SYLLABUS

Vector Optimization

University year 2025-2026

1. Information regarding the programme

1.1. Higher education institution	Babeş-Bolyai University
1.2. Faculty	Mathematics and Computer Science
1.3. Department	Mathematics
1.4. Field of study	Mathematics
1.5. Study cycle	Master
1.6. Study programme/Qualification	Mathematics and Computer Science
1.7. Form of education	Part-time education

2. Information regarding the discipline

2.1. Name of the discipline	**Vector Optimization**					Discipline code	**MME3403**
2.2. Course coordinator					Conf. dr. Trif Tiberiu		
2.3. Seminar coordinator					Conf. dr. Trif Tiberiu		
2.4. Year of study	**2**	2.5. Semester	**3**	2.6. Type of evaluation	**VP**	2.7. Discipline regime	DS

3. Total estimated time (hours/semester of didactic activities)

3.1. Hours per week	**3**	of which: 3.2 course	**2**	3.3 seminar/laboratory	**1**
3.4. Total hours in the curriculum	**42**	of which: 3.5 course	**28**	3.6 seminar/laborator	**14**
Time allotment for individual study (ID) and self-study activities (SA)					**hours**
Learning using manual, course support, bibliography, course notes (SA)					30
Additional documentation (in libraries, on electronic platforms, field documentation)					30
Preparation for seminars/labs, homework, papers, portfolios and essays					30
Tutorship					14
Evaluations					29
Other activities					
3.7. Total individual study hours	**133**				
3.8. Total hours per semester	**175**				
3.9. Number of ECTS credits	**7**				

4. Prerequisites (if necessary)

4.1. curriculum	Mathematical analysis 1 (Analysis on R); Mathematical analysis 2 (Differential Calculus on R^n).
4.2. competencies	Ability to use abstract notions, theoretical results and practical methods of Mathematical Analysis.

5. Conditions (if necessary)

5.1. for the course	blackboard, chalk, video projector
5.2. for the seminar /lab activities	blackboard, chalk

6.1. Specific competencies acquired [1]

Professional/essential competencies	• C1.1 Identifying concepts, describing theories and using specific language. • C1.4 Recognizing the main classes/types of mathematical problems and selecting appropriate methods and techniques for solving them. • C2.1 Identifying the basic concepts used in describing phenomena and processes. • C2.3 Applying appropriate theoretical analysis methods to the given issue.
Transversal competencies	• CT1. Applying rigorous and efficient work rules, demonstrating responsible attitudes towards the scientific and teaching field, for the optimal and creative use of one's own potential in specific situations, while respecting the principles and norms of professional ethics.

6.2. Learning outcomes

Knowledge	The student: - has acquired the specific skills of mathematics-related disciplines. - knows fundamental notions related to Vector Optimization.
Skills	The student is able to: - construct clear and well-supported mathematical arguments to explain mathematical problems, topics and ideas in writing. - prove theorems using mathematical language in theoretical courses and will be able to present these results both orally and in writing.
Responsibility and autonomy:	The student has the ability to - independently explore certain mathematical contents, drawing on previously acquired ideas and tools, in order to extend his/her knowledge. - independently extend previously acquired mathematical ideas and arguments to a mathematical topic that has not been previously studied.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	Students should acquire knowledge about vector (multicriteria) optimization.
7.2 Specific objective of the discipline	Students will study several classes of practical vector optimization problems.

[1] One can choose either competences or learning outcomes, or both. If only one option is chosen, the row related to the other option will be deleted, and the kept one will be numbered 6.

8. Content

8.1 Course	Teaching methods	Remarks
Week 1. Preorder relations; maximal elements of a set with respect to a preference relation; formulation of general optimization problems. Linear prorder relations (compatible with the vector addition and multiplication of vectors by scalars).	lecture, proof, examples	
Week 2. Cones; characterizations of (convex, pointed, generating, totally-generating) cones; the relationship between linear preorder relations and convex cones. Topological properties of convex cones: (relative) solid and closed convex cones; the polar cone of a set; polyhedral cones.	lecture, proof, examples	
Week 3. Concepts of efficiency in vector optimization; efficient points and weakly efficient points w.r.t. a convex cone; efficient solutions and weakly efficient solutions of vector optimization problems.	lecture, proof, examples	
Week 4. Monotone and strictly monotone scalar functions (w.r.t. a preorder relation) and the their extremum points; examples of linear/nonlinear monotone functions; conical sections of a set; the existence of efficient/weakly efficient points.	lecture, proof, examples	
Week 5. Sufficient conditions for efficiency and weak efficiency. Cone-convex sets; necessary conditions for weak-efficiency. Proper efficient points.	lecture, proof, examples	
Week 6. Cone-convex vector-valued functions, their characterizations by means of the epigraph and the polar cone; the cone-convexity of the images of convex sets by cone-convex functions.	lecture, proof, examples	
Week 7. Explicitly cone-quasiconvex functions and lexicographic quasiconvex vector-valued functions, their characterization and some of important properties; the relationship between explicit cone- convexity and lexicographic quasiconvexity.	lecture, proof, examples	
Week 8. Scalarization methods for vector optimization problems: the weighting method (for convex objective functions); the parametric method (for quasiconvex/, explicitly quasiconvex/ explicitly quasiaffine objective functions).	lecture, proof, examples	
Week 9. The geometric and topological structure of the boundary of a closed radiant set (the homeomorphism of Bonnisseau-Cornet).	lecture, proof, examples	
Week 10. Simply shaded and completely shaded sets (w.r.t. a convex cone) and their characterizations. The connectedness /contractibility of the sets of efficient points.	lecture, proof, examples	
Week 11. The role of Helly's Theorem in reducing the number of criteria involved in vector optimization with convex/quasiconvex objective functions.	lecture, proof, examples	
Week 12. Pareto reducible vector optimization problems involving explicitly / lexicographic	lecture, proof, examples	

quasiconvex objective functions.		
Week 13. Approximate efficient / weakly efficient solutions and their role in numerical methods.	lecture, proof, examples	
Week 14. Efficient sequences and their relationship with the minimizing sequences of certain scalarization functions.	lecture, proof, examples	

