SYLLABUS

Applied Functional Analysis

University year 2025-2026

1. Information regarding the programme

1.1. Higher education institution	Babeş-Bolyai University
1.2. Faculty	Mathematics and Computer Science
1.3. Department	Mathematics
1.4. Field of study	Mathematics
1.5. Study cycle	Bachelor
1.6. Study programme/Qualification	Mathematics and Computer Science/Mathematician
1.7. Form of education	Attendance

2. Information regarding the discipline

2.1. Name of the discipline	Functional Analysis					Discipline code	**MLE0004**
2.2. Course coordinator Assoc Prof. PHD Brigitte Breckner							
2.3. Seminar coordinator Assoc. Prof. PHD Brigitte Breckner							
2.4. Year of study	3	2.5. Semester	5	2.6. Type of evaluation	E	2.7. Discipline regime	elective

3. Total estimated time (hours/semester of didactic activities)

3.1. Hours per week	**4**	of which: 3.2 course	**2**	3.3 seminar/laboratory	**2**
3.4. Total hours in the curriculum	56	of which: 3.5 course	28	3.6 seminar/laborator	**28**
Time allotment for individual study (ID) and self-study activities (SA)					**hours**
Learning using manual, course support, bibliography, course notes (SA)					45
Additional documentation (in libraries, on electronic platforms, field documentation)					10
Preparation for seminars/labs, homework, papers, portfolios and essays					20
Tutorship					15
Evaluations					4
Other activities:					-
3.7. Total individual study hours	**94**				
3.8. Total hours per semester	**150**				
3.9. Number of ECTS credits	**6**				

4. Prerequisites (if necessary)

4.1. curriculum	linear algebra; general topology; mathematical analysis
4.2. competencies	abstract and logical thinking

5. Conditions (if necessary)

5.1. for the course	blackboard, chalk, video projector
5.2. for the seminar /lab activities	blackboard, chalk

6.1. Specific competencies acquired [1]

[1] One can choose either competences or learning outcomes, or both. If only one option is chosen, the row related to the other option will be deleted, and the kept one will be numbered 6.

Professional/essential competencies	C1.1 To identify the appropriate notions, to describe the speficic topic and to use an appropriate language. C1.3 To apply correctly basic methods and principles in order to solve mathematical problems.
Transversal competencies	CT1 To apply efficient and rigorous working rules, to manifest responsible attitudes towards the scientific and didactic fields, respecting the professional and ethical principles.

6.2. Learning outcomes

Knowledge	The student - has acquired the mathematics discipline-specific skills needed to complete homework, - knows fundamental concepts related to functional analysis as well as methods of applying them in areas of science related to mathematics and computer science.
Skills	The student is able to: - construct clear and well-supported mathematical arguments to explain mathematical problems, topics and ideas in writing, - prove theorems using mathematical language in theoretical courses and will be able to present these results both orally and in writing.
Responsibility and autonomy:	The student has the ability to work independently to obtain: - independently explore some mathematical content, building on ideas and tools already learned to extend their knowledge, - independently extend mathematical ideas and arguments already learned to a mathematical topic not previously studied.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	Presentation of the basic notions and results of Functional Analysis
7.2 Specific objective of the discipline	To become familiar with the abstract thinking and the problematization specific to Functional Analysis

8. Content

8.1 Course	Teaching methods	Remarks
1. Complements of linear algebra (linear spaces, linear subspaces, linear hull, base,	Lecture with mathematical proofs, problematization, discussion	

