COURSE DESCRIPTION

Special topics in fluid mechanics / Capitole speciale de mecanica fluidelor

Academic year 2026-2027

1. Programme-related data

1.1. Higher Education Institution	**Babeş-Bolyai University of Cluj-Napoca**
1.2. Faculty	**Mathematics and Computer Science**
1.3. Doctoral School	**Mathematics and Computer Science**
1.4. Field of study	**Mathematics**
1.5. Level of study	**Doctoral studies**

2. Course-related data

2.1. Course title	**Special topics in fluid mechanics / Capitole speciale de mecanica fluidelor**				Course code	**MDE3141**
2.2. Course coordinator	**Professor PhD Mirela KOHR**					
2.3. Seminar coordinator	**Professor PhD Mirela KOHR**					
2.4. Year of study	1	2.5. Semester	1	2.6. Type of assessment	Exam	
2.7. Course status	Optional			2.8. Course type	Specialisation subject	

3. Total estimated time (hours per semester of teaching activities)

3.1. Number of hours per week	**3**	of which: 3.2. course	**2**	3.3. seminar/ laboratory/ project	**1**
3.4. Total of hours in the curriculum	36	of which: 3.5. course	24	3.6. seminar/ laboratory	**12**
Time allocation for individual study (IS) and self-taught activities (ST)					**hours**
Learning from textbooks, course materials, bibliography, and notes (IS)					40
Additional research in the library, on subject-specific electronic platforms, and on-site					40
Preparing seminars/ laboratories/ projects, assignments, reports, portfolios, and essays					44
Tutoring (professional guidance)					50
Examinations					40
Other activities					-
3.7. Total hours of individual study (IS) and self-taught activities (ST)				214	
3.8. Total hours per semester				250	
3.9. Number of credits				10	

4. Prerequisites (where applicable)

4.1. curriculum-related	In-depth knowledge of the following disciplines: • Theoretical Mechanics; • Partial Differential Equations; • Real Analysis; • Functional Analysis; • Numerical Analysis.
4.2 skills-related	There are useful logical thinking and mathematical notions and results from the above-mentioned fields.

5. Specific conditions (where applicable)

5.1. course-related	Classroom with blackboard, video projector
5.2. seminar/laboratory-related	Classroom with blackboard, video projector

6. Subject-specific learning outcomes

Knowledge
1. Graduated PhD student knows, understands and uses concepts, individual results and advanced mathematical theories in the field of Mathematical Fluid Mechanics and related mathematical areas, having the capacity to handle and communicate them.
2. Graduated PhD student has the capacity to develop and use efficient research skills, to formulate new problems, to initiate new mathematical research in advanced mathematics topics of Fluid Mechanics and also in other related areas of Mathematics, preparing reports and scientific papers.
3. Graduated PhD student has the capacity to model and analyse from the mathematical point of view real processes and phenomena from other sciences, physics, medicine, engineering, and computer science.
4. Graduated PhD student has the capacity of advanced skills to develop and manage research projects, applying a wide range of quantitative and qualitative methods.
5. Graduated PhD student has the capacity to conduct rigorous literature reviews, critically evaluate scientific publications, and identify innovative research directions.
6 Graduated PhD student has competence in preparing scientific manuscripts according to international standards, presenting results at scientific conferences, and effectively communicating research findings to the scientific community.
7. Graduated PhD student has the ability to integrate concepts and methodologies from related fields (physics, chemistry, biology, engineering, computer science) to address complex scientific problems.

Skills
1. Graduated PhD student has the ability to identify and state significant problems which can be the basis for subsequent research.
2. Graduated PhD student has the ability to use scientific language and to perform research in Mathematics.
3. Graduated PhD student has the ability for continuous self-perfecting and study.
4. Graduated PhD student has the ability to inform themselves, to work independently or in a team in order to carry out studies and to solve complex problems.
5. Application of organized and efficient work rules, a responsible attitude towards the didactic-scientific field, to bring creative value to own potential respecting professional ethics principles.
6. Graduated PhD student has the ability to adopt and integrate in different environments from education and research.
7. Graduated PhD student has the ability to adapt to the requirements of a dynamical society and to communicate efficiently in an international language.

