SYLLABUS

Calculus 2 (Differential and integral calculus in R^n)

University year 2025-2026

1. Information regarding the programme

1.1. Higher education institution	Babeş-Bolyai University
1.2. Faculty	Mathematics and Computer Science
1.3. Department	Mathematics
1.4. Field of study	Mathematics
1.5. Study cycle	Licence
1.6. Study programme/Qualification	Mathematics and Computer Science
1.7. Form of education	Part-time education

2. Information regarding the discipline

2.1. Name of the discipline	**Calculus 2 (Differential and integral calculus in R^n)**					Discipline code	**MLE0071**
2.2. Course coordinator					Conf. dr. Trif Tiberiu		
2.3. Seminar coordinator					Conf. dr. Trif Tiberiu		
2.4. Year of study	**1**	2.5. Semester	**2**	2.6. Type of evaluation	**E**	2.7. Discipline regime	Fundamental

3. Total estimated time (hours/semester of didactic activities)

3.1. Hours per week	**6**	of which: 3.2 course	**3**	3.3 seminar/laboratory	**3**
3.4. Total hours in the curriculum	**84**	of which: 3.5 course	**42**	3.6 seminar/laborator	**42**
Time allotment for individual study (ID) and self-study activities (SA)					**hours**
Learning using manual, course support, bibliography, course notes (SA)					20
Additional documentation (in libraries, on electronic platforms, field documentation)					10
Preparation for seminars/labs, homework, papers, portfolios and essays					20
Tutorship					6
Evaluations					10
Other activities					
3.7. Total individual study hours	**66**				
3.8. Total hours per semester	**150**				
3.9. Number of ECTS credits	**6**				

4. Prerequisites (if necessary)

4.1. curriculum	Calculus 1 (Calculus in R)
4.2. competencies	- the ability to do algebraic calculations - operating with abstract concepts and the ability to make logical deductions - the ability to solve mathematical problems based on learned concepts

5. Conditions (if necessary)

5.1. for the course	blackboard, chalk, video projector
5.2. for the seminar /lab activities	blackboard, chalk

6.1. Specific competencies acquired [1]

Professional/essential competencies	• C1.1 Identifying concepts, describing theories and using specific language. • C1.4 Recognizing the main classes/types of mathematical problems and selecting appropriate methods and techniques for solving them. • C2.1 Identifying the basic concepts used in describing phenomena and processes. • C2.3 Applying appropriate theoretical analysis methods to the given issue.
Transversal competencies	• CT1. Applying rigorous and efficient work rules, demonstrating responsible attitudes towards the scientific and teaching field, for the optimal and creative use of one's own potential in specific situations, while respecting the principles and norms of professional ethics.

6.2. Learning outcomes

Knowledge	The student: - has acquired the specific skills of mathematics-related disciplines. - knows fundamental notions related to the topology of the Euclidean space Rn, the differential calculus of functions of several variables, as well as different types of integrals for functions of several variables (multiple integrals, line and surface integrals).
Skills	The student is able to: - construct clear and well-supported mathematical arguments to explain mathematical problems, topics and ideas in writing. - prove theorems using mathematical language in theoretical courses and will be able to present these results both orally and in writing.
Responsibility and autonomy:	The student has the ability to - independently explore certain mathematical contents, drawing on previously acquired ideas and tools, in order to extend his/her knowledge. - independently extend previously acquired mathematical ideas and arguments to a mathematical topic that has not been previously studied.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	• Knowledge of the topology of the Euclidean space Rn, of the differential calculus of functions of several variables, of functions with bounded variation, as well as of the different types of integrals for functions of several variables (multiple integrals, line and surface integrals).

[1] One can choose either competences or learning outcomes, or both. If only one option is chosen, the row related to the other option will be deleted, and the kept one will be numbered 6.

7.2 Specific objective of the discipline	• Presentation of fundamental notions and some basic results regarding the topology of the Euclidean space Rn • Presentation of fundamental notions and some basic results regarding the differential calculus of functions of several variables • Presentation of functions with bounded variation and their main properties • Presentation of different types of integrals for functions of several variables (multiple integrals, curvilinear and surface integrals), as well as methods for their calculation.

