

## SYLLABUS

### Special chapters of ordinary differential equations

University year 2025-2026

#### 1. Information regarding the programme

1.1. Higher education institution	Babeş-Bolyai University
1.2. Faculty	Mathematics and Computer Science
1.3. Department	Mathematics
1.4. Field of study	Mathematics
1.5. Study cycle	Bachelor
1.6. Study programme/Qualification	Mathematics and Informatics (in Romanian)
1.7. Form of education	Full attendance

#### 2. Information regarding the discipline

2.1. Name of the discipline		Special chapters of ordinary differential equations					Discipline code	MLE0038
2.2. Course coordinator					Conf. Dr. Adriana Buică			
2.3. Seminar coordinator					Conf. Dr. Adriana Buică			
2.4. Year of study	2	2.5. Semester	4	2.6. Type of evaluation	V	2.7. Discipline regime		Optional

#### 3. Total estimated time (hours/semester of didactic activities)

3.1. Hours per week	<b>4</b>	of which: 3.2 course	<b>2</b>	3.3 seminar/laboratory	<b>2</b>
3.4. Total hours in the curriculum	<b>56</b>	of which: 3.5 course	<b>28</b>	3.6 seminar/laborator	<b>28</b>
<b>Time allotment for individual study (ID) and self-study activities (SA)</b>					<b>hours</b>
Learning using manual, course support, bibliography, course notes (SA)					30
Additional documentation (in libraries, on electronic platforms, field documentation)					10
Preparation for seminars/labs, homework, papers, portfolios and essays					40
Tutorship					4
Evaluations					4
Other activities:					6
<b>3.7. Total individual study hours</b>	<b>94</b>				
<b>3.8. Total hours per semester</b>	<b>150</b>				
<b>3.9. Number of ECTS credits</b>	<b>6</b>				

#### 4. Prerequisites (if necessary)

4.1. curriculum	Mathematical Analysis I, II, Linear Algebra I, Differential Equations I
4.2. competencies	Continuous, Lipschitz, class $C^n$ functions, rules for derivation and integration, separable differential equations, the fundamental theorems for linear differential systems, the characteristic equation method in the case of constant coefficients

#### 5. Conditions (if necessary)

5.1. for the course	Classroom with blackboard.
5.2. for the seminar /lab activities	Classroom with blackboard.

## 6. Specific competencies acquired <sup>1</sup>

Professional/essential competencies	<ul style="list-style-type: none"> <li>C1.1 Understanding the notions, the theories and the proper usage of the specific scientific language</li> <li>C2.3 The application of the theoretical methods to the specific problems.</li> </ul>
Transversal competencies	<ul style="list-style-type: none"> <li>CT1. The application of the efficient and rigorous work methods, a responsible attitude towards the scientific and teaching domain in order to reach the full own potential in specific situations, respecting the ethical rules and principles.</li> </ul>

## 7. Objectives of the discipline (outcome of the acquired competencies)

<b>7.1 General objective of the discipline</b>	<ul style="list-style-type: none"> <li>To introduce the fundamental notions and results of the qualitative theory of differential equations and dynamical systems</li> </ul>
<b>7.2 Specific objective of the discipline</b>	<ul style="list-style-type: none"> <li>To introduce the qualitative study of a differential equation</li> <li>To understand the different phenomena of dependence on the initial values or parameters of the solutions of a differential equation: the continuous dependence, stability, topological equivalence, structural stability, bifurcations.</li> </ul>

## 8. Content

8.1 Course	Teaching methods	Remarks
1.The harmonic oscillator. The pendulum equation. Periodic solutions. Oscillatory solutions	Exposition, proofs, examples	
2. Zeros for the solutions of second order linear differential equations. The Sturm separation theorem and the Sturm comparison theorem	Exposition, proofs, examples	
3. Oscillatory equations: sufficient conditions.	Exposition, proofs, examples	
4. The boundary value problem. The Green function. Eigenvalues and eigenfunctions.	Exposition, proofs, examples	
5. Nonlinear differential equations. Maximal solutions.	Exposition, proofs, examples	
6. Continuity and differentiability with respect to the initial data and parameters. The stability notions.	Exposition, proofs, examples	
7. The first notions of continuous dynamical systems. The stability of the equilibria for scalar equations. The first test.	Exposition, proofs, examples	
8. Planar autonomous systems. The properties of the flow. First integrals and conservative systems.	Exposition, proofs, examples	
9. The topological equivalence of planar autonomous linear systems.	Exposition, proofs, examples	