Responsibility and autonomy
1. Graduated PhD student has the ability to inform themselves, to work independently or in a team in order to carry out studies and to solve complex problems.
2. Graduated PhD student has the ability of critical investigation of specific literature.
3. Graduated PhD student has the ability to use international data bases of academic research.

7. Contents

7.1. Course	**Teaching and learning methods**	**Remarks[1]**
1. The fundamental spaces of the theory of distributions. Distributions.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
2. Sobolev spaces on $\mathbf{R}^n$. Sobolev spaces on Lipschitz domains in $\mathbf{R}^n$ and on Lipschitz boundaries. The dual of a Sobolev space. The Sobolev continuous embedding theorem and the Rellich - Kondrachov compact embedding theorem.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
3. Fundamental notions of the Kinematics of fluids.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

[1] For example, organisational aspects, recommendations for students, specific aspects relating to the course/seminar, such as inviting experts in the field, etc.

4. Fluid Dynamics: Principle of mass conservation. The continuity equation.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
5. Fluid Dynamics: The Cauchy stress tensor. The Cauchy equations.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
6. The constitutive equation of ideal fluid and the Euler equations. The constitutive equation of viscous Newtonian fluid and the Navier-Stokes equations. Special forms of the Navier-Stokes equations. Fundamental properties of the Stokes system.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
7. Variational approach for the weak solution of the Stokes system in bounded Lipschitz domains with Dirichlet boundary conditions.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
8. The fundamental solution of the Stokes system in $\mathbf{R}^n$. Applications.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
9. The layer potential theory for the Stokes system. Bounded, compact, and Fredholm operators on Banach spaces. Stokes layer potentials and their properties in Sobolev spaces.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
10. Well-posedness results in Sobolev spaces for boundary value problems for the Stokes system in bounded Lipschitz domains in $\mathbf{R}^n$. Existence and uniqueness in Sobolev spaces for the Dirichlet problem for the Navier-Stokes system in bounded Lipschitz domains in $\mathbf{R}^3$. Variational and layer potential approaches.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
11. Well-posedness results in weighted Sobolev spaces for the exterior Dirichlet problem for the Stokes system in $\mathbf{R}^n$. Existence results in weighted Sobolev spaces for the exterior Dirichlet problem for the Navier-Stokes system in $\mathbf{R}^3$. Variational and layer potential approaches.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
12. Layer potentials for the Stokes type systems with variable coefficients on domains in $\mathbf{R}^n$ or on manifolds with bounded geometry. Boundary problems for the Stokes and Navier-Stokes type systems with variable coefficients on Lipschitz domains in $\mathbf{R}^n$ or on manifolds with bounded geometry. Well-posedness results in Sobolev spaces. Applications to viscous flows in porous media. New research directions.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

Bibliography

1. Kohr, M., Mikhailov, S.E., Nistor, V., Wendland, W.L, *Stationary Stokes and Navier-Stokes Equations with Variable Coefficients - Integral Operators and Variational Approaches*, vol. 2380 of Lecture Notes in Mathematics, Springer, Cham, 2026.

2. Kohr, M., Pop, I., *Viscous Incompressible Flow for Low Reynolds Numbers*, WIT Press (Wessex Institute of Technology Press), Southampton (UK) – Boston, 2004.

3. Kohr, M., *Modern Problems in Viscous Fluid Mechanics*, Cluj University Press, Cluj-Napoca, 2 vols. 2000 (in Romanian).

4. Kohr, M., *Special Topics of Mechanics*, Cluj University Press, Cluj-Napoca, 2005 (in Romanian).

5. Kohr, M., Nistor, V., Wendland, W.L., *The Stokes operator on manifolds with cylindrical ends*, Journal of Differential Equations, **407** (204), 345-373.

6. Truesdell, C., Rajagopal, K.R., *An Introduction to the Mechanics of Fluids*, Birkhäuser, Basel, 2000.

7. Boyer, F., Fabrie, P., *Mathematical Tools for the Study of the Incompressible Navier-Stokes Equations and Related Models*, Springer, New York, 2013.

8. Galdi, G.P., *An Introduction to the Mathematical Theory of the Navier–Stokes Equations*, 2nd Edition, Springer, Berlin, 2011.

