SYLLABUS

Real Analysis

University year 2025-2026

1. Information regarding the programme

1.1. Higher education institution	Babeş-Bolyai University
1.2. Faculty	Mathematics and Computer Science
1.3. Department	Mathematics
1.4. Field of study	Mathematics
1.5. Study cycle	Bachelor of Science
1.6. Study programme/Qualification	Mathematics Computer Science (in English)
1.7. Form of education	Full-time

2. Information regarding the discipline

2.1. Name of the discipline	**Real Analysis**					Discipline code	**MLE0074**
2.2. Course coordinator	Conf. dr. Adriana Nicolae						
2.3. Seminar coordinator	Conf. dr. Adriana Nicolae						
2.4. Year of study	**2**	2.5. Semester	**3**	2.6. Type of evaluation	**C**	2.7. Discipline regime	Compulsory

3. Total estimated time (hours/semester of didactic activities)

3.1. Hours per week	**4**	of which: 3.2 course	**2**	3.3 seminar/laboratory	**2**
3.4. Total hours in the curriculum	**56**	of which: 3.5 course	**28**	3.6 seminar/laborator	**28**
Time allotment for individual study (ID) and self-study activities (SA)					**hours**
Learning using manual, course support, bibliography, course notes (SA)					25
Additional documentation (in libraries, on electronic platforms, field documentation)					10
Preparation for seminars/labs, homework, papers, portfolios and essays					20
Tutorship					4
Evaluations					10
Other activities:					
3.7. Total individual study hours	**69**				
3.8. Total hours per semester	**125**				
3.9. Number of ECTS credits	**5**				

4. Prerequisites (if necessary)

4.1. curriculum	Calculus 1, 2; Mathematical Logic and Set Theory
4.2. competencies	Analytic thinking

5. Conditions (if necessary)

5.1. for the course	Lecture hall equipped with blackboard and chalk
5.2. for the seminar /lab activities	Classroom equipped with blackboard and chalk

6.1. Specific competencies acquired [1]

Professional/essential competencies	• C1.1 Identification of notions, description of theories and use of specific language. • C2.3 Application of appropriate theoretical analysis methods to the given problem. • C5.2 Use of mathematical arguments to prove mathematical results.
Transversal competencies	• CT1 Application of efficient and rigorous working rules by adopting responsible attitudes towards the scientific and didactic fields for the development of the own creative potential respecting professional and ethical principles.

6.2. Learning outcomes

Knowledge	The student: - aquired skills specific to the Mathematics-related disciplines needed to complete the assignments. - knows fundamental notions related to Real Analysis and methods of applying them to areas of science related to Mathematics and Computer Science.
Skills	The student is able to: - construct clear and well-supported mathematical arguments to explain mathematical problems, topics, and ideas in writing. - prove theorems using the language of mathematics in theoretical courses and present those results both orally and in writing.
Responsibility and autonomy:	The student has the ability to: - explore some mathematical content independently, drawing on ideas and tools from previous coursework to extend their understanding. - extend mathematical ideas and arguments from previous coursework to a mathematical topic not previously studied.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	• To acquire fundamental knowledge about general measure theory and integration, and to apply it in solving problems.
7.2 Specific objective of the discipline	• To acquire knowledge about elements of general measure theory and integration (e.g., σ-algebras, measures, the Lebesque exterior measure, the Lebesgue measure, integration of measurable functions, limit theorems, L^p spaces, Fubini's theorem).

[1] One can choose either competences or learning outcomes, or both. If only one option is chosen, the row related to the other option will be deleted, and the kept one will be numbered 6.

8. Content

8.1 Course	**Teaching methods**	**Remarks**
1. Introduction: the problem of measure. Measurable spaces and measure spaces	Lecture, discussion, didactical demonstration, problematisation	
2. The Lebesgue exterior measure	Lecture, discussion, didactical demonstration, problematisation	
3. The Lebesgue measure	Lecture, discussion, didactical demonstration, problematisation	
4. Properties of the Lebesgue measure	Lecture, discussion, didactical demonstration, problematisation	
5. Measurable functions	Lecture, discussion, didactical demonstration, problematisation	
6. Approximation of measurable functions	Lecture, discussion, didactical demonstration, problematisation	
7. Integration of measurable functions (I)	Lecture, discussion, didactical demonstration, problematisation	
8. Integration of measurable functions (II)	Lecture, discussion, didactical demonstration, problematisation	
9. Limit theorems and applications (I)	Lecture, discussion, didactical demonstration, problematisation	
10. Limit theorems and applications (II). The relation between the Riemann and Lebesgue integrals.	Lecture, discussion, didactical demonstration, problematisation	
11. L^p spaces (I)	Lecture, discussion, didactical demonstration, problematisation	
12. L^p spaces (II)	Lecture, discussion, didactical demonstration, problematisation	
13. Types of convergence	Lecture, discussion, didactical demonstration, problematisation	
14. Measure and integration on product spaces	Lecture, discussion, didactical demonstration, problematisation	

