COURSE DESCRIPTION

Mathematical Methods in Fluid Mechanics

Academic year 2026-2027

1. Programme-related data

1.1. Higher Education Institution	**Babeş-Bolyai University**
1.2. Faculty	**Mathematics and Computer Science**
1.3. Department	**Mathematics**
1.4. Field	**Mathematics**
1.5. Level of study	**Master**
1.6. Degree programme / Qualification	**Advanced Mathematics**
1.7. Form of education	**Full-time**

2. Course-related data

2.1. Course title		**Mathematical Methods in Fluid Mechanics**				Course code	**MME3104**
2.2. Course coordinator		**Professor PhD Mirela KOHR**					
2.3. Seminar coordinator		**Professor PhD Mirela KOHR**					
2.4. Year of study	**1**	2.5. Semester	**1**	2.6. Type of assessment		Viva voce	
2.7. Course status		Compulsory		2.8. Course type		Core subject	

3. Total estimated time (hours per semester of teaching activities)

3.1. Number of hours per week	**3**	of which: 3.2. course	**2**	3.3. seminar/ laboratory/ project	**1 sem**
3.4. Total of hours in the curriculum	**42**	of which: 3.5. course	**28**	3.6. seminar/ laboratory	**14**
Time allocation for individual study (IS) and self-taught activities (ST)					**hours**
Learning from textbooks, course materials, bibliography, and notes (IS)					38
Additional research in the library, on subject-specific electronic platforms, and on-site					38
Preparing seminars/ laboratories/ projects, assignments, reports, portfolios, and essays					38
Tutoring (professional guidance)					10
Examinations					9
Other activities					-

3.7. Total hours of individual study (IS) and self-taught activities (ST)	**133**
3.8. Total hours per semester	**175**
3.9. Number of credits	**7**

4. Prerequisites (where applicable)

4.1. curriculum-related	In-depth knowledge of the following disciplines: • Theoretical Mechanics; • Partial Differential Equations; • Real Analysis; • Numerical Analysis.
4.2 skills-related	There are useful logical thinking and mathematical notions and results from the above-mentioned fields.

5. Specific conditions (where applicable)

5.1. course-related	Classroom with blackboard, video projector
5.2. seminar/laboratory-related	Classroom with blackboard, video projector

6.1. Competencies resulting from the completion of the degree programme ***(as referred to in the curriculum)***[1]

[1] The professional and/or transversal skills targeted by the subject for which the course description is prepared will be copied from the curriculum of the degree programme. For each competency, the complete entry, including

Professional competencies	
Competency code	**Competency**
PC1	develop problem-solving strategies
PC3	perform analytical mathematical calculations
PC6	disseminate results among the scientific community
PC5	apply the principles of ethics and scientific integrity in research activities
Transversal competencies	
Competency code	**Competency**
TC3	work independently
TC6	think analytically

6.2. Learning outcomes relevant to the degree programme (as referred to in the curriculum)[2]

Learning outcomes targeted by the subject		
Competency code	**Knowledge and comprehension**	**Specific academic skills**
PC1	1. The graduate analyses the hypotheses and conclusions from mathematical assertions and links them within the demonstration.	1. The graduate demonstrates the acquisition and use of effective research methods and techniques.
PC3	5. The graduate formulates observations and differentiates notions, properties and assertions from advanced disciplines of mathematics through examples and counterexamples.	5. The graduate verifies, on particular cases or by constructing examples or counterexamples, the validity of mathematical statements. The graduate translates a practical situation into mathematical language, solves the problem obtained and interprets the results obtained.
PC6	2. The graduate defines the basic concepts from advanced mathematics disciplines in the curriculum.	2. The graduate correctly and rigorously formulates the statements of mathematical assertions (lemmas, propositions, theorems) from the disciplines in the curriculum.
TC3	3. The graduate compares and distinguishes related notions and their properties from advanced mathematics disciplines in the curriculum.	3. The graduate is able to identify and formulate significant problems which form the basis for further research.
PC5, TC6	4. The graduate critically studies the specialized literature, including by using international databases, identifying fundamental concepts.	4. The graduate applies appropriate techniques for solving advanced problems.

7. Subject-specific learning outcomes

Knowledge and comprehension	
1.	The student/graduate has acquired the knowledge specific to the discipline studied necessary for solving problems, communicating concepts, fundamental and advanced theories in the field of Mathematics.
2.	The student/graduate knows, understands and uses fundamental notions of Fluid Mechanics, as well as advanced mathematical theories necessary in fields of science related to Mathematics, Physics, Medicine, and Engineering.
3.	The student/graduate is able to use acquired knowledge to pursue a doctoral program in Mathematics or in other fields of science that use mathematical models.

the competency code, will be copied with the exact wording that appears in the curriculum, without any changes. If no competency is copied from either of the two categories, the row corresponding to that category is deleted from the table.

