**SYLLABUS**

**Integral Equations with Applications**

**University year 2025-2026**

**1. Information regarding the programme**

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| 1.1 Higher education institution | **Babeş-Bolyai University Cluj-Napoca** |
| 1.2 Faculty | **Faculty of Mathematics and Computer Science** |
| 1.3 Department | **Department of Mathematics** |
| 1.4 Field of study | **Mathematics** |
| 1.5 Study cycle | **Master** |
| 1.6 Study programme / Qualification | **Advanced Mathematics** |
| 1.7 Form of education | **Full-Time** |

**2. Information regarding the discipline**

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|---|---|---|---|---|---|---|---|
| 2.1 Name of the discipline | | | **Integral Equations with Applications (Ecuaţii integrale cu aplicaţii)** | | | Discipline code | **MME3160** |
| 2.2 Course coordinator | | | **Prof. Sanda Micula, PhD. Habil.** | | | | |
| 2.3 Seminar coordinator | | | **Prof. Sanda Micula, PhD. Habil.** | | | | |
| 2.4. Year of study | **2** | 2.5 Semester | **3** | 2.6. Type of evaluation | **E** | 2.7 Type of discipline | **DS Optional** |

**3. Total estimated time** (hours/semester of didactic activities)

| | | | | | |
|---|---|---|---|---|---|
| 3.1 Hours per week | **3** | Of which: 3.2 course | **2** | 3.3 seminar/laboratory | **1 sem** |
| 3.4 Total hours in the curriculum | **42** | Of which: 3.5 course | **28** | 3.6 seminar/laboratory | **14** |
| **Time allotment for individual study (ID) and self-study activities (SA)** | | | | | hours |
| Learning using manual, course support, bibliography, course notes (SA) | | | | | 40 |
| Additional documentation (in libraries, on electronic platforms, field documentation) | | | | | 30 |
| Preparation for seminars/labs, homework, papers, portfolios and essays | | | | | 40 |
| Tutorship | | | | | 14 |
| Evaluations | | | | | 9 |
| Other activities: ..... | | | | | |

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| 3.7 Total individual study hours | 133 |
| 3.8 Total hours per semester | 175 |
| 3.9 Number of ECTS credits | 7 |

**4. Prerequisites** (if necessary)

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| 4.1. curriculum | • Mathematical Analysis, Numerical Analysis |
| 4.2. competencies | • Knowledge of basic notions of operator theory<br>• Average programming skills |

**5. Conditions** (if necessary)

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| 5.1. for the course | • Classroom with large blackboard and video projector |
| 5.2. for the seminar /lab activities | • Classroom with large blackboard and video projector/computers with Matlab |

**6. Specific competencies acquired**

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| **Professional competencies** | C4.1 Defining basic concepts, theory and mathematical models<br>C4.2 Interpretation of mathematical models<br>C4.3 Identifying the appropriate models and methods for solving real-life problems<br>C4.5 Embedding formal models in applications from various areas |
| **Transversal competencies** | CT1 Application of efficient and organized work rules, of responsible attitudes towards the didactic-scientific domain, to creatively value one's own potential, with the respect towards the principles and norms of professional etic.<br>CT3 Use of efficient methods and techniques to learn, inform, research and develop the abilities to value the knowledge, to adapt to requirements of a dynamic society and to communicate in Romanian language and in a language of international circulation. |

**7. Objectives of the discipline** (outcome of the acquired competencies)

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| 7.1 General objective of the discipline | • Acquire knowledge of the general theory of integral equations, with focus on applications.<br>• Gain the ability to apply concepts and results from integral equations theory to specific problems. |
| 7.2 Specific objective of the discipline | • Understand and be able to use main concepts and results from general integral equations theory.<br>• Be able to analyze the solvability of specific integral equations arising in applications.<br>• Understand, use and be able to derive numerical methods for the approximate solution of integral equations arising in applications from various fields. |

**8. Content**

| 8.1 Course | Teaching methods | Remarks |
|---|---|---|
| **1. Introduction**. Basic concepts. History of integral equations. Classifications and examples. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Description | |
| **2.** Types of integral equations with exact solutions. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Description | |
| **3.** Relationship between initial value/boundary value problems and integral equations. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Description | |
| **4. Volterra integral equations**. The method of successive approximations. Laplace transforms. Adomian decomposition. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Description | |
| **5.** Series solution. Volterra integral equations of the first kind. Integral equations of the convolution type. Abel integral equation. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Description | |
| **6. Fredholm integral equations.** The method of | • Interactive exposure | |

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|---|---|---|
| successive approximations, Neumann series. Adomian decomposition. Compact integral operators. Properties. The Fredholm alternative theorem. | • Explanation<br>• Conversation<br>• Description | |
| **7.** Homogeneous Fredholm equations. Fredholm integral equations of the first kind. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Description | |
| **8. Numerical methods**. Degenerate kernel methods. Taylor series approximation. Interpolatory degenerate kernel approximation. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Description | |
| **9.** Projection methods, collocation and Galerkin methods. Iterated collocation and Galerkin methods. Error analysis. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Description | |
| **10.** Nyström methods. Product integration methods. Error analysis. Discrete collocation and discrete Galerkin methods. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Description | |
| **11. Applications**. Volterra's population model. Diffraction problems, Fresnel integrals. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Description | |
| **12.** Applications to potential theory. The Thomas-Fermi equation. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Description | |
| **13.** Applications to ocean waves. Green's function method for waves. Seismic response of dams. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Description | |
| **14.** Heat transfer and heat radiation. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Description | |

