SYLLABUS

Geometric Function Theory in Several Complex Variables

University year 2025-2026

1. Information regarding the programme

1.1. Higher education institution	Babeş-Bolyai University
1.2. Faculty	Mathematics and Computer Science
1.3. Department	Mathematics
1.4. Field of study	Mathematics
1.5. Study cycle	Master
1.6. Study programme/Qualification	Advanced Mathematics
1.7. Form of education	Full time

2. Information regarding the discipline

2.1. Name of the discipline	**Geometric Function Theory in Several Complex Variables**					Discipline code	**MME3115**
2.2. Course coordinator					**Professor PhD Mirela KOHR**		
2.3. Seminar coordinator					**Professor PhD Mirela KOHR**		
2.4. Year of study	**2**	2.5. Semester	**4**	2.6. Type of evaluation	**E**	2.7. Discipline regime	**Optional/DS**

3. Total estimated time (hours/semester of didactic activities)

3.1. Hours per week	**3**	of which: 3.2 course	**2**	3.3 seminar/laboratory	**1 sem**
3.4. Total hours in the curriculum	**36**	of which: 3.5 course	**24**	3.6 seminar/laborator	**12**
Time allotment for individual study (ID) and self-study activities (SA)					**hours**
Learning using manual, course support, bibliography, course notes (SA)					45
Additional documentation (in libraries, on electronic platforms, field documentation)					45
Preparation for seminars/labs, homework, papers, portfolios and essays					45
Tutorship					34
Evaluations					20
Other activities:					-
3.7. Total individual study hours	189				
3.8. Total hours per semester	225				
3.9. Number of ECTS credits	9				

4. Prerequisites (if necessary)

4.1. curriculum	In-depth knowledge of the following disciplines: • Complex analysis; • Real analysis; • Topology; • Partial differential equations.
4.2. competencies	There are useful logical thinking and mathematical notions and results from the above-mentioned fields

5. Conditions (if necessary)

5.1. for the course	Classroom with blackboard, video projector
5.2. for the seminar /lab activities	Classroom with blackboard, video projector

6.1. Specific competencies acquired [1]

Professional/essential competencies	• Ability to understand, handle and communicate concepts, fundamental and advanced theories in the field of mathematics. • Ability to understand scientific papers in the field of mathematics, to formulate new problems and to initiate new mathematical research, preparing reports and scientific papers. • Use of advanced skills to develop and manage mathematical projects of research nature, applying a wide range of quantitative and qualitative methods. • Ability to use acquired knowledge in pursuing a doctoral program in Pure Mathematics, Applied Mathematics, or in other fields that use concepts of complex analysis. • Ability for continuous self-perfecting and study.
Transversal competencies	• Ability to inform themselves, to work independently or in a team in order to carry out studies and to solve complex problems. • Application of organized and efficient work rules, a responsible attitude towards the didactic-scientific field, to bring creative value to own potential respecting professional ethics principles. • Ability to adopt and integrate in different environments from education and research. • Ability to adapt to the requirements of a dynamical society and to communicate efficiently in an international language.

6.2. Learning outcomes

Knowledge	• The master student knows, understand and use concepts, individual results and advanced mathematical theories. • The master student has the ability to develop and use efficient research skills. • The master student has the ability to model and analyze from the mathematical point of view real processes and phenomena from other sciences, physics, medicine, and engineering.
Skills	• The master student has the ability to identify and state significant problems which can be the basis for subsequent research. • The master student has the ability to use scientific language and to perform research in Mathematics.
Responsibility and autonomy:	• The master student has the ability to inform themselves, to work independently or in a team in order to carry out studies and to solve complex problems. • The master student has the ability of critical investigation of specific literature. • The master student has the ability to use international data bases of academic research.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	• Knowledge, understanding and use of main concepts and results of geometric function theory of several complex variables. • Knowledge, understanding and use of methods of complex analysis in one or higher dimensions in the study of special problems in pure and applied mathematics. • Ability to use and apply concepts and fundamental results of advanced mathematics in the study of specific problems of function theory in $\mathbf{C}^n$.
7.2 Specific objective of the discipline	• Acquiring basic and advanced knowledge in geometric function theory in $\mathbf{C}^n$. • Understanding of main concepts and results in the theory of holomorphic mappings on the unit ball in $\mathbf{C}^n$. • Knowledge, understanding and use of advanced topics in mathematics in the study of special problems in several complex variables. • Ability student involvement in scientific research.

[1] One can choose either competences or learning outcomes, or both. If only one option is chosen, the row related to the other option will be deleted, and the kept one will be numbered 6.