8. Content

8.1 Course	**Teaching methods**	**Remarks**
Week 1. Topology in $\mathbf{R}^n$: the Euclidean space $\mathbf{R}^n$ (the inner product, the Euclidean norm, the Euclidean distance), the topological structure of $\mathbf{R}^n$ (balls, neighbourhoods, interior points, adherent points, boundary points, and limit points, open and closed sets). Sequences in $\mathbf{R}^n$: convergent and Cauchy sequences, characterization of adherent points, of limit points, and of closed sets by means of sequences.	lecture, proof, examples	
Week 2. Compact sets in $\mathbf{R}^n$: definition of compact sets, examples of compact sets in $\mathbf{R}^n$, characterization of compact sets in $\mathbf{R}^n$. Limits of vector functions of vector variable: definition of the limit, characterization of the limit by means of sequences, operations with functions having a limit.	lecture, proof, examples	
Week 3. Continuity of vector functions of vector variable: definition of the continuity at a point, characterization of the continuity by means of sequences, operations with continuous functions, the Weierstrass theorem. Linear mappings and their norm.	lecture, proof, examples	
Week 4. Differentiability in $\mathbf{R}^n$: the derivative of a vector function of a real variable, the mean value theorem for vector functions of a real variable. Differentiability of vector functions of vector variable (definition of the Frechet differential, continuity of Frechet differentiable functions, derivative vs differential for vector functions of a real variable).	lecture, proof, examples	
Week 5. Differentiability in $\mathbf{R}^n$: the directional derivative of a vector function of vector variable and its relationship with the Frechet differential, partial derivatives and their relationship with the Frechet differential. The chain rule, the differentiability of the inverse function.	lecture, proof, examples	
Week 6. Differentiabilty in $\mathbf{R}^n$: mean value theorems for functions of several variables. Functions of the class C^1. The local inversion theorem, the implicit function theorem.	lecture, proof, examples	
Week 7. Differentiability in $\mathbf{R}^n$: Lagrange multipliers, second order partial derivatives, the Schwarz and Young theorems concerning the mixed partial derivatives. Necessary and sufficient conditions for extrema. Higher order partial derivatives, Taylor's formula.	lecture, proof, examples	

Week 8. Riemann integral on a compact interval in $\mathbf{R}^n$: definition of the Riemann integral on a compact interval in $\mathbf{R}^n$, Riemann integrability tests on a compact interval in $\mathbf{R}^n$ (the Heine, Cauchy, and Darboux tests). Computation of Riemann integrals on compact intervals by means of iterated integrals (the Fubini theorem).	lecture, proof, examples	
Week 9. Riemann integral on bounded sets in $\mathbf{R}^n$: computation of Riemann integrals on bounded sets in $\mathbf{R}^n$ by means of iterated integrals (the Fubini theorem). Change of variables in multiple integrals. Applications in physics of multiple integrals: centres of gravity and moments of inertia.	lecture, proof, examples	
Week 10. Vector functions of bounded variation: definition, examples, properties of the total variation. Additivity of the total variation with respect to the interval, the Jordan reprezentation theorem, computation of the total variation for functions of the class C^1.	lecture, proof, examples	
Week 11. Line integrals: paths, examples, equivalent paths, curves and oriented curves. First degree differential forms. Integration of first degree differential forms along a path (the line integral of the second kind), mechanical work.	lecture, proof, examples	
Week 12. Line integrals: the Green formula, integration of exact differential forms, the Leibniz-Newton formula, the Poincaré theorem concerning the integration of exact differential forms, mechanical work in the gravitational field.	lecture, proof, examples	
Week 13. Surface integrals: parametrized surfaces, examples. Differential forms of the second degree and their integrals over parametrized surfaces (surface integrals of the second kind).	lecture, proof, examples	
Week 14. Stokes and Gauss-Ostrogradski formulas.	lecture, proof, examples	