Bibliography

1. V. Anisiu, Topologie şi teoria măsurii, Universitatea "Babeş-Bolyai", Cluj-Napoca, 1993.
2. J.J. Benedetto, W. Czaja, Integration and modern analysis, Birkhäuser, Boston, MA, 2009.
3. D.L. Cohn, Measure theory, 2nd ed., Birkhäuser/Springer, New York, 2013.
4. G.B. Folland, Real analysis. Modern techniques and their applications, 2nd ed., John Wiley & Sons, Inc., New York, 1999.
5. F. Jones, Lebesgue integration on Euclidean space, Jones and Bartlett Publishers, Boston, MA, 1993.
6. H.L. Royden, P.M. Fitzpatrick, Real analysis, 4th ed., Pearson, 2010.
7. W. Rudin, Real and complex analysis, 3rd ed., McGraw-Hill Book Co., New York, 1987.
8. E. Stein, R. Shakarchi, Real analysis. Measure theory, integration, and Hilbert spaces, Princeton University Press, Princeton, NJ, 2005.
9. D.W. Stroock, A concise introduction to the theory of integration, 2nd ed., Birkhäuser Boston, Inc., Boston, MA, 1994.
10. T. Tao, An introduction to measure theory, American Mathematical Society, Providence, RI, 2011.

8.2 Seminar / laboratory	**Teaching methods**	**Remarks**
1. Introduction: the problem of measure. Measurable spaces and measure spaces	Discussion, problem solving, didactical demonstration	
2. The Lebesgue exterior measure	Discussion, problem solving, didactical demonstration	
3. The Lebesgue measure	Discussion, problem solving, didactical demonstration	
4. Properties of the Lebesgue measure	Discussion, problem solving, didactical demonstration	
5. Measurable functions	Discussion, problem solving, didactical demonstration	
6. Approximation of measurable functions	Discussion, problem solving, didactical demonstration.	
7. Integration of measurable functions (I)	Discussion, problem solving, didactical demonstration	
8. Integration of measurable functions (II)	Discussion, problem solving, didactical demonstration	

9. Limit theorems and applications (I)	Discussion, problem solving, didactical demonstration	
10. Limit theorems and applications (II). The relation between the Riemann and Lebesgue integrals.	Discussion, problem solving, didactical demonstration	
11. L^p spaces (I)	Discussion, problem solving, didactical demonstration	
12. L^p spaces (II)	Discussion, problem solving, didactical demonstration	
13. Types of convergence	Discussion, problem solving, didactical demonstration	
14. Measure and integration on product spaces	Discussion, problem solving, didactical demonstration	

Bibliography (in addition to the books mentioned before which also contain exercises)

1. R.L. Schilling, Measures, integrals and martingales, Cambridge University Press, New York, 2005.
2. W.J. Kaczor, M.T. Nowak, Problems in Mathematical Analysis III. Integration, American Mathematical Society, Providence, RI, 2003.
3. A. Torchinsky, Problems in real and functional analysis, American Mathematical Society, Providence, RI, 2015.

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

The course ensures a solid theoretical background, according to national and international standards, being essential for the study of other courses such as Partial Differential Equations, Functional Analysis, or Probability Theory. This discipline is useful in preparing future teachers and researchers in mathematics, but is also addressed to those who use various modern mathematical methods and techniques in other areas.

10. Evaluation

Activity type	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Percentage of final grade
10.4 Course	- Knowledge of basic notions, examples and results - Ability to prove theoretical results	- Test, exam - Lecture and seminar activity	- Test: 35% - Exam: 65% - Lecture and seminar activity: bonus max. 5%
10.5 Seminar/laboratory	- Problem solving using concepts and results acquired during the lecture classes		
10.6 Minimum standard of performance			
- The accumulation of at least 10 attendances at the seminar. - Both the test grade at the end of the semester and the final grade should be at least 5. The bonus points are only awarded in this case.			

11. Labels ODD (Sustainable Development Goals)[2]

	General label for Sustainable Development							
								

[2] Keep only the labels that, according to the *Procedure for applying ODD labels in the academic process*, suit the discipline and delete the others, including the general one for *Sustainable Development* – if not applicable. If no label describes the discipline, delete them all and write „*Not applicable*.".

Date:
11.04.2025

Signature of course coordinator

Conf. dr. Adriana Nicolae

Signature of seminar coordinator

Conf. dr. Adriana Nicolae

Date of approval:
25.04.2025

Signature of the head of department

Prof. dr. Andrei Mărcuș