linear operators, linear functionals, sublinear functionals)		
2. Complements of linear algebra (Helly's Lemma; the Hahn-Banach Theorem for real linear spaces)	Lecture with mathematical proofs, problematization, discussion	
3. Complements of linear algebra (the Hahn-Banach Theorem for complex linear spaces; seminorms; the Bohnenblust-Sobczyk-Suhomlinov Theorem and its consequence). Normed spaces (definition of the norm and of the metric induced by a norm)	Lecture with mathematical proofs, problematization, discussion	
4. Normed spaces (definition of the topology compatible with the algebraic structure of a linear space; definition of the topological linear space; proof of the fact that every normed space is a topological linear space; topological properties of the balls in a normed space)	Lecture with mathematical proofs, problematization, discussion	
5. Normed spaces (series and absolutely convergent series in a normed space; the notion of Cauchy sequence and of complete metric space; the notion of Banach space; the characterization of the completeness of a normed space with the aid of series). Finite dimensional normed spaces (equivalent norms; the characterization of equivalent norms)	Lecture with mathematical proofs, problematization, discussion	
6. Finite dimensional normed spaces (the equivalence of norms in finite dimensional linear spaces; the completeness of finite dimensional normed spaces; the Riesz Lemma)	Lecture with mathematical proofs, problematization, discussion	
7. Finite dimensional normed spaces (characterizations of finite dimensional normed spaces). Inner product spaces (the definition of the inner product; properties of inner products; the notion of inner product space)	Lecture with mathematical proofs, problematization, discussion	
8. Inner product spaces (the continuity of the inner product; the characterization of the norms induced by an inner product; the notion of orthogonality of two vectors; Pythagora's equality)	Lecture with mathematical proofs, problematization, discussion	
9. Inner product spaces (orthonormal families; properties of orthonormal families; orthonormal bases; characterizations of orthonormal bases; the notions of Fourier coefficients and of Fourier expansion)	Lecture with mathematical proofs, problematization, discussion	
10. Inner product spaces (proof of the fact that nonempty, complete and convex subsets of inner product spaces are Chebyshev sets; the notion of the orthogonal complement of a subset of an inner product space; characterizations of the points of best approximation in a linear subspace; the orthogonal decomposition of a Hilbert space; chracterizations of orthonormal bases in Hilbert spaces)	Lecture with mathematical proofs, problematization, discussion	
11. Linear continuous operators between normed spaces (characterizations of the continuity of linear operators between normed spaces; the normed space of linear continuous operators between normed spaces; linear	Lecture with mathematical proofs, problematization, discussion	

continuous functionals on normed spaces; the dual of a normed space)		
12. Linear continuous operators between normed spaces (isomorphisms and isometric isomorphisms between normed spaces; the Neumann series associated to a linear continuous operator)	Lecture with mathematical proofs, problematization, discussion	
13. Fundamental results in functional analysis (the uniform boundedness and the pointwise boundedness of a family of linear continuous operators between two normed spaces; the definition of Baire space; the uniform boundedness principle; the open mapping theorem; the bounded inverse theorem)	Lecture with mathematical proofs, problematization, discussion	
14. The extension of linear continuous functionals (the two theorems of Hahn and their consequences). Recap	Lecture with mathematical proofs, problematization, discussion	

Bibliography

1. BRECKNER W. W.: Analiză funcțională, Presa Universitară Clujeană, Cluj-Napoca, 2009.

2. BREZIS H.: Analiză funcțională. Teorie și aplicații, Ed. Academiei Române, București, 2002.

3. CONWAY J. B.: A Course in Functional Analysis. Second Edition, Springer-Verlag, New-York –Berlin – Heidelberg, 1999.

4. HEUSER H.: Funktionalanalysis. Theorie und Anwendung, 3. Auflage, B. G. Teubner, Stuttgart, 1992.

5. KANTOROVICI L.V., AKILOV G. P.: Analiză funcțională. Editura Științifică și Enciclopedică, București, 1986.

6. MUNTEAN I.: Analiză funcțională, Universitatea "Babeș-Bolyai", Cluj-Napoca, 1993.

7. PRECUPANU T.: Analiză funcțională pe spații liniare normate, Editura Universității "Alexandru Ioan Cuza", Iași, 2005.

8. WERNER D.: Funktionalanalysis, Vierte, überarbeitete Auflage., Springer-Verlag, Berlin - Heidelberg - New York, 2002.

8.2 Seminar / laboratory	**Teaching methods**	**Remarks**
1. Complements of linear algebra	Problematization, discussion, team work	
2. Complements of linear algebra (the relationship between complex linear functionals and real linear functionals; applications of the Hahn-Banach Theorem for real linear spaces)	Problematization, discussion, team work	
3. Complements of linear algebra (seminorms; an application of the Hahn-Banach Theorem); Minkowski's inequality. Examples of norms (the $\|\cdot\|_p$ norms on $\mathbf{K}^m$; the supremum norm on B(T, **K**); the norm on l^p, where p is a real number greater or equal than 1)	Problematization, discussion, team work	
4. Normed spaces (the continuity of the norm; the characterization of the distances on a linear space that are induced by norms; examples of distances that are not induced by norms; properties of the spaces l^p)	Problematization, discussion, team work	
5. Normed spaces (the notion of the sum of a family of points and notions related to this concept; unconditionally convergent series; bounded sets and sequences; examples of equivalent norms on $\mathbf{K}^{m)}$	Problematization, discussion, team work	
6. Examples of Banach spaces (the spaces B(T, **K**), CB(T, **K**) and C(T, **K**)). Examples of equivalent norms. An example of a non-complete normed space	Problematization, discussion, team work	

