relevant for the degree programme and targeted by the subject for which the course description is prepared will be listed. The entries, copied without any changes from the Curriculum by subject type (Core Subject/Specialisation Subject/Complementary Subject), are listed under the corresponding competency.

8. Contents

8.1. Course	Teaching and learning methods	Remarks[3]
1. Introduction. Basic concepts. History of integral equations. Classifications and examples.	• Interactive exposure • Explanation • Conversation • Description	
2. Types of integral equations with exact solutions.	• Interactive exposure • Explanation • Conversation • Description	
3. Relationship between initial value/boundary value problems and integral equations.	• Interactive exposure • Explanation • Conversation • Description	
4. Volterra integral equations. The method of successive approximations. Laplace transforms. Adomian decomposition.	• Interactive exposure • Explanation • Conversation • Description	
5. Series solution. Volterra integral equations of the first kind. Integral equations of the convolution type. Abel integral equation.	• Interactive exposure • Explanation • Conversation • Description	
6. Fredholm integral equations. The method of successive approximations, Neumann series. Adomian decomposition. Compact integral operators. Properties. The Fredholm alternative theorem.	• Interactive exposure • Explanation • Conversation • Description	
7. Homogeneous Fredholm equations. Fredholm integral equations of the first kind.	• Interactive exposure • Explanation • Conversation • Description	
8. Numerical methods. Degenerate kernel methods. Taylor series approximation. Interpolatory degenerate kernel approximation.	• Interactive exposure • Explanation • Conversation • Description	
9. Projection methods, collocation and Galerkin methods. Iterated collocation and Galerkin methods. Error analysis.	• Interactive exposure • Explanation • Conversation • Description	
10. Nyström methods. Product integration methods. Error analysis. Discrete collocation and discrete Galerkin methods.	• Interactive exposure • Explanation • Conversation • Description	
11. Applications. Volterra's population model. Diffraction problems, Fresnel integrals.	• Interactive exposure • Explanation	

[3] For example, organisational aspects, recommendations for students, specific aspects relating to the course/seminar, such as inviting experts in the field, etc.

	• Conversation • Description	
12. Applications to potential theory. The Thomas-Fermi equation.	• Interactive exposure • Explanation • Conversation • Description	
13. Applications to ocean waves. Green's function method for waves. Seismic response of dams.	• Interactive exposure • Explanation • Conversation • Description	
14. Heat transfer and heat radiation.	• Interactive exposure • Explanation • Conversation • Description	

Bibliography

1. M. Rahman, Integral Equations and their Applications, WIT Press, Ashurst, Southampton, 2007.
2. A. M. Wazwaz, Linear and Nonlinear Integral Equations, Methods and Applications. Higher Education Press, Beijing. Springer, New York, 2011.
3. K. E. Atkinson, The Numerical Solution of Integral Equations of the Second Kind, Cambridge University Press, Cambridge,1997.
4. S. Micula, G. V. Milovanović, Chapter 16: Iterative Processes and Integral Equations of the Second Kind, Book: Matrix and Operator Equations and Applications, Birkhäuser, Springer Nature, Heidelberg, 2023.
5. A. D. Polyanin, A. V. Manzhirov, Handbook of Integral Equations, 2nd ed., CRC Press, Boca Raton, 2008.
6. S. Prössdorf, B. Silbermann, Numerical Analysis for Integral and Related Operator Equations, Wiley, Oxford, 1991.

8.2. Seminar/ laboratory	**Teaching and learning methods**	**Remarks**
1. Relationship between initial value/boundary value problems and integral equations.	• Interactive exposure • Explanation • Conversation • Individual and group work	The seminar is structured as 2 hours per week, every other week
2. Solvable integral equations.	• Interactive exposure • Explanation • Conversation • Individual and group work	
3. Volterra integral equations. Abel's integral equation.	• Interactive exposure • Explanation • Conversation • Individual and group work	
4. Fredholm integral equations. Mixed integral equations.	• Interactive exposure • Explanation • Conversation • Individual and group work	
5. Interpolation-based collocation and Galerkin methods. Iterated solutions.	• Interactive exposure • Explanation • Conversation • Individual and group work	
6. Nyström methods. Product integration. Discrete projection methods.	• Interactive exposure • Explanation • Conversation • Individual and group work	
7. Various applications.	• Interactive exposure	

	• Explanation • Conversation • Individual and group work	

Bibliography

1. M. Rahman, Integral Equations and their Applications, WIT Press, Ashurst, Southampton, 2007.
2. A. M. Wazwaz, Linear and Nonlinear Integral Equations, Methods and Applications. Higher Education Press, Beijing. Springer, New York, 2011.
3. K. E. Atkinson, The Numerical Solution of Integral Equations of the Second Kind, Cambridge University Press, Cambridge,1997.
4. S. Micula, G. V. Milovanović, Chapter 16: Iterative Processes and Integral Equations of the Second Kind, Book: Matrix and Operator Equations and Applications, Birkhäuser, Springer Nature, Heidelberg, 2023.
5. A. D. Polyanin, A. V. Manzhirov, Handbook of Integral Equations, 2nd ed., CRC Press, Boca Raton, 2008.
6. S. Prössdorf, B. Silbermann, Numerical Analysis for Integral and Related Operator Equations, Wiley, Oxford, 1991.

9. Evaluation

Activity type	9.1 Evaluation criteria	9.2 Evaluation methods	9.3 Percentage of final grade
9.4 Course	- acquire the basic principles and notions in Integral Equations theory - apply correctly various course concepts and methods	**Written exam**	**70%**
9.5 Seminar/laboratory	- understand and be able to use theory and results in problems and applications - apply numerical procedures and algorithms to solve practical and real-life problems	- active participation in discussing and solving problems throughout the semester - individual presentation of solutions	**30%**
9.6 Minimum standard of performance			
A grade of 5 or above (on a scale from 1 to 10) on **each** of the activities mentioned above (written exam, seminar evaluation)			

10. SDG labels (Sustainable Development Goals)[4]

	○	Sustainable Development Generic Label

								
○	○	○	○	○	○	○	○	X
								No label applies

[4] Select a single label which, according to the *Implementation of SDG labels in the academic process*, best matches the subject. If the subject addresses sustainable development in a generic manner (i.e. by presenting/introducing the general framework of sustainable development, etc.), then the Sustainable Development generic label may be applied. If none of the labels describe the subject, select the last option: *"No label applies."*

Date	Signature of course coordinator	Signature of seminar coordinator
10.04.2026	Prof. Sanda Micula, PhD. Habil.	Prof. Sanda Micula, PhD. Habil.

Date of approval

Signature of the head of department

Prof. dr. Andrei Mărcuş