SYLLABUS

MULTI-VALUED ANALYSIS AND APPLICATIONS

University year 2025-2026

1. Information regarding the programme

1.1. Higher education institution	Babeş-Bolyai University
1.2. Faculty	Mathematics and Computer Science
1.3. Department	Mathematics
1.4. Field of study	Mathematics
1.5. Study cycle	Master
1.6. Study programme/Qualification	Advanced Mathematics/Master
1.7. Form of education	with frecvency

2. Information regarding the discipline

2.1. Name of the discipline	**Multi-valued Analysis and Applications**					Discipline code	**MME3402**
2.2. Course coordinator				Prof.dr. Adrian Petrusel			
2.3. Seminar coordinator				Prof.dr. Adrian Petrusel			
2.4. Year of study	2	2.5. Semester	3	2.6. Type of evaluation	VP	2.7. Discipline regime	compulsory

3. Total estimated time (hours/semester of didactic activities)

3.1. Hours per week	**3**	of which: 3.2 course	**2**	3.3 seminar/laboratory	**1**
3.4. Total hours in the curriculum	42	of which: 3.5 course	28	3.6 seminar/laborator	**14**
Time allotment for individual study (ID) and self-study activities (SA)					**hours**
Learning using manual, course support, bibliography, course notes (SA)					32
Additional documentation (in libraries, on electronic platforms, field documentation)					23
Preparation for seminars/labs, homework, papers, portfolios and essays					32
Tutorship					21
Evaluations					8
Other activities:					17
3.7. Total individual study hours	133				
3.8. Total hours per semester	175				
3.9. Number of ECTS credits	7				

4. Prerequisites (if necessary)

4.1. curriculum	Differential Equations, Mathematical Analysis (I-III), Topology, Nonlinear Applied Analysis
4.2. competencies	• operation with abstract concepts • the ability to make logical deductions • the ability to solve mathematical problems based on the learned concepts

5. Conditions (if necessary)

5.1. for the course	Black board, Video projector
5.2. for the seminar /lab activities	Black board, Video projector

6.1. Specific competencies acquired [1]

Professional/essential Competencies	- Ability to understand and manipulate concepts, results and advanced mathematical theories. - Ability to model and analyze from the mathematical point of view real processes from other sciences, economics, and engineering. - Ability to use the scientific language and to write scientific reports and papers. - Acquiring specific methods of multi-valued analysis theory (mainly related to nonlinear analysis) and its applications
Transversal competencies	- Ability to inform themselves, to work independently or in a team in order to realize studies and to solve complex problems. - Ability for continuous self-perfecting and study. - Ability to use advanced and complementary knowledge in order to obtain a PhD in Pure Mathematics and Applied Mathematics.

6.2. Learning outcomes

Knowledge	The student knows: the basic concepts and results in multi-valued analysis (general properties, metrizability, various notions of continuity), as well as the metric and topological fixed point theory for multi-valued operators and its applications to differential and integral inclusions
Skills	The student is able to: - construct clear and well-supported mathematical arguments to explain mathematical problems, topics and ideas in writing. - - demonstrate theorems using mathematical language in theoretical courses and will be able to present these results both orally and in writing.
Responsibility and autonomy:	The student has the ability to: - explore independently certain mathematical contents, relying on the ideas and tools already acquired, to expand their knowledge. - to work independently and to extend already acquired mathematical ideas and arguments to a mathematical topic that has not been previously studied.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	• to present the basic concepts and results in multi-valued analysis and fixed point theory for multi-valued operators and its applications to differential and integral inclusions
7.2 Specific objective of the discipline	To understand and and use creatively: • basic concepts and tools of metric spaces and Hausdorff-Pompeiu metric theory • main concepts concerning multi-valued operator theory • main concepts and results of metric fixed point theory, coincidence point theory and coupled fixed point theory for multi-valued operators • applications of the fixed point theory for multi-valued operators to differential and integral inclusions

[1] One can choose either competences or learning outcomes, or both. If only one option is chosen, the row related to the other option will be deleted, and the kept one will be numbered 6.