10. The stability of equilibria of planar systems: the linearization method and the Lyapunov functions method.	Exposition, proofs, examples	
11. The topological equivalence of nonlinear planar systems in a neighborhood of an equilibrium point. The Hartman-Grosman theorem.	Exposition, proofs, examples	
12. The analysis of the solutions of scalar nonautonomous differential equations.	Exposition, proofs, examples	
13. . The analysis of the solutions of scalar nonautonomous differential equations with applications to planar autonomous systems. The second test.	Exposition, proofs, examples	
14. Discrete scalar dynamical systems: the stability of the fixed points. Conclusions. The announcement of the final marks.	Exposition, proofs, examples	
Bibliography <ol style="list-style-type: none"> <li>1. A. Buică, <i>Introduction to the qualitative theory of ordinary differential equations</i>, Notițe de curs postate în Teams.</li> <li>2. J. Hale, H. Koçak, <i>Dynamics and bifurcations</i>, Springer-Verlag, 1991.</li> <li>3. M.W. Hirsch, S. Smale, <i>Differential equations, dynamical systems, and linear algebra</i>, Academic Press, 1974.</li> <li>4. R. Precup, <i>Ecuatii diferențiale</i>, Risoprint, Cluj-Napoca, 2011. <i>Ordinary Differential Equations</i>, De Gruyter, 2018.</li> <li>5. Ioan A. Rus, <i>Ecuatii diferențiale, ecuatii integrale si sisteme dinamice</i>, Transilvania Press, 1996.</li> </ol>		
<b>8.2 Seminar / laboratory</b>	<b>Teaching methods</b>	<b>Remarks</b>
1. Qualitative behaviour of the solutions of second order linear differential equations with constant coefficients	Examples, dialogue, explanations, proofs, critical thinking	
2. The zeros of the solutions of second order linear differential equations with variable coefficients	Examples, dialogue, explanations, proofs, critical thinking	
3. Airy and Bessel equations. Other examples.	Examples, dialogue, explanations, proofs, critical thinking	
4. The boundary value problem. The Green function. Eigenvalues and eigenfunctions. Exercises	Examples, dialogue, explanations, proofs, critical thinking	
5. Nonlinear differential equations. Maximal solutions. Exercises	Examples, dialogue, explanations, proofs, critical thinking	
6. Continuity and differentiability with respect to the initial data and parameters. The stability notions. Exercises	Examples, dialogue, explanations, proofs, critical thinking	
7. Phase portraits for scalar autonomous equations. Exercises	Examples, dialogue, explanations, proofs, critical thinking	
8. Phase portraits for planar linear autonomous systems. Exercises	Examples, dialogue, explanations, proofs, critical thinking	
9. Phase portraits for planar autonomous systems using polar coordinates. Exercises	Examples, dialogue, explanations, proofs, critical thinking	
10. The stability of the equilibria of planar autonomous systems. Exercises	Examples, dialogue, explanations, proofs, critical thinking	
11. Phase portraits of planar autonomous systems in a neighborhood of an equilibrium point. Exercises	Examples, dialogue, explanations, proofs, critical thinking	
12. The analysis of the solutions of scalar nonautonomous differential equations.	Examples, dialogue, explanations, proofs, critical thinking	
13. The analysis of the prey-predator type planar systems.	Examples, dialogue, explanations, proofs, critical thinking	
14. The stability of the fixed points of scalar discrete dynamical systems.	Examples, dialogue, explanations, proofs, critical thinking	
Bibliography <ol style="list-style-type: none"> <li>1. A. Buică, <i>Introduction to the qualitative theory of ordinary differential equations</i>, Notițe de curs postate în Teams.</li> <li>2. J. Hale, H. Koçak, <i>Dynamics and bifurcations</i>, Springer-Verlag, 1991.</li> <li>3. M.W. Hirsch, S. Smale, <i>Differential equations, dynamical systems, and linear algebra</i>, Academic Press, 1974.</li> <li>4. R. Precup, <i>Ecuatii diferențiale</i>, Risoprint, Cluj-Napoca, 2011. <i>Ordinary Differential Equations</i>, De Gruyter, 2018.</li> <li>5. Ioan A. Rus, <i>Ecuatii diferențiale, ecuatii integrale si sisteme dinamice</i>, Transilvania Press, 1996.</li> </ol>		


**9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program**

- this topic is covered in the main universities worldwide and, in particular, also of our country
- in this course the students apply and refine their knowledges on the analysis of real functions, which starts to be studied in high schools in Romania
- in this course the students learn the fundamentals of the differential equations theory, having the opportunity to deepen the studies at master and doctoral level

**10. Evaluation**

Activity type	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Percentage of final grade
10.4 Course	The evaluation of the knowledges and the competencies to apply them	2 tests, the first one in the 7th week, and the second one in the 13th week	60%
10.5 Seminar/laboratory	The in-class activity	Conversation, team and individual work	10%
	The second test is conditioned by the prior attendance to at least 10 seminars.		
	Homeworks	Problems solving	20%
10.6 Minimum standard of performance			
<ul style="list-style-type: none"> <li>• At least 12 points (from the maximum of 30) on each test, at least 12 points (from the maximum of 30) on the seminar evaluation, the final mark to be at least 5.</li> </ul>			

**11. Labels ODD (Sustainable Development Goals)<sup>2</sup>**

General label for Sustainable Development								
								

Date:  
11.04.2025

Signature of course coordinator

Conf. Dr. Adriana Buică

Signature of seminar coordinator

Conf. Dr. Adriana Buică

<sup>2</sup> Keep only the labels that, according to the [Procedure for applying ODD labels in the academic process](#), suit the discipline and delete the others, including the general one for *Sustainable Development* – if not applicable. If no label describes the discipline, delete them all and write „Not applicable.”.

Date of approval:  
25.04.2025

Signature of the head of department

Prof. dr. Andrei Mărcuș