Bibliography

1. BRECKNER, B.E., POPOVICI, N.: Convexity and Optimization. An Introduction, EFES, Cluj-Napoca, 2006.
2. EHRGOT, M.: Multicriteria Optimization. Springer, Berlin Heidelberg New York, 2005.
3. GOEPFERT, A., RIAHI, H., TAMMER, C., ZALINESCU, C.: Variational Methods in Partially Ordered Spaces. Springer-Verlag, New York, 2003.
4. JAHN, J.: Vector Optimization. Theory, Applications, and Extensions. Springer, Berlin, 2004.
5. LUC, D.T.: Theory of Vector Optimization. Springer Verlag, Berlin, 1989.
6. POPOVICI, N.: Optimizare vectoriala, Casa Cartii de Stiinta, Cluj-Napoca, 2005.

8.2 Seminar / laboratory	**Teaching methods**	**Remarks**
Week 1. Geometric interpretation of the preference relations induced by the objective functions of some practical optimization problems (Fermat-Weber-type location problems, resource allocation problems, etc.)	Examples, dialogue, explanation, proof, problematization	
Week 2. Particular classes of convex cones in the *n*- dimensional Euclidean space (polyhedral cones, the lexicographic cone, Phelps-type cones).	Examples, dialogue, explanation, proof, problematization	
Week 3. Exercises involving the concepts of: polar cone, basis of a convex cone, the (relative) interior, and the facial structure of a convex cone.	Examples, dialogue, explanation, proof, problematization	
Week 4. Finding the efficient / weakly efficient solutions of certain vector optimization problems by a geometric approach.	Examples, dialogue, explanation, proof, problematization	
Week 5. Exercises concerning the (strict) monotony of certain scalar functions.	Examples, dialogue, explanation, proof, problematization	
Week 6. Identifying the (weakly) efficient solutions of some concrete vector optimization problems in R^2 by means of the necessary and sufficient conditions of (weakly) efficiency.	Examples, dialogue, explanation, proof, problematization	
Week 7. Geometric representations of the direct images of convex/polyhedral sets by certain cone-convex functions and their (weakly) efifcient points.	Examples, dialogue, explanation, proof, problematization	
Week 8. Geometric representation of the level sets of certain cone-quasiconvex vector-valued functions.	Examples, dialogue, explanation, proof, problematization	
Week 9. Exercises concerning explicitly quasiconvex functions (in particular, lexicographic convex functions and linear-fractional functions).	Examples, dialogue, explanation, proof, problematization	
Week 10. Bicriteria optimization problems solved by a geometrical approach.	Examples, dialogue, explanation, proof, problematization	
Week 11. Linear vector optimization problems solved by the weighting scalarization method.	Examples, dialogue, explanation, proof, problematization	
Week 12. Nonlinear vector optimization problems solved by the weighting scalarization method. Integration of some exact differential	Examples, dialogue, explanation, proof, problematization	

forms. Applications to the Green formula.		
Week 13. Linear vector optimization problems solved by the parametric method.	Examples, dialogue, explanation, proof, problematization	
Week 14. Nonlinear vector optimization problems solved by the parametric method.	Examples, dialogue, explanation, proof, problematization	

Bibliography

1. ALZORBA, S., GUNTHER, C., POPOVICI, N., TAMMER, C.: A new algorithm for solving planar multiobjective location problems involving the Manhattan norm, European Journal of Operational Research, Vol. 258 (1) 2017, pp. 35-46.
2. EHRGOT, M.: Multicriteria Optimization. Springer, Berlin Heidelberg New York, 2005.
3. POPOVICI, N.: Pareto reducible multicriteria optimization problems, Optimization, Vol. 54 (2005), pp. 253-263.
4. SAWARAGI, Y., NAKAYAMA, H., TANINO, T.: Theory of Multiobjective Optimization. Academic Press, New York, 1985.
5. YU, P.L.: Multiple criteria decision making: concepts, techniques and extensions. Plenum Press, New York - London, 1985.

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

The course ensures a solid theoretical background, according to national and international standards.

10. Evaluation

Activity type	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Percentage of final grade
10.4 Course	Knowledge of fundamental notions and results	Written paper	90%
10.5 Seminar/laboratory	Solving problems based on learned notions and theorems	Solving the exercises on the board	10%
10.6 Minimum standard of performance			

- Accumulation of 5 points on the exam (for a final grade of 5).

11. Labels ODD (Sustainable Development Goals)[2]

	General label for Sustainable Development							
								9 INDUSTRY, INNOVATION AND INFRASTRUCTURE

[2] Keep only the labels that, according to the *Procedure for applying ODD labels in the academic process*, suit the discipline and delete the others, including the general one for *Sustainable Development* – if not applicable. If no label describes the discipline, delete them all and write „*Not applicable*.".

Date: 11.04.2025	Signature of course coordinator Conf. dr. Trif Tiberiu	Signature of seminar coordinator Conf. dr. Trif Tiberiu

Date of approval: 25.04.2025	Signature of the head of department Prof. dr. Andrei Mărcuș