8. Content

8.1 Course	**Teaching methods**	**Remarks**
Functionals on the family of all subsets of a metric space: gap functional, excess functional, Hausdorff-Pompeiu functional, diameter functional	**Expositions**: description, explanation, class lectures, dialog-based lectures, lectures with demonstrations, introductive lectures, synthesis lectures. **Conversations**: debate, dialog, introductive conversations, conversations for knowledge consolidation, conversations to systematize and synthesize knowledge **Use of problems**: use of problem questions, problems and problem situations.	
Hausdorff-Pompeiu functional: basic properties	the same as before	
Continuity notions for multi-valued operators	the same as before	
Fixed point theorems for multi-valued operators: the multi-valued contraction principle	the same as before	
Generalizations of Nadler's Contraction Principle	the same as before	
Weakly Picard operator theory. Examples	the same as before	
Qualitative properties of the fixed point set	the same as before	
Coincidence point theory for multi-valued operators	the same as before	
Coupled fixed point theorems for multi-valued operators	the same as before	
Applications of the multi-valued analysis to integral and differential inclusions	the same as before	
Open problems in the theory of multi-valued operators	the same as before	

Bibliography

1. J.-P. Aubin, H. Frankowska, Set-Valued Analysis, Birkhauser, Basel, 1990.
2. S. Hu, N.S. Papageorgiou, Handbook of Multivalued Analysis, Vol. I and II, Kluwer Acad. Publ., Dordrecht, 1997 and 1999.
3. I.A. Rus, A. Petruşel, G. Petruşel, Fixed Point Theory, Presa Universitara Clujeana, 2008.
4. A. Granas, J. Dugundji, Fixed Point Theory, Springer, 2003.
5. A. Petruşel, Gh. Mot, G. Petruşel, Topics in Nonlinear Analysis and Applications to Mathematical Economics, House of the Book of Science, Cluj-Napoca, 2007.

8.2 Seminar / laboratory	**Teaching methods**	**Remarks**
Examples and exercises concerning gap functional, excess functional, Pompeiu-Hausdorff functional, diameter functional	**Conversations**: debate, dialog, introductive conversations, conversations for knowledge consolidation, conversations to systematize and synthesize knowledge	

	Use of problems: use of problem questions, problems and problem situations	
Examples and exercises concerning Hausdorff-Pompeiu functional	the same as before	
Examples and exercises concerning Hausdorff-Pompeiu functional (II)	the same as before	
Examples and exercises concerning continuity notions for multi-valued operators	the same as before	
Examples and exercises concerning continuity notions for multi-valued operators	the same as before	
Examples and exercises concerning the multi-valued contraction principle	the same as before	
Examples and exercises concerning generalizations of the multi-valued contraction principle	the same as before	
Examples and exercises concerning weakly Picard operators	the same as before	
Examples and exercises concerning coincidence point theorems	the same as before	
Examples and exercises concerning coupled fixed point theorems	the same as before	
Examples and exercises concerning applications of the fixed point theory for multi-valued operators	the same as before	
Examples and exercises concerning applications of the fixed point theory for multi-valued operators (II)	the same as before	

Bibliography

1. K. Deimling, Multivalued Differential Equations, W. de Gruyter, Basel, 1992.
2. L. Gorniewicz, Topological Fixed Point Theory of Multivalued Mappings, Kluwer Acad. Publ., Dordrecht, 1999.
3. A. Petruşel, Operatorial Inclusions, House of the Book of Science Cluj-Napoca, 2003
4. A. Granas, J. Dugundji, Fixed Point Theory, Springer, 2003.
5. I.A. Rus, A. Petruşel, G. Petruşel, Fixed Point Theory, Presa Univesritara Clujeana, 2008.

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

- The syllabus of this course is focused on the multivalued operator theory, as a basis for a good research activity through the Doctoral School in Mathematics.
- The content of this discipline is in accordance with the curricula of the most important universities in Romania and abroad, where multi-valued analysis plays an essential role. This discipline is useful in preparing future teachers and researchers in pure and applied mathematics, as well as those who use mathematical models and advanced methods of study in other areas.

10. Evaluation

Activity type	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Percentage of final grade
10.4 Course	Knowledge of concepts and basic results	Middle term written test	40%
	Ability to justify by proofs theoretical results	Final written test	40%
10.5 Seminar/laboratory	Ability to apply concepts and results	Oral reports	10%

	Ability to prove new results in multi-valued analysis	Oral reports	10%
10.6 Minimum standard of performance			
• Successful passing of the exam is conditioned by the final grade that has to be at least 5. • All university official rules with respect to students attendance of academic activities, as well as to cheating and plagiarism are valid and enforced.			

11. Labels ODD (Sustainable Development Goals)[2]

	General label for Sustainable Development							
								

Date:
11.04.2025

Signature of course coordinator

Prof.dr. Adrian Petrusel

Signature of seminar coordinator

Prof.dr. Adrian Petrusel

Date of approval:
25.04.2025

Signature of the head of department

Prof. dr. Andrei Mărcuș

[2] Keep only the labels that, according to the *Procedure for applying ODD labels in the academic process*, suit the discipline and delete the others, including the general one for *Sustainable Development* – if not applicable. If no label describes the discipline, delete them all and write „*Not applicable.*”.