[2] The learning outcomes relevant for the degree programme and targeted by the subject for which the course description is prepared will be listed. The entries, copied without any changes from the Curriculum by subject type (Core Subject/Specialisation Subject/Complementary Subject), are listed under the corresponding competency.

4.	The student/graduate is able to use advanced skills to develop and manage mathematical projects of research nature, applying a wide range of quantitative and qualitative methods.
	Specific academic skills
1.	The student/graduate is able to identify and state significant problems which can be the basis for subsequent research.
2.	The student/graduate is able to use scientific language and to perform research in Mathematics.
3.	The student/graduate is able to construct clear and well-supported mathematical arguments to explain mathematical problems, topics, and ideas in writing.

8. Contents

8.1. Course	**Teaching and learning methods**	**Remarks**[3]
1. Introduction in the theory of Sobolev spaces (I): The fundamental spaces of the theory of distributions. Distributions.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
2. Introduction in the theory of Sobolev spaces (II): Sobolev spaces on $\mathbf{R}^n$. Sobolev spaces on Lipschitz domains in $\mathbf{R}^n$ and on Lipschitz boundaries. The dual of a Sobolev space. The Sobolev continuous embedding theorem and the Rellich - Kondrachov compact embedding theorem.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
3. Kinematics of fluids: fluid, configuration, motion. Velocity and acceleration fields. Spatial description of the motion of a fluid.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
4. Fluid Dynamics: Principle of mass conservation. The continuity equation.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
5. Fluid Dynamics: The Cauchy stress tensor. The Cauchy equations.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
6. The constitutive equation of ideal fluid. The Euler equations.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
7. The mathematical model of viscous Newtonian fluid: The constitutive equation and the Navier-Stokes equations. Special forms of the Navier-Stokes equations.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
8. Uniqueness results for the Dirichlet and Neumann problems for the Stokes system in bounded Lipschitz domains in $\mathbf{R}^n$. Variational approach for the weak solution of the Stokes problem in a bounded Lipschitz domain with Dirichlet boundary condition.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
9. The method of fundamental solutions in fluid mechanics: The Oseen-Burgers tensor and the fundamental pressure vector for the Stokes system in $\mathbf{R}^n$ (n=2, 3).	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
10. The layer potential theory for the Stokes system (I): Bounded and compact operators, Fredholm operators on Banach spaces. The Fredholm alternative.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

[3] For example, organisational aspects, recommendations for students, specific aspects relating to the course/seminar, such as inviting experts in the field, etc.

11. The layer potential theory for the Stokes system (II): Single- and double layer potentials for the Stokes system. Boundedness, compactness, and Fredholm properties in Sobolev spaces.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
12. Applications of the Stokes layer potential methods: Well-posedness results in Sobolev spaces for boundary value problems for the Stokes system in bounded Lipschitz domains in $\mathbf{R}^n$. Existence and uniqueness in Sobolev spaces for the Dirichlet problem for the Navier-Stokes system in bounded Lipschitz domains in $\mathbf{R}^3$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
13. Applications of Stokes layer potential methods and variational techniques: Well-posedness results in weighted Sobolev spaces for the exterior Dirichlet problem for the Stokes system in $\mathbf{R}^n$. Existence results in weighted Sobolev spaces for the exterior Dirichlet problem for the Navier-Stokes system in $\mathbf{R}^3$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
14. Layer potentials, boundary and transmission problems for the Stokes and Navier-Stokes systems with variable coefficients in Lipschitz domains: Variational and layer potential approaches. Well-posedness results in Sobolev spaces. Applications to viscous flow problems in the presence of interfaces and in porous media. Numerical results. Research directions.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

