SYLLABUS

Stochastic Analysis and Applications

University year 2025-2026

1. Information regarding the programme

1.1. Higher education institution	Babeş-Bolyai University
1.2. Faculty	Mathematics and Computer Science
1.3. Department	Mathematics
1.4. Field of study	Mathematics
1.5. Study cycle	Undergraduate
1.6. Study programme/Qualification	Mathematics
1.7. Form of education	Full-time

2. Information regarding the discipline

2.1. Name of the discipline	**Stochastic Analysis and Applications**						Discipline code	**MLE0105**
2.2. Course coordinator					Lect. Dr. Oana-Andrea Lang			
2.3. Seminar coordinator					Lect. Dr. Oana-Andrea Lang			
2.4. Year of study	3	2.5. Semester	6	2.6. Type of evaluation	E/C		2.7. Discipline regime	Optional

3. Total estimated time (hours/semester of didactic activities)

3.1. Hours per week	**4**	of which: 3.2 course	**2**	3.3 seminar/laboratory	**2**
3.4. Total hours in the curriculum	48	of which: 3.5 course	24	3.6 seminar/laborator	**24**
Time allotment for individual study (ID) and self-study activities (SA)					**hours**
Learning using manual, course support, bibliography, course notes (SA)					50
Additional documentation (in libraries, on electronic platforms, field documentation)					20
Preparation for seminars/labs, homework, papers, portfolios and essays					12
Tutorship					10
Evaluations					10
Other activities:					-

3.7. Total individual study hours	**102**
3.8. Total hours per semester	**150**
3.9. Number of ECTS credits	**6**

4. Prerequisites (if necessary)

4.1. curriculum	• Real Analysis • Probability Theory • Basics in Ordinary Differential Equations (ODEs)
4.2. competencies	• Understanding of fundamental calculus (e.g. Riemann-Stieltjes integral) • Ability to work with conditional probabilities and probability distributions • Knowledge of elementary principles in the theory of ODEs

5. Conditions (if necessary)

5.1. for the course	Classroom with blackboard/video projector
5.2. for the seminar /lab activities	Classroom with blackboard/video projector

6.1. Specific competencies acquired [1]

Professional/essentialcompetencies	**C1.1** Identification of concepts, description of theories, and use of specific terminology. **C2.3** Application of appropriate theoretical analysis methods to the given issue. **C4.2** Explanation and interpretation of mathematical models.
Transversal competencies	**CT1.** Applying the rules of rigorous and efficient work, demonstrating responsible attitudes toward the scientific and educational fields, in order to optimally and creatively harness one's own potential in specific situations, while respecting the principles and norms of professional ethics.

6.2. Learning outcomes

Knowledge	The student knows • fundamental notions related to Stochastic Analysis and methods of applying them to areas of science related to Mathematics, Mechanics and Engineering. • how to use at least a programming and editing environment to create attractive mathematical texts with formulas, diagrams and images.
Skills	The student is able to • ensure the formation of skills specific to the Mathematics-related disciplines needed to complete the assignments. • explore some mathematical content independently, drawing on ideas and tools from previous coursework to extend their understanding.
Responsibility and autonomy:	The student has the ability to work independently to • extend mathematical ideas and arguments from previous coursework to a mathematical topic not previously studied. • interpret articles or books from the mathematical literature and incorporate ideas and results from the literature in their written and oral presentations.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	• To provide students with a solid foundation in stochastic analysis and filtering theory, equipping them with the mathematical tools and concepts necessary to model and analyse complex systems under uncertainty, with applications in life sciences.
7.2 Specific objective of the discipline	• To enable students to understand and apply the principles of stochastic integration, stochastic differential equations (SDEs), linear and nonlinear filtering methods to real-world problems. • To gain the ability to formulate, solve, and interpret stochastic mathematical models related to different random processes.

[1] One can choose either competences or learning outcomes, or both. If only one option is chosen, the row related to the other option will be deleted, and the kept one will be numbered 6.

8. Content

8.1 Course	Teaching methods	Remarks
Week 1: Preliminaries in Probability Theory: probability spaces, sigma-algebras, filtrations; Stochastic processes (definition and examples); Stopping times; Brownian motion.	Interactive exposure, explanation, conversation, demonstration	
Week 2: Stochastic integration: construction of the Itô integral, parallel with the construction of the Riemann-Stieltjes integral; Quadratic variation.	Interactive exposure, explanation, conversation, demonstration	
Week 3: Itô formula and applications; Martingales; Martingale representation theorem.	Interactive exposure, explanation, conversation, demonstration	
Week 4: Introduction to stochastic differential equations (SDEs); Formal definitions of solutions.	Interactive exposure, explanation, conversation, demonstration	
Week 5: SDEs: existence and uniqueness in the case of Lipschitz coefficients; Gronwall's Lemma.	Interactive exposure, explanation, conversation, demonstration	
Week 6: SDEs: existence and uniqueness in the case of Lipschitz coefficients (cont.); Connections to stochastic partial differential equations (SPDEs).	Interactive exposure, explanation, conversation, demonstration	
Week 7: Introduction to filtering; Posterior distribution, prior distribution; Kalman-Bucy filter; Girsanov's Theorem.	Interactive exposure, explanation, conversation, demonstration	
Week 8: The Filtering Equations I: Zakai Equation; Novikov's condition.	Interactive exposure, explanation, conversation, demonstration	
Week 9: The Filtering Equations II: Kushner-Stratonovich Equation; The innovation process approach.	Interactive exposure, explanation, conversation, demonstration	
Week 10: Particle filters.	Interactive exposure, explanation, conversation, demonstration	
Week 11: Stochastic climate models; Principles of Data Assimilation.	Interactive exposure, explanation, conversation, demonstration	
Week 12: Stochastic models in Neuroscience.	Interactive exposure, explanation, conversation, demonstration	