8. Content

8.1 Course	**Teaching methods**	**Remarks**
1. The Carathéodory family *M* of holomorphic mappings in several complex variables. Growth and distortion results, coefficient bounds. Compactness of the family *M*.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
2. Starlike mappings on the unit ball in $\mathbf{C}^n$. Necessary and sufficient conditions for starlikeness. Growth and distortion results and coefficient bounds.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
3. Convex mappings on the unit ball in $\mathbf{C}^n$. Necessary and sufficient conditions for convexity on the Euclidean unit ball and the unit polydisc in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
4. Growth, distortion and coefficient bounds for convex mappings on the unit ball in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
5. Loewner chains and transition mappings (evolution families) in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
6. Loewner chains, Herglotz vector fields and the generalized Loewner differential equation in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
7. Kernel convergence and biholomorphic mappings on the unit ball in $\mathbf{C}^n$. Applications in the theory of Loewner chains.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
8. The solutions of the generalized Loewner differential equation in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
9. The family $S^0(B^n)$ of biholomorphic mappings with parametric representation on the unit ball in $\mathbf{C}^n$. Characterizations in terms of Loewner chains. Compactness of the family $S^0(B^n)$. The Runge property. Open problems.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
10. Extreme points and support points associated with the family $S^0(B^n)$. Approximation properties by automorphisms of the space $\mathbf{C}^n$. Open problems and conjectures.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
11. Univalence criteria on the unit ball in $\mathbf{C}^n$ via the theory of Loewner chains. Parametric representation and asymptotic starlikeness in higher dimensions.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
12. Extension operators that preserve analytic and geometric properties (starlikeness, convexity, Loewner chains, parametric representation). Open problems, conjectures, and research directions.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

Bibliography

1. I. Graham, G. Kohr, *Geometric Function Theory in One and Higher Dimensions*, Marcel Dekker Inc., New York, 2003.
2. G. Kohr, *Basic Topics in Holomorphic Functions of Several Complex Variables*, Cluj University Press, Cluj-Napoca, 2003.
3. G. Kohr, *Geometric Function Theory in Several Complex Variables*, Lecture Notes, 2020.
4. P. Duren, I. Graham, H. Hamada, G. Kohr, *Solutions for the generalized Loewner differential equation in several complex variables*, Mathematische Annalen, **347** (2010), 411-435.

5. I. Graham, H. Hamada, G. Kohr, M. Kohr, *Extremal properties associated with univalent subordination chains in* $\mathbf{C}^n$, Mathematische Annalen, **359** (2014), 61-99.
6. I. Graham, H. Hamada, G Kohr, M Kohr, *Loewner PDE in infinite dimensions*, Computational Methods and Function Theory, **25** (2025), 151–171.
7. S. Gong, *Convex and Starlike Mappings in Several Complex Variables*, Kluwer Acad. Publ., Dordrecht, 1998.
8. P. Duren, *Univalent Functions*, Springer-Verlag, New York, 1983.
9. M. Elin, S. Reich. D. Shoikhet, *Numerical Range of Holomorphic Mappings and Applications*, Birkhäuser, Springer, Cham, 2019.
10. S.G. Krantz, *Function Theory of Several Complex Variables*, Reprint of the 1992 Edition, AMS Chelsea Publishing, Providence, Rhode Island, 2001.
11. Ch. Pommerenke, *Univalent Functions*, Vandenhoeck & Ruprecht, Göttingen, 1975.
12. T. Poreda, *On generalized differential equations in Banach spaces*, Dissertationes Mathematicae, **310** (1991), 1-50.
13. M. Range, *Holomorphic Functions and Integral Representations in Several Complex Variables* Springer-Verlag, New York, 1986.
14. W. Rudin, *Function Theory in the Unit Ball of* $\mathbf{C}^n$, Springer-Verlag, New York, 1980.

8.2 Seminar	**Teaching methods**	**Remarks**
1. Examples of mappings in the Carathéodory family *M*. Special subclasses of *M*. Distortion and coefficient bounds.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
2. Sufficient conditions of starlikeness on the unit ball in $\mathbf{C}^n$. Examples of starlike mappings.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
3. Sufficient conditions of convexity on the unit ball in $\mathbf{C}^n$. Examples of convex mappings.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
4. Starlike mappings of order α on the Euclidean unit ball in $\mathbf{C}^n$, $0\leq \alpha<1$. Growth and coefficient bounds. Examples.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
5. Loewner chains and transition mappings (evolution families) in several complex variables. Examples.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
6. Loewner chains and the associated Loewer PDE in higher dimensions. Applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
7. The analytical characterizations of starlikeness and spirallikeness of type α on the unit ball in $\mathbf{C}^n$ in terms of Loewner chains.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week