Bibliography

1. Kohr, M., Mikhailov, S.E., Nistor, V., Wendland, W.L., *Stationary Stokes and Navier-Stokes Equations with Variable Coefficients- Integral Operators and Variational Approaches*, Springer, Cham, 2026.
2. Kohr, M., Pop, I., *Viscous Incompressible Flow for Low Reynolds Numbers*, WIT Press (Wessex Institute of Technology Press), Southampton (UK) – Boston, 2004.
3. Kohr, M., *Modern Problems in Viscous Fluid Mechanics*, Cluj University Press, Cluj-Napoca, 2 vols. 2000 (in Romanian).
4. Kohr, M., *Mathematical Methods in Fluid Mechanics*, Lecture Notes, 2025/2026.
5. Truesdell, C., Rajagopal, K.R., *An Introduction to the Mechanics of Fluids*, Birkhäuser, Basel, 2000.
6. Boyer, F., Fabrie, P., *Mathematical Tools for the Study of the Incompressible Navier-Stokes Equations and Related Models*, Springer, New York, 2013.
7. Galdi, G.P., *An Introduction to the Mathematical Theory of the Navier–Stokes Equations*, 2nd Edition, Springer, Berlin, 2011.
8. Adams, R. Fournier, J., *Sobolev Spaces*, 2nd Edition, Pure and Applied Mathematics, vol. 140, Elsevier/Academic Press, Amsterdam, 2003.
9. Agranovich, M.S., *Sobolev Spaces, Their Generalizations, and Elliptic Problems in Smooth and Lipschitz Domains,* Springer, Heidelberg, 2015.
10. Hsiao, G.C., Wendland W.L., *Boundary Integral Equations*, Springer-Verlag, Heidelberg, 1st Edition 2008, 2nd Edition 2021.
11. Mitrea, M. Wright, M., *Boundary value problems for the Stokes system in arbitrary Lipschitz domains*, Astérisque, France, 344 (2012): viii+241 pp.
12. Temam, R., *Navier-Stokes Equations. Theory and Numerical Analysis*, AMS Chelsea Edition, 2001.
13. Sayas, F-J., Brown, T.S., Hassell, M.E., *Variational Techniques for Elliptic Partial Differential Equations: Theoretical Tools and Advanced Applications*, CRC Press, Boca Raton, FL, 2019.
14. Rieutord, M., *Fluid Dynamics. An Introduction*, Springer Cham Heidelberg, 2015.

8.2. Seminar	**Teaching and learning methods**	**Remarks**
1. Introduction in the theory of Sobolev spaces (I): The fundamental spaces of the theory of distributions. Distributions.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week

2.	Introduction in the theory of Sobolev spaces (II): Sobolev spaces over $\mathbf{R}^n$. Sobolev spaces on Lipschitz domains in $\mathbf{R}^n$ and on Lipschitz boundaries. Trace theorems.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
3.	Differential operators. Material derivatives. The Euler theorem. Applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
4.	Second order Cartesian tensors in $\mathbf{R}^n$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
5.	Properties of the Cauchy stress tensor: Cauchy's fundamental theorem, and the symmetry property.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
6.	The mathematical model of incompressible fluid.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
7.	The Killing theorem. Applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
8.	Variational approach for the weak solution of the Stokes problem in a bounded Lipschitz domain with homogeneous Dirichlet boundary condition.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
9.	The exterior Dirichlet problem for the Stokes system in Lipschitz domains in $\mathbf{R}^n$ (n=2,3). Well-posedness results and applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
10.	The method of fundamental solutions in fluid mechanics (I): Layer potential representations for the Stokes flow.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
11.	The method of fundamental solutions in fluid mechanics (II): Applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week

12. Well-posedness results in Sobolev spaces for boundary value problems for the Stokes and Navier-Stokes systems in bounded Lipschitz domains in $\mathbf{R}^3$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
13. Exterior Dirichlet problems for the Stokes and Navier-Stokes systems in $\mathbf{R}^3$, with data in weighted Sobolev spaces.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
14. Stokes and Navier-Stokes systems with variable coefficients in Lipschitz domains: Variational and layer potential approaches. Applications to flow problems in porous media. Numerical results. Research directions.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week

