SYLLABUS

RINGS, MODULES AND REPRESENTATIONS

University year 2025-2026

1. Information regarding the programme

1.1. Higher education institution	Babeș-Bolyai University
1.2. Faculty	Mathematics and Computer Science
1.3. Department	Mathematics
1.4. Field of study	Mathematics
1.5. Study cycle	Master
1.6. Study programme/Qualification	Advanced Mathematics
1.7. Form of education	With frequency

2. Information regarding the discipline

2.1. Name of the discipline	**Rings, Modules, and Representations**					Discipline code	**MME3159**
2.2. Course coordinator				Conf. Dr. George Ciprian Modoi			
2.3. Seminar coordinator				Conf. Dr. George Ciprian Modoi			
2.4. Year of study	2	2.5. Semester	4	2.6. Type of evaluation	E	2.7. Discipline regime	Optional

3. Total estimated time (hours/semester of didactic activities)

3.1. Hours per week	**3**	of which: 3.2 course	**2**	3.3 seminar/laboratory	**1**
3.4. Total hours in the curriculum	36	of which: 3.5 course	24	3.6 seminar/laborator	**12**
Time allotment for individual study (ID) and self-study activities (SA)					**hours**
Learning using manual, course support, bibliography, course notes (SA)					45
Additional documentation (in libraries, on electronic platforms, field documentation)					45
Preparation for seminars/labs, homework, papers, portfolios and essays					45
Tutorship					34
Evaluations					20
Other activities:					
3.7. Total individual study hours	**189**				
3.8. Total hours per semester	**225**				
3.9. Number of ECTS credits	**9**				

4. Prerequisites (if necessary)

4.1. curriculum	Category Theory (MME3123); Group Theory and Applications (MME3103); Homological Algebra (MME3112)
4.2. competencies	Linear algebra, basics about rings, modules, categories, functors.

5. Conditions (if necessary)

5.1. for the course	N/A
5.2. for the seminar /lab activities	N/A

6.1. Specific competencies acquired [1]

[1] One can choose either competences or learning outcomes, or both. If only one option is chosen, the row related to the other option will be deleted, and the kept one will be numbered 6.

Professional/essential competencies	• Understanding and use of main concepts and results concerning rings, modules and quiver representations; • Ability to use fundamental theoretical concepts and to apply them in various fields of mathematics fields of mathematics (Algebra, Geometry etc.); • Ability to use scientific language and to write scientific reports and papers.
Transversal competencies	• Ability to inform themselves, to work independently or in a team; • Ability to identify and use advanced techniques and methods to do advanced research; • Ability for continuous self-perfecting and study.

6.2. Learning outcomes

Knowledge	The student knows: the notions involved in the discipline, namely quiver, path algebra, module and the main results related to them as Morita theory.
Skills	The student is able to identify projective, injective and simple objects in the category of modules over a path algebra, to compute homological dimensions etc.
Responsibility and autonomy:	The student has the ability to work independently and to do advanced research in order to pursue PhD in Mathematics.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	• To get advanced knowledge on rings, modules and quiver representations. • To build the ability to use fundamental theoretical concepts and to apply them in various fields of mathematics
7.2 Specific objective of the discipline	• To construct some specific examples of rings, algebras and modules using the mechanism of quiver representations. • To use the language of categories and functors in this particular case of the theory of modules and quiver representations.

8. Content

8.1 Course	**Teaching methods**	**Remarks**
1. Quivers and their representations	Lectures, didactical demonstration, conversation	

2. Rings, algebras and modules	Lectures, didactical demonstration, conversation	
3. Quiver representations vs modules; path algebras	Lectures, didactical demonstration, conversation	
4. Kernels, cokernels, exact sequences	Lectures, didactical demonstration, conversation	
5. Hom functors	Lectures, didactical demonstration, conversation	
6. Simples, projectives and injectives	Lectures, didactical demonstration, conversation	
7. Projective resolutions and injective coresolutions	Lectures, didactical demonstration, conversation	
8. Duality and Nakayama functor	Lectures, didactical demonstration, conversation	
9. Admissible ideals and quotients of path algebras	Lectures, didactical demonstration, conversation	
10. Homological dimensions	Lectures, didactical demonstration, conversation	
11. Morita theory	Lectures, didactical demonstration, conversation	
12. Tilted algebras	Lectures, didactical demonstration, conversation	

Bibliography

1. 1. F.W. Anderson, K.R. Fuller, *Rings and Categories of Modules*, Springer, 1992.
2. H. Derksen, J. Weyman – *An Introduction to Quiver Representations*, Graduate Studies in Mathematics 184, American Mathematical Society, 2017.
3. R. Schiffler – *Quiver Representations*, CMS Books in Mathematcs, Springer, 2014.
4. S. Mac Lane – *Categories for the Working Mathematician*, Graduate Text in Mathematics, Second Edition, Springer Verlag, 1998.

8.2 Seminar / laboratory	**Teaching methods**	**Remarks**
1. Examples of quiver representations	problematization, exercises, problem solving	
2. Universal properties; categorical (re)formulations of kernels, cokernels, sums, products etc.	problematization, exercises, problem solving	
3. Example of modules over path algebras	problematization, exercises, problem solving	
4. Examples of simple, projective and injective objects	problematization, exercises, problem solving	
5. Computing some homological dimensions	problematization, exercises, problem solving	
6. Examples of Morita equivalent rings and algebras	problematization, exercises, problem solving	

Bibliography

1. S. Breaz, G. Calugareanu, G. Modoi, C. Pelea, D. Valcan: *Exercices in Abelian Group Theory*, Kluwer 2003.
2. H. Derksen, J. Weyman – *An Introduction to Quiver Representations*, Graduate Studies in Mathematics 184, American Mathematical Society, 2017.
3. T.Y. Lam, *Exercises in classical ring theory*, Springer, 2003.
4. R. Schiffler – *Quiver Representations*, CMS Books in Mathematcs, Springer, 2014.

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

- The content of this discipline is in accordance with the curricula of many important universities.
- The mechanism of quiver representations is a relatively simple and intuitive method to construct examples of rings and modules satisfying various abstract properies, provinding a useful requisite for anyone which is interested in algebra, geometry and connex subiects of mathematics.
- The methods and tools presented here are often used in specifical PhD research activities.

10. Evaluation

Activity type	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Percentage of final grade
10.4 Course	Concepts and basic results	Final exam	20%
	Examples	Final exam	20%
10.5 Seminar/laboratory	Ability to use the concepts in order to solve standard problems	Final exam	20%
	Ability to solve advanced problems	Homework	40%
10.6 Minimum standard of performance			
• The minimum standard of performance (grade 5 out 10) is acquired by understanding definitions and knowing examples for the main studied notions.			

11. Labels ODD (Sustainable Development Goals)[2]

	General label for Sustainable Development							
								

[2] Keep only the labels that, according to the *Procedure for applying ODD labels in the academic process*, suit the discipline and delete the others, including the general one for *Sustainable Development* – if not applicable. If no label describes the discipline, delete them all and write „*Not applicable*.".

Date:
11.04.2025

Signature of course coordinator

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Signature of seminar coordinator

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Date of approval:
25.04.2025

Signature of the head of department

Prof. dr. Andrei Mărcuș