8. Variation of Loewner chains in **C**^{n}. Applications to extremal problems for univalent mappings with parametric representation on the unit ball in **C**^{n}.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
9. Bounded mappings with parametric representation on the unit ball in **C**^{n}. Growth and coefficient bounds. Applications to extremal problems.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
10. Univalence criteria on the unit ball in **C**^{n} via the theory of Loewner chains.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
11. Kernel convergence and Loewner chains in **C**^{n}.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
12. Extension operators that preserve analytic and geometric properties. Open problems, conjectures, and research directions.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week

Bibliography

1. I. Graham, G. Kohr, *Geometric Function Theory in One and Higher Dimensions*, Marcel Dekker Inc., New York, 2003.
2. G. Kohr, *Basic Topics in Holomorphic Functions of Several Complex Variables*, Cluj University Press, Cluj-Napoca, 2003.
3. G. Kohr, *Geometric Function Theory in Several Complex Variables*, Seminar Notes, 2020.
4. F. Bracci, I. Graham, H. Hamada, G. Kohr, *Variation of Loewner chains, extreme and support points in the class* S^0 *in higher dimensions*, Constructive Approximation, **43** (2016), 231-251.
5. P. Duren, I. Graham, H. Hamada, G. Kohr, *Solutions for the generalized Loewner differential equation in several complex variables*, Mathematische Annalen, **347** (2010), 411-435.
6. I. Graham, H. Hamada, G. Kohr, M. Kohr, *Extremal properties associated with univalent subordination chains in* **C**^{n}, Mathematische Annalen, **359** (2014), 61-99.
7. I. Graham, H. Hamada, G Kohr, M Kohr, *Loewner PDE in infinite dimensions*, Computational Methods and Function Theory, **25** (2025), 151–171.
8. G. Kohr, P. Liczberski, *Univalent Mappings of Several Complex Variables*, Cluj University Press, Cluj-Napoca, 1998.
9. P. Curt, *Special Chapters in Geometric Function Theory of Several Complex Variables*, Editura Albastră, Cluj-Napoca, 2001 (in Romanian).
10. S. Gong, *Convex and Starlike Mappings in Several Complex Variables*, Kluwer Acad. Publ., Dordrecht, 1998.
11. S. Gong, The Bieberbach Conjecture, Amer. Math. Soc. Intern. Press, Providence, R.I., 1999.
12. S.G. Krantz, *Function Theory of Several Complex Variables*, Reprint of the 1992 Edition, AMS Chelsea Publishing, Providence, Rhode Island, 2001.
13. Ch. Pommerenke, *Univalent Functions*, Vandenhoeck & Ruprecht, Göttingen, 1975.
14. F. Bracci (Ed.), *Geometric Function Theory in Higher Dimension*, Springer INdAM Series, vol. **26** (2017), Springer International Publishing AG, Cham, Switzerland.

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

- The content of this discipline is in accordance with the curricula of the most important universities in Romania and abroad, where the advanced mathematics plays an essential role.
- This discipline is useful in specific PhD research activities, in preparing future researchers in pure and applied mathematics.

10. Evaluation

Activity type	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Percentage of final grade
10.4 Course	Knowledge of concepts and basic results.	Written exam.	60%
	Ability to justify by proofs theoretical results.		
10.5 Seminar	Ability to apply concepts and results acquired in the course in the study of advanced topics of geometric function theory in $\mathbf{C}^n$ and related area.	Evaluation of reports and homework during the semester, and active participation in the seminar activity. A midterm written test.	15% 25%
There are valid the official rules of the faculty concerning the attendance of students to teaching activities.			
10.6 Minimum standard of performance			
• The final grade should be at least 5 (from a scale of 1 to 10).			

11. Labels ODD (Sustainable Development Goals)[2]

	General label for Sustainable Development							
								9 INDUSTRY, INNOVATION AND INFRASTRUCTURE

Date:
11.04.2025

Signature of course coordinator

Prof.PhD. Mirela KOHR

Signature of seminar coordinator

Prof.PhD. Mirela KOHR

Date of approval:
25.04.2025

Signature of the head of department

Prof.PhD. Andrei MĂRCUŞ

[2] Keep only the labels that, according to the *Procedure for applying ODD labels in the academic process*, suit the discipline and delete the others, including the general one for *Sustainable Development* – if not applicable. If no label describes the discipline, delete them all and write „*Not applicable*.”.