7. Examples of Banach spaces (the spaces C^1([a,b]), l^∞, c, c_0 and l^p). An example of a bounded sequence without any convergent subsequence. An exemple of a non-complete normed space	Problematization, discussion, team work	
8. Inner product spaces (examples and properties). Hilbert spaces (definition andexamples). Orthogonality in inner product spaces	Problematization, discussion, team work	
9. The Chebyshev approximation problem. The notions of best approximation point, Chebyshev set, proximinal set, and best approximation problem in a normed space. Determination of the best approximation points in concrete cases. The Bessel inequality in inner product spaces	Problematization, discussion, team work	
10. Examples of orthonormal bases. The orthogonal decomposition of Hilbert spaces	Problematization, discussion, team work	
11. The determination of the norm of linear continuous operators/functionals	Problematization, discussion, team work	
12. The pointwise convergence of a sequence of linear continuos operators between two normed spaces. The Frechet-Riesz Theorem concerning the general form of linear continuous functionals on Hilbert spaces. Applications of the Frechet-Riesz Theorem	Problematization, discussion, team work	
13. The general form of linear continuous functionals on the l^p normed spaces, where p is a real number greater or equal than 1. Applications of the uniform boundedness principle and of the open mapping theorem	Problematization, discussion, team work	
14. Applications of the theorems of Hahn. Applications of the bounded inverse theorem. An example of a linear operator which is not continuous	Problematization, discussion, team work	

Bibliography

1. BREZIS H.: Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, 2011.

2. HEUSER H.: Funktionalanalysis. Theorie und Anwendung, 3. Auflage. B. G. Teubner, Stuttgart, 1992.

3. POPA E.: Culegere de probleme de analiză funcțională, Editura Didactică și Pedagogică, București, 1981.

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9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

- Functional analysis is one of the most important branches of mathematics, having applications in various domains (numerical analysis, approximation theory, optimization, PDEs, probability theory, mathematical and theoretical physics). This discipline both provides the theoretical background for such applications and gives samples of them.

10. Evaluation

Activity type	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Percentage of final grade
10.4 Course	Knowledge of concepts and basic results	There will be a midterm test concerning the contents of the first 7 lectures and seminars. Those who are pleased	
	Ability to perform proofs		

		with their grade, will perform the written exam at the end of the semester concerning only the contents of the last 7 lectures and seminars. In this case, the final grade will be the arithmetic mean of the two grades. Those who are not pleased with the grade obtained in the midterm test, will perform the exam concerning the whole contents of the lectures and seminars. In this case, the final grade will be the grade obtained in the exam. The retaken exam will be about the whole contents of the lectures and seminars.	
10.5 Seminar/laboratory	Ability to apply concepts and results acquired in the lecture		
	There are valid the official rules of the faculty concerning the attendance of students to teaching activities.		
10.6Minimum standard of performance			

- The ability to prove that a functional is a norm/seminorm
- The ability to prove that a linear operator/functional is continuous and to determine its norm
- Basic knowledge on the topics from the lectures and seminars

11. Labels ODD (Sustainable Development Goals)[2]

	General label for Sustainable Development							
								

[2] Keep only the labels that, according to the *Procedure for applying ODD labels in the academic process*, suit the discipline and delete the others, including the general one for *Sustainable Development* – if not applicable. If no label describes the discipline, delete them all and write „*Not applicable.*”.

Date:
11.04.2025

Signature of course coordinator

Assoc. Prof. PHD Brigitte Breckner

Signature of seminar coordinator

Assoc. Prof. PHD Brigitte Breckner

Date of approval:
25.04.2025

Signature of the head of department

Prof. dr. Andrei Mărcuș