Bibliography

1. Kohr, M., Mikhailov, S.E., Nistor, V., Wendland, W.L., *Stationary Stokes and Navier-Stokes Equations with Variable Coefficients- Integral Operators and Variational Approaches*, Springer, Cham, 2026.
2. Kohr, M., Pop, I., *Viscous Incompressible Flow for Low Reynolds Numbers*, WIT Press (Wessex Institute of Technology Press), Southampton (UK) – Boston, 2004.
3. Kohr, M., *Modern Problems in Viscous Fluid Mechanics*, Cluj University Press, Cluj-Napoca, 2 vols. 2000 (in Romanian).
4. Kohr, M., *Mathematical Methods in Fluid Mechanics*, Seminar Notes, 2025/2026.
5. Kohr, M., Mikhailov, S.E., Wendland, W.L., *Non-homogeneous Dirichlet-transmission problems for the anisotropic Stokes and Navier-Stokes systems in Lipschitz domains with transversal interfaces*, Calculus of Variations and Partial Differential Equations, 61:198 (2022), 47 pp.
6. Kohr, M., Nistor, V., Wendland, W. *The Stokes operator on manifolds with cylindrical ends*, Journal of Differential Equations, **407** (2024), 345–373.
7. Kiselev, S.P., Vorozhtsov, E.V., Fomin, V.M., *Foundations of Fluid Mechanics with Applications. Problem Solving Using Mathematica*, Birkhäuser, Boston, 1999.
8. Hsiao, G.C., Wendland W.L., *Boundary Integral Equations*, Springer-Verlag, Heidelberg, 1st Edition 2008, 2nd Edition 2021.
9. Mitrea, M. Wright, M., *Boundary value problems for the Stokes system in arbitrary Lipschitz domains*, Astérisque, France, 344 (2012): viii+241 pp.
10. Precup, R., *Linear and Semilinear Partial Differential Equations.* De Gruyter, Berlin, 2013.
11. Galdi, G.P., *An Introduction to the Mathematical Theory of the Navier–Stokes Equations*, 2nd Edition. Springer, Berlin, 2011.
12. Boyer, F., Fabrie, P., *Mathematical Tools for the Study of the Incompressible Navier-Stokes Equations and Related Models*, Springer, New York, 2013.
13. McLean, W., *Strongly Elliptic Systems and Boundary Integral Equations*, Cambridge University Press, Cambridge, UK, 2000.
14. Wloka, J. T., Rowley, B., Lawruk, B., *Boundary Value Problems for Elliptic Systems*, Cambridge University Press, Cambridge, 1995.

9. Evaluation

Type of activity	9.1 Evaluation criteria[4]	9.2 Evaluation methods[5]	9.3 Percentage in the final grade
9.4. Course	Knowledge of concepts and basic results.	Colloquium	60%
	Ability to justify by proofs theoretical results.		
9.5. Seminar	Ability to apply concepts and results acquired in the course in mathematical modeling and analysis of problems in Fluid Mechanics.	Evaluation of reports and homework during the semester, and active participation in the seminar activity. A midterm written test.	15% 25%
There are valid the official rules of the faculty concerning the attendance of students at teaching activities.			
9.6 Minimum standard for passing			
• The final grade should be at least 5 (from a scale of 1 to 10).			

10. SDG labels (Sustainable Development Goals)[6]

	○	Sustainable Development Generic Label

1 FĂRĂ SĂRĂCIE	2 FOAMETE „ZERO"	3 SĂNĂTATE ȘI BUNĂSTARE	4 EDUCAȚIE DE CALITATE	5 EGALITATE DE GEN	6 APĂ CURATĂ ȘI SANITAȚIE	7 ENERGIE CURATĂ ȘI LA PREȚURI ACCESIBILE	8 MUNCĂ DECENTĂ ȘI CREȘTERE ECONOMICĂ	9 INDUSTRIE, INOVAȚIE ȘI INFRASTRUCTURĂ
○	○	○	○	○	○	○	○	X
10 INEGALITĂȚI REDUSE	11 ORAȘE ȘI COMUNITĂȚI DURABILE	12 CONSUM ȘI PRODUCȚIE RESPONSABILE	13 ACȚIUNE CLIMATICĂ	14 VIAȚA ACVATICĂ	15 VIAȚA TERESTRĂ	16 PACE, JUSTIȚIE ȘI INSTITUȚII EFICIENTE	17 PARTENERIATE PENTRU REALIZAREA OBIECTIVELOR	No label applies
○	○	○	○	○	○	○	○	○

Date of entry:
10.04.2026

Signature of course coordinator
Prof.PhD. Mirela KOHR

Signature of seminar coordinator
Prof.PhD. Mirela KOHR

Date of approval in the department:
24.04.2026

Signature of the head of department
Prof.PhD. Andrei MĂRCUŞ

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[4] The evaluation criteria must directly reflect the learning outcomes targeted at the level of the degree programme respectively at the level of the subject. More specifically, the learning outcomes set out in the expected learning outcomes are assessed.

[5] Both final evaluation methods and ongoing evaluation strategies should be established.

[6] Select a single label which, according to the *Implementation of SDG labels in the academic process*, best matches the subject. If the subject addresses sustainable development in a generic manner (i.e. by presenting/introducing the general framework of sustainable development, etc.), then the Sustainable Development generic label may be applied. If none of the labels describe the subject, select the last option: *"No label applies."*