COURSE DESCRIPTION

Special Topics in Complex Analysis (Capitole speciale de analiză complexă)

Academic year 2026-2027

1. Programme-related data

1.1. Higher Education Institution	**Babeș-Bolyai University of Cluj-Napoca**
1.2. Faculty	**Mathematics and Computer Science**
1.3. Doctoral School	**Mathematics and Computer Science**
1.4. Field of study	**Mathematics**
1.5. Level of study	**Doctoral studies**

2. Course-related data

2.1. Course title	**Special topics in complex analysis / Capitole speciale de analiză complexă**			Course code	**MDE3139**
2.2. Course coordinator	**Professor PhD Mirela KOHR**				
2.3. Seminar coordinator	**Professor PhD Mirela KOHR**				
2.4. Year of study	1	2.5. Semester	1	2.6. Type of assessment	Exam
2.7. Course status	Optional			2.8. Course type	Specialisation subject

3. Total estimated time (hours per semester of teaching activities)

3.1. Number of hours per week	**3**	of which: 3.2. course	**2**	3.3. seminar/ laboratory/ project	**1**
3.4. Total of hours in the curriculum	36	of which: 3.5. course	24	3.6. seminar/ laboratory	12
Time allocation for individual study (IS) and self-taught activities (ST)					**hours**
Learning from textbooks, course materials, bibliography, and notes (IS)					40
Additional research in the library, on subject-specific electronic platforms, and on-site					40
Preparing seminars/ laboratories/ projects, assignments, reports, portfolios, and essays					44
Tutoring (professional guidance)					50
Examinations					40
Other activities					-
3.7. Total hours of individual study (IS) and self-taught activities (ST)				214	
3.8. Total hours per semester				250	
3.9. Number of credits				10	

4. Prerequisites (where applicable)

4.1. curriculum-related	In-depth knowledge of the following disciplines: • Complex analysis; • Real analysis; • Topology; • Partial differential equations.
4.2 skills-related	There are useful logical thinking and mathematical notions and results from the above-mentioned fields

5. Specific conditions (where applicable)

5.1. course-related	Classroom with blackboard, video projector
5.2. seminar/laboratory-related	Classroom with blackboard, video projector

6. Subject-specific learning outcomes

Knowledge
1. Graduated PhD student knows, understands and uses concepts, individual results and advanced mathematical theories in the field of Complex Analysis and related mathematical areas, having the capacity to handle and communicate them.
2. Graduated PhD student has the capacity to develop and use efficient research skills, to formulate new problems, to initiate new mathematical research in advanced topics of Complex Analysis and also in other related areas of Mathematics, preparing reports and scientific papers.
3. Graduated PhD student has the capacity to model and analyse from the mathematical point of view real processes and phenomena from other sciences, physics, medicine, engineering, and computer science.
4. Graduated PhD student has the capacity of advanced skills to develop and manage research projects, applying a wide range of quantitative and qualitative methods.
5. Graduated PhD student has the capacity to conduct rigorous literature reviews, critically evaluate scientific publications, and identify innovative research directions.
6. Graduated PhD student has competence in preparing scientific manuscripts according to international standards, presenting results at scientific conferences, and effectively communicating research findings to the scientific community.
7. Graduated PhD student has the ability to integrate concepts and methodologies from related fields (physics, chemistry, biology, engineering, computer science) to address complex scientific problems.
Skills
1. Graduated PhD student has the ability to identify and state significant problems which can be the basis for subsequent research.
2. Graduated PhD student has the ability to use scientific language and to perform research in Mathematics.
3. Graduated PhD student has the ability for continuous self-perfecting and study.
4. Graduated PhD student has the ability to inform themselves, to work independently or in a team in order to carry out studies and to solve complex problems.
5. Application of organized and efficient work rules, a responsible attitude towards the didactic-scientific field, to bring creative value to own potential respecting professional ethics principles.
6. Graduated PhD student has the ability to adopt and integrate in different environments from education and research.
7. Graduated PhD student has the ability to adapt to the requirements of a dynamical society and to communicate efficiently in an international language.
Responsibility and autonomy
1. Graduated PhD student has the ability to inform themselves, to work independently or in a team in order to carry out studies and to solve complex problems.
2. Graduated PhD student has the ability of critical investigation of specific literature.
3. Graduated PhD student has the ability to use international data bases of academic research.

7. Contents

7.1. Course	**Teaching and learning methods**	**Remarks[1]**
Part I		
1. Analytic branches. Index. General properties. The Cauchy integral formulas. Applications.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
2. Cauchy's theorem related to zeros and poles of meromorphic functions. The argument principle.. Rouché's theorem. Open mapping theorem and Hurwitz's theorem. Applications.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
3. The Fréchet space H(Ω). Families of holomorphic functions. Montel and Vitali's theorems. Extremal	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative	

[1] For example, organisational aspects, recommendations for students, specific aspects relating to the course/seminar, such as inviting experts in the field, etc.