**Bibliography**

1. M. Rahman, Integral Equations and their Applications, WIT Press, Ashurst, Southampton, 2007.
2. A. M. Wazwaz, Linear and Nonlinear Integral Equations, Methods and Applications. Higher Education Press, Beijing. Springer, New York, 2011.
3. K. E. Atkinson, The Numerical Solution of Integral Equations of the Second Kind, Cambridge University Press, Cambridge,1997.
4. S. Micula, G. V. Milovanović, Chapter 16: Iterative Processes and Integral Equations of the Second Kind, Book: Matrix and Operator Equations and Applications, Birkhäuser, Springer Nature, Heidelberg, 2023.
5. A. D. Polyanin, A. V. Manzhirov, Handbook of Integral Equations, 2nd ed., CRC Press, Boca Raton, 2008.
6. S. Prössdorf, B. Silbermann, Numerical Analysis for Integral and Related Operator Equations, Wiley, Oxford, 1991.

| 8.2 Seminar / laboratory | Teaching methods | Remarks |
|---|---|---|
| **1.** Relationship between initial value/boundary value problems and integral equations. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Individual and group work | The seminar is structured as 2 hours per week, every other week |
| **2.** Solvable integral equations. | • Interactive exposure<br>• Explanation<br>• Conversation | |

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|---|---|---|
| | • Individual and group work | |
| **3.** Volterra integral equations. Abel's integral equation. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Individual and group work | |
| **4.** Fredholm integral equations. Mixed integral equations. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Individual and group work | |
| **5.** Interpolation-based collocation and Galerkin methods. Iterated solutions. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Individual and group work | |
| **6.** Nyström methods. Product integration. Discrete projection methods. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Individual and group work | |
| **7.** Various applications. | • Interactive exposure<br>• Explanation<br>• Conversation<br>• Individual and group work | |

**Bibliography**

1. M. Rahman, Integral Equations and their Applications, WIT Press, Ashurst, Southampton, 2007.
2. A. M. Wazwaz, Linear and Nonlinear Integral Equations, Methods and Applications. Higher Education Press, Beijing. Springer, New York, 2011.
3. K. E. Atkinson, The Numerical Solution of Integral Equations of the Second Kind, Cambridge University Press, Cambridge,1997.
4. S. Micula, G. V. Milovanović, Chapter 16: Iterative Processes and Integral Equations of the Second Kind, Book: Matrix and Operator Equations and Applications, Birkhäuser, Springer Nature, Heidelberg, 2023.
5. A. D. Polyanin, A. V. Manzhirov, Handbook of Integral Equations, 2nd ed., CRC Press, Boca Raton, 2008.
6. S. Prössdorf, B. Silbermann, Numerical Analysis for Integral and Related Operator Equations, Wiley, Oxford, 1991.

**9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program**

- Courses with similar content exist in the studying program of major universities in Romania and abroad, for Mathematics and Applied Mathematics students at the Master's level;
- The knowledge and skills acquired in this course give students a foundation for launching a career in scientific research;
- The analysis and modeling abilities acquired in this course are useful in any career path students may choose.

**10. Evaluation**

| Type of activity | 10.1 Evaluation criteria | 10.2 Evaluation methods | 10.3 Share in the grade (%) |
|---|---|---|---|
| 10.4 Course | - acquire the basic principles and notions in Integral Equations theory<br>- apply correctly various course concepts and methods | **Written exam** | **70%** |
| 10.5 Seminar/lab activities | - understand and be able to use theory and results in problems and applications<br>- apply numerical procedures and algorithms to solve practical and | - active participation in discussing and solving problems throughout the semester<br>- individual presentation | **30%** |

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|---|---|---|---|
| | real-life problems | of solutions | |
| 10.6 Minimum performance standards | | | |
| A grade of 5 or above (on a scale from 1 to 10) on **each** of the activities mentioned above (written exam, seminar evaluation) | | | |

**11. Labels ODD (Sustainable Development Goals)[1]**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | General label for Sustainable Development | | | | | | | |
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| Date | Signature of course coordinator | Signature of seminar coordinator |
|---|---|---|
| 29.04.2025 | Prof. Sanda Micula, PhD. Habil. | Prof. Sanda Micula, PhD. Habil. |

Date of approval

Signature of the head of department

Prof. dr. Andrei Mărcuş

[1] Keep only the labels that, according to the *Procedure for applying ODD labels in the academic process*, suit the discipline and delete the others, including the general one for *Sustainable Development* – if not applicable. If no label describes the discipline, delete them all and write „*Not applicable.*".