9. Adams, R. Fournier, J., *Sobolev Spaces*, 2nd Edition, Pure and Applied Mathematics, vol. 140, Elsevier/Academic Press, Amsterdam, 2003.

10. Agranovich, M.S., *Sobolev Spaces, Their Generalizations, and Elliptic Problems in Smooth and Lipschitz Domains,* Springer, Heidelberg, 2015.

11. Hsiao, G.C., Wendland W.L., *Boundary Integral Equations*, Springer-Verlag, Heidelberg, 1th Edition 2008, 2nd Edition 2021.

12. Mitrea, M. Wright, M., *Boundary value problems for the Stokes system in arbitrary Lipschitz domains*, Astérisque, France, 344 (2012): viii+241 pp.

13. Temam, R., *Navier-Stokes Equations. Theory and Numerical Analysis*, AMS Chelsea Edition, 2001.

14. Sayas, F-J., Brown, T.S., Hassell, M.E., *Variational Techniques for Elliptic Partial Differential Equations: Theoretical Tools and Advanced Applications*, CRC Press, Boca Raton, FL, 2019.

15. Rieutord, M., *Fluid Dynamics. An Introduction*, Springer Cham Heidelberg, 2015.

7.2. Seminar	Teaching and learning methods	Remarks
1. The fundamental spaces of the theory of distributions. Distributions.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
2. Sobolev spaces over $\mathbf{R}^n$. Sobolev spaces on Lipschitz domains in $\mathbf{R}^n$ and on Lipschitz boundaries. Trace and conormal derivative operators.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
3. Differential operators. Material derivatives. The Reynolds transport theorem. Applications. Second order Cartesian tensors in $\mathbf{R}^n$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
4. Cauchy's fundamental Theorem for the Cauchy stress tensor. The symmetry property of the Cauchy stress tensor.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
5. The mathematical model of incompressible fluid. The Killing theorem. Applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
6. Variational approach for the weak solution of the Stokes system in a bounded Lipschitz domain with homogeneous Dirichlet boundary condition.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
7. The Dirichlet problem for the Stokes system in exterior Lipschitz domains in $\mathbf{R}^n$, with data in weighted Sobolev spaces. Well-posedness results and applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
8. The fundamental solution of the Stokes system in $\mathbf{R}^n$. Stokes layer potentials and their properties in Sobolev spaces. Applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
9. Well-posedness results in Sobolev spaces for boundary value problems for the Stokes system in bounded Lipschitz domains in $\mathbf{R}^n$. Layer potential approach. Applications and numerical results.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
10. Boundary value problems for the Navier-Stokes system in bounded Lipschitz domains in $\mathbf{R}^3$. Exterior Dirichlet problems for the Navier-Stokes systems in $\mathbf{R}^3$, with data in weighted Sobolev spaces. Applications and numerical results.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	

11. Boundary value problems for the Stokes and Navier-Stokes systems with variable coefficients in Lipschitz domains. Variational and layer potential approach (I).	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
12. Stokes and Navier-Stokes systems with variable coefficients in Lipschitz domains: Variational and layer potential approach (II). Applications to flow problems in porous media. Research directions.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	

Bibliography

1. Kohr, M., Mikhailov, S.E., Nistor, V., Wendland, W.L, *Stationary Stokes and Navier-Stokes Equations with Variable Coefficients - Integral Operators and Variational Approaches*, vol. 2380 of Lecture Notes in Mathematics, Springer, Cham, 2026.

2. Kohr, M., Pop, I., *Viscous Incompressible Flow for Low Reynolds Numbers*, WIT Press (Wessex Institute of Technology Press), Southampton (UK) – Boston, 2004.

3. Kohr, M., *Modern Problems in Viscous Fluid Mechanics*, Cluj University Press, Cluj-Napoca, 2 vols. 2000 (in Romanian).

4. Kohr, M., Mikhailov, S.E., Wendland, W.L., *Non-homogeneous Dirichlet-transmission problems for the anisotropic Stokes and Navier-Stokes systems in Lipschitz domains with transversal interfaces*, Calculus of Variations and Partial Differential Equations, 61:198 (2022), 47 pp.