problems on compact subsets of $H(\Omega)$.	explanations.	
4. Conformal mappings. The automorphisms of the unit disc and the upper half-plane. The automorphisms of the complex plane.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
5. The Riemann mapping theorem. Extension to the boundary.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
6. Univalent functions. General properties. The family S. Necessary and sufficient conditions for univalence on the unit disc of the complex plane.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
Part II		
7. Introduction in the theory of functions of several complex variables. Holomorphic functions in $\mathbf{C}^n$. The generalized Cauchy-Riemann equations. Integral representation of holomorphic functions on the polydisc. Sequences and series of holomorphic functions in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
8. Sets of uniqueness for the holomorphic functions in $\mathbf{C}^n$. The Montel and Vitali theorems. Holomorphic mappings. Biholomorphic mappings in $\mathbf{C}^n$. Fatou-Bieberbach domains. Poincaré's theorem.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
9. Cartan's uniqueness theorems. The automorphisms of the Euclidean unit ball and the unit polydisc in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
10. The Carathéodory family *M* of holomorphic mappings on the unit ball in $\mathbf{C}^n$. Compactness of the family *M*. Starlike and convex mappings on the unit ball in $\mathbf{C}^n$. Necessary and sufficient conditions for starlikeness and convexity.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
11. Loewner chains, transition mappings, Herglotz vector fields, and the Loewner differential equation in $\mathbf{C}^n$. The solutions of the Loewner differential equation on the unit ball in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
12. The family $S^0(B^n)$ of biholomorphic mappings with parametric representation on the unit ball in $\mathbf{C}^n$. Characterizations in terms of Loewner chains. Compactness of the family $S^0(B^n)$. Open problems and new research directions.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

Bibliography

1. I. Graham, G. Kohr, *Geometric Function Theory in One and Higher Dimensions*, Marcel Dekker Inc., New York, 2003.
2. G. Kohr, *Basic Topics in Holomorphic Functions of Several Complex Variables*, Cluj University Press, Cluj-Napoca, 2003.
3. G. Kohr, P.T. Mocanu, *Special Topics of Complex Analysis*, Cluj University Press, Cluj-Napoca, 2005 (in Romanian).
4. P. Hamburg, P.T. Mocanu, N. Negoescu, *Mathematical Analysis* (*Complex Functions*), Editura Didactică şi Pedagogică, Bucureşti, 1982 (in Romanian).
5. C.A. Berenstein, R. Gay, *Complex Variables: An Introduction*, Springer-Verlag New York Inc., 1991.
6. Conway, J.B., *Functions of One Complex Variable,* vol. I, Graduate Texts in Mathematics, Springer Verlag, New York, 1978 (Second Edition).
7. P. Duren, *Univalent Functions*, Springer-Verlag, New York, 1983.
8. Ch. Pommerenke, *Univalent Functions*, Vandenhoeck & Ruprecht, Göttingen, 1975.
9. P. Duren, I. Graham, H. Hamada, G. Kohr, *Solutions for the generalized Loewner differential equation in several complex*

variables, Mathematische Annalen, **347** (2010), 411-435.

10. I. Graham, H. Hamada, G. Kohr, M. Kohr, *Extremal properties associated with univalent subordination chains in* $\mathbf{C}^n$, Mathematische Annalen, **359** (2014), 61-99.

11. K. Güerlebeck, K. Habetha, W. Spröβig, *Holomorphic Functions in the Plane and n-Dimensional Space,* Birkhäuser, Basel-Boston-Berlin, 2008.

12. R.C. Gunning, *Introduction to Holomorphic Functions of Several Variables*, vol. I. *Function Theory*, Wadsworth & Brooks/Cole, Monterey, CA, 1990.

13. S.G. Krantz, *Function Theory of Several Complex Variables*, Reprint of the 1992 Edition, AMS Chelsea Publishing, Providence, Rhode Island, 2001.

14. M. Range, *Holomorphic Functions and Integral Representations in Several Complex Variables* Springer-Verlag, New York, 1986.

15. W. Rudin, *Function Theory in the Unit Ball of* $\mathbf{C}^n$, Springer-Verlag, New York, 1980.

7.2. Seminar	**Teaching and learning methods**	**Remarks**
Part I		
1. Applications of the Residue Theorem to the computation of some real integrals.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
2. Applications of the argument principle and Rouché's Theorem.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
3. Examples of compact families of holomorphic functions. Extremal problems on compact subsets of $H(\Omega)$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
4. Applications of the Riemann mapping theorem. Conformal mappings of special simply connected domains in $\mathbf{C}$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
5. Necessary and sufficient conditions of univalence for holomorphic functions on the unit disc of the complex plane.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
6. The family S of normalized univalent functions on the unit disc of the complex plane. Properties and examples of functions of the family S.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
Part II		
7. Applications of the Cauchy integral representations on the unit polydisc in $\mathbf{C}^n$. Applications of the maximum modulus theorem and the Schwarz Lemma for holomorphic functions of several complex variables.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
8. Pluriharmonic functions and plurisubharmonic functions. Examples.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
9. Sufficient conditions of univalence for holomorphic mappings on the unit ball in $\mathbf{C}^n$. Examples of locally biholomorphic mappings. Examples of univalent mappings. Examples of biholomorphic automorphisms of the n-	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	

dimensional complex space $\mathbf{C}^n$. Fatou-Bieberbach domains and Runge domains in $\mathbf{C}^n$.		
10. The Carathéodory family *M* of holomorphic mappings on the unit ball in $\mathbf{C}^n$. Starlike and convex mappings on the unit ball in $\mathbf{C}^n$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
11. The generalized Loewner differential equation in $\mathbf{C}^n$. The solutions of the generalized Loewner differential equation in $\mathbf{C}^n$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
12. The family $S^0(B^n)$ of biholomorphic mappings with parametric representation on the unit ball in $\mathbf{C}^n$. Characterizations in terms of Loewner chains. Compactness of the family $S^0(B^n)$. Extremal problems and open problems.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	