Bibliography

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9. HEUSER H.: Lehrbuch der Analysis, Teil 1, 11. Auflage, B. G. Teubner, Stuttgart, 1994; Teil 2, 9. Auflage, B. G. Teubner, Stuttgart, 1995.
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11. NICULESCU C. P.: Calculul integral al funcţiilor de mai multe variabile. Teorie şi aplicaţii. Editura Universitaria, Craiova, 2002.
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8.2 Seminar / laboratory	**Teaching methods**	**Remarks**
Week 1. The Euclidean space $\mathbf{R}^n$: problems concerning the Euclidean space $\mathbf{R}^n$.	Examples, dialogue, explanation, proof, problematization	
Week 2. Compact sets in $\mathbf{R}^n$: problems concerning compact sets in $\mathbf{R}^n$.	Examples, dialogue, explanation, proof, problematization	
Week 3. Limits of vector functions of vector variable, continuity of vector functions of vector variable. Linear mappings and their norm: computation of the norm for some concrete linear mappings.	Examples, dialogue, explanation, proof, problematization	
Week 4. Computation of directional derivatives, partial derivatives, and differentials for concrete functions.	Examples, dialogue, explanation, proof, problematization	
Week 5. Differentials: study of the Frechet differentiability for concrete functions. Applications to the chain rule.	Examples, dialogue, explanation, proof, problematization	
Week 6. Mean value theorems for functions of several variables. Diffeomorphisms and implicit functions.	Examples, dialogue, explanation, proof, problematization	
Week 7. Extrema for functions of several variables, higher order partial derivatives.	Examples, dialogue, explanation, proof, problematization	
Week 8. Calculation of double integrals over rectangles. Calculation of triple integrals over parallelepipeds. Double and triple integrals over simple sets with respect to an axis.	Examples, dialogue, explanation, proof, problematization	
Week 9. Calculation of double integrals by means of change of variables (polar coordinates).	Examples, dialogue, explanation, proof, problematization	
Week 10. Calculation of triple integrals by means of change of variables (spherical coordinates, cylindrical coordinates).	Examples, dialogue, explanation, proof, problematization	
Week 11. Problems concerning functions of bounded variation. Line integrals of the first kind: definition, main theoretical results, calculation of line integrals of the first kind along concrete paths.	Examples, dialogue, explanation, proof, problematization	
Week 12. Line integrals of the second kind: calculation of the integrals of certaind first degree differential forms along concrete paths. Integration of some exact differential forms. Applications to the Green formula.	Examples, dialogue, explanation, proof, problematization	
Week 13. Calculation of surface integrals of the first and of the second kind.	Examples, dialogue, explanation, proof, problematization	
Week 14. Problems concerning the Stokes and the Gauss-Ostrogradski formulas.	Examples, dialogue, explanation, proof, problematization	

Bibliography

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2. CĂTINAȘ D. et al.: Calcul integral. Culegere de probleme pentru seminarii, examene şi concursuri. Editura U. T. Pres, Cluj-Napoca, 2000.
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9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

The theme of this course (the topology of the Euclidian $\mathbf{R}^n$, the differential calculus of functions of several variables, functions of bounded variation, and various types of integrals for functions of several variables - multiple integrals, line integrals, and surface integrals) is provided in the study program of to all major universities in Romania and the world. It is an indispensable part of preparing future math teachers, future mathematics researchers, and those working in other fields that directly apply mathematical methods.

10. Evaluation

Activity type	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Percentage of final grade
10.4 Course	Knowledge of fundamental notions and results	Written paper	90%
10.5 Seminar/laboratory	Solving problems based on learned notions and theorems	Solving the exercises on the board	10%
10.6 Minimum standard of performance			
• Accumulation of 5 points on the exam (for a final grade of 5).			

11. Labels ODD (Sustainable Development Goals)[2]

	General label for Sustainable Development							
								9 INDUSTRY, INNOVATION AND INFRASTRUCTURE

Date:
11.04.2025

Signature of course coordinator

Conf. dr. Trif Tiberiu

Signature of seminar coordinator

Conf. dr. Trif Tiberiu

Date of approval:
25.04.2025

Signature of the head of department

Prof. dr. Andrei Mărcuș

[2] Keep only the labels that, according to the *Procedure for applying ODD labels in the academic process*, suit the discipline and delete the others, including the general one for *Sustainable Development* – if not applicable. If no label describes the discipline, delete them all and write „*Not applicable*.".