Bibliography
[1] Revuz, D., & Yor, M. (1999), *Continuous Martingales and Brownian Motion* (2nd ed.). Springer.
[2] Gawarecki, L., & Mandrekar, V. (2011), *Stochastic Differential Equations in Infinite Dimensions.* Springer.
[3] Crisan, D., & Bain, A. (2002), *Fundamentals of Stochastic Filtering.* Springer.
[4] Slingo, J. M., & Palmer, T. N. (2011), *Uncertainty in Weather and Climate Prediction.* Cambridge University Press.
[5] Gabbiani, F., & Cox, S. (2011), *Mathematical Neuroscience: From Discrete to Continuous Models and Neural Computation.* MIT Press.

8.2 Seminar / laboratory	Teaching methods	Remarks
Week 1: Exercises on stochastic processes; Problems on stopping times and their properties; Proofs of key properties of Brownian motion (e.g., Markov property, continuity).	Interactive exposure, explanation, conversation	
Week 2: Exercises on constructing the Itô integral step by step; Proofs and computations involving quadratic variation	Interactive exposure, explanation, conversation	
Week 3: Exercises on applying Itô's formula to different stochastic processes; Proofs and examples illustrating key properties of martingales; Applications of the Martingale Representation Theorem	Interactive exposure, explanation, conversation	

Week 4: Exercises on writing and interpreting basic SDEs; Problems on verifying whether a given process satisfies an SDE; Examples of explicit solutions to simple SDEs; Comparison of SDEs with ordinary differential equations (ODEs).	Interactive exposure, explanation, conversation	
Week 5: Exercises on checking Lipschitz conditions for SDE coefficients; Problems on applying Gronwall's Lemma in the context of SDEs.	Interactive exposure, explanation, conversation	
Week 6: Exercises on advanced examples of SDEs with Lipschitz coefficients; Problems on verifying existence and uniqueness for more complex SDEs.	Interactive exposure, explanation, conversation	
Week 7: Exercises on the concept of prior and posterior distributions in filtering; Hands-on exercises on filtering and updating distributions.	Interactive exposure, explanation, conversation	
Week 8: Problems on verifying Novikov's condition in filtering models; Hands-on exercises on using the Zakai Equation for specific stochastic processes.	Interactive exposure, explanation, conversation	
Week 9: Problems on the application of the innovation process approach in filtering; Discussion on the differences between the Zakai and Kushner-Stratonovich equation	Interactive exposure, explanation, conversation	
Week 10: Additional procedures in particle filtering – examples: tempering, jittering, etc.	Interactive exposure, explanation, conversation	
Week 11: Data assimilation exercises for systems from weather prediction and climate modelling.	Interactive exposure, explanation, conversation	
Week 12: Examples of stochastic models used in Neuroscience and discussion.	Interactive exposure, explanation, conversation	

Bibliography
[1] Revuz, D., & Yor, M. (1999), *Continuous Martingales and Brownian Motion* (2nd ed.). Springer.
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[4] Slingo, J. M., & Palmer, T. N. (2011), *Uncertainty in Weather and Climate Prediction.* Cambridge University Press.
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9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

- The course content is designed to meet the evolving needs of the professional community and employers by incorporating current research, methodologies, and practical applications in stochastic processes, filtering theory, and their real-world implementations in fields like life sciences and climate modeling.
- The program is regularly updated to reflect the expectations of professional associations and academic communities, ensuring students are equipped with the skills and knowledge required for advanced roles in academia, research institutions, and industries where stochastic modeling and analysis are applied.

10. Evaluation

Activity type	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Percentage of final grade
10.4 Course	Knowledge and understanding of fundamental concepts and results	Exam	60%

10.5 Seminar/laboratory	Problem solving based on the concepts and theorems learnt	Coursework	40%
10.6 Minimum standard of performance			
• Over 50% overall.			

11. Labels ODD (Sustainable Development Goals)[2]

	General label for Sustainable Development							
								9 INDUSTRY, INNOVATION AND INFRASTRUCTURE

Date:
11.04.2025

Signature of course coordinator

O Lang

Signature of seminar coordinator

O Lang

Date of approval:
25.04.2025

Signature of the head of department

Prof. dr. Andrei Mărcuș

[2] Keep only the labels that, according to the *Procedure for applying ODD labels in the academic process*, suit the discipline and delete the others, including the general one for *Sustainable Development* – if not applicable. If no label describes the discipline, delete them all and write „*Not applicable*.”.