5. Truesdell, C., Rajagopal, K.R., *An Introduction to the Mechanics of Fluids*, Birkhäuser, Basel, 2000.

6. Kiselev, S.P., Vorozhtsov, E.V., Fomin, V.M., *Foundations of Fluid Mechanics with Applications. Problem Solving Using Mathematica*, Birkhäuser, Boston, 1999.

7. Hsiao, G.C., Wendland W.L., *Boundary Integral Equations*, Springer-Verlag, Heidelberg, 1th Edition 2008, 2nd Edition 2021.

8. Mitrea, M. Wright, M., *Boundary value problems for the Stokes system in arbitrary Lipschitz domains*, Astérisque, France, 344 (2012): viii+241 pp.

9. Precup, R., *Linear and Semilinear Partial Differential Equations.* De Gruyter, Berlin, 2013.

10. Galdi, G.P., *An Introduction to the Mathematical Theory of the Navier–Stokes Equations*, 2nd Edition. Springer, Berlin, 2011.

11. Boyer, F., Fabrie, P., *Mathematical Tools for the Study of the Incompressible Navier-Stokes Equations and Related Models*, Springer, New York, 2013.

12. McLean, W., *Strongly Elliptic Systems and Boundary Integral Equations*, Cambridge University Press, Cambridge, UK, 2000.

13. Wloka, J. T. , Rowley, B., Lawruk, B., *Boundary Value Problems for Elliptic Systems*, Cambridge University Press, Cambridge, 1995.

3. Evaluation

Type of activity	8.1 Evaluation criteria[2]	8.2 Evaluation methods[3]	8.3 Percentage in the final grade
8.4. Course	Knowledge of concepts and basic results.	Written exam.	60%
	Ability to justify by proofs theoretical results.		
8.5. Seminar	Ability to apply concepts and results acquired in the course in mathematical modeling and analysis of problems in Fluid Mechanics.	Evaluation the activity of PhD students during the semester.	40%
8.6 Minimum standard for passing			
Ability to apply concepts and results acquired in the course to study special problems in complex analysis.			

[2] The evaluation criteria must directly reflect the learning outcomes targeted at the level of the degree programme respectively at the level of the subject. More specifically, the learning outcomes set out in the expected learning outcomes are assessed.

[3] Both final evaluation methods and ongoing evaluation strategies should be established.

9. SDG labels (Sustainable Development Goals)[4]

	○	Sustainable Development Generic Label						
1 FĂRĂ SĂRĂCIE	2 FOAMETE „ZERO"	3 SĂNĂTATE ŞI BUNĂSTARE	4 EDUCAŢIE DE CALITATE	5 EGALITATE DE GEN	6 APĂ CURATĂ ŞI SANITAŢIE	7 ENERGIE CURATĂ ŞI LA PREŢURI ACCESIBILE	8 MUNCĂ DECENTĂ ŞI CREŞTERE ECONOMICĂ	9 INDUSTRIE, INOVAŢIE ŞI INFRASTRUCTURĂ
○	○	○	○	○	○	○	○	○
10 INEGALITĂŢI REDUSE	11 ORAŞE ŞI COMUNITĂŢI DURABILE	12 CONSUM ŞI PRODUCŢIE RESPONSABILE	13 ACŢIUNE CLIMATICĂ	14 VIAŢA ACVATICĂ	15 VIAŢA TERESTRĂ	16 PACE, JUSTIŢIE ŞI INSTITUŢII EFICIENTE	17 PARTENERIATE PENTRU REALIZAREA OBIECTIVELOR	No label applies
○	○	○	○	○	○	○	○	○

Date of entry:
11.02.2026

Signature of course coordinator
Prof.PhD. Mirela KOHR

Signature of seminar coordinator
Prof.PhD. Mirela KOHR

Date of approval in the department:

Signature of the head of department

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[4] Select a single label which, according to the *Implementation of SDG labels in the academic process*, best matches the subject. If the subject addresses sustainable development in a generic manner (i.e. by presenting/introducing the general framework of sustainable development, etc.), then the Sustainable Development generic label may be applied. If none of the labels describe the subject, select the last option: *"No label applies."*