Bibliography

1. I. Graham, G. Kohr, *Geometric Function Theory in One and Higher Dimensions*, Marcel Dekker Inc., New York, 2003.
2. G. Kohr, *Basic Topics in Holomorphic Functions of Several Complex Variables*, Cluj University Press, Cluj-Napoca, 2003.
3. G. Kohr, P.T. Mocanu, *Special Topics of Complex Analysis*, Cluj University Press, Cluj-Napoca, 2005 (in Romanian).
4. P. Hamburg, P.T. Mocanu, N. Negoescu, *Mathmatical Analysis* (*Complex Functions*), Editura Didactică şi Pedagogică, Bucureşti, 1982 (in Romanian).
5. Bulboacă, T., Joshi, S.B., Goswami, P., *Complex Analysis. Theory and Applications*, de Gruyter, Berlin, Boston, 2019.
6. Conway, J.B., *Functions of One Complex Variable,* vol. I, Graduate Texts in Mathematics, Springer Verlag, New York, 1978 (Second Edition).
7. F. Bracci, I. Graham, H. Hamada, G. Kohr, *Variation of Loewner chains, extreme and support points in the class* S^0 *in higher dimensions*, Constructive Approximation, **43** (2016), 231-251.
8. P. Duren, I. Graham, H. Hamada, G. Kohr, *Solutions for the generalized Loewner differential equation in several complex variables*, Mathematische Annalen, **347** (2010), 411-435.
9. I. Graham, H. Hamada, G Kohr, M Kohr, *Loewner PDE in infinite dimensions*, Computational Methods and Function Theory, **25** (2025), 151–171.
10. K. Güerlebeck, K. Habetha, W. Spröβig, *Holomorphic Functions in the Plane and n-Dimensional Space,* Birkhäuser, Basel-Boston-Berlin, 2008.
11. R.C. Gunning, *Introduction to Holomorphic Functions of Several Variables*, vol.I. *Function Theory*, Wadsworth & Brooks/Cole, Monterey, CA, 1990.
12. S.G. Krantz, *Function Theory of Several Complex Variables*, Reprint of the 1992 Edition, AMS Chelsea Publishing, Providence, Rhode Island, 2001.
13. R. Narasimhan, *Several Complex Variables*, The University of Chicago Press, Chicago, 1971.
14. R. Narasimhan, Y. Nievergelt, *Complex Analysis in One Variable*, Birkhäser, 2001.
15. M. Range, *Holomorphic Functions and Integral Representations in Several Complex Variables* Springer-Verlag, New York, 1986.

8. Evaluation

Type of activity	8.1 Evaluation criteria[2]	8.2 Evaluation methods[3]	8.3 Percentage in the final grade
8.4. Course	Knowledge of concepts and basic results	Written exam.	60%
	Ability to justify by proofs theoretical results		
8.5. Seminar	Ability to apply concepts and results acquired in the course at mathematical modeling and analysis of current research problems in pure and applied mathematics.	Evaluation the activity of PhD students during the semester.	40%
8.6 Minimum standard for passing			
Ability to apply concepts and results acquired in the course to study special problems in complex analysis.			

9. SDG labels (Sustainable Development Goals)[4]

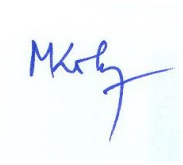
	○	Sustainable Development Generic Label						
								
○	○	○	○	○	○	○	○	○
								No label applies
○	○	○	○	○	○	○	○	○

Date of entry:

11.02.2026

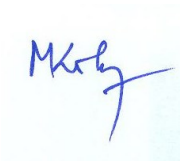
Signature of course coordinator

Prof.PhD. Mirela KOHR



Signature of seminar coordinator

Prof.PhD. Mirela KOHR



Date of approval in the department:

Signature of the head of department

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[2] The evaluation criteria must directly reflect the learning outcomes targeted at the level of the degree programme respectively at the level of the subject. More specifically, the learning outcomes set out in the expected learning outcomes are assessed.

[3] Both final evaluation methods and ongoing evaluation strategies should be established.

[4] Select a single label which, according to the *Implementation of SDG labels in the academic process*, best matches the subject. If the subject addresses sustainable development in a generic manner (i.e. by presenting/introducing the general framework of sustainable development, etc.), then the Sustainable Development generic label may be applied. If none of the labels describe the subject, select the last option: *"No label applies."*