

## SYLLABUS

### Homological Algebra

University year 2025-2026

#### 1. Information regarding the programme

1.1 Higher education institution	Universitatea Babeş-Bolyai Cluj-Napoca
1.2 Faculty	Matematică şi Informatică
1.3 Department	Matematică
1.4 Field of study	Matematică
1.5 Study cycle	Master
1.6 Study programme / Qualification	Advanced Mathematics
1.7. Form of education	Cu frecvenţă

#### 2. Information regarding the discipline

2.1. Name of the discipline		Homological Algebra				Discipline code		MME3112	
2.2. Course coordinator					Prof. Simion Breaz				
2.3. Seminar coordinator					Prof. Simion Breaz				
2.4. Year of study	2	2.5. Semester	3	2.6. Type of evaluation		E	2.7. Discipline regime		compulsory

#### 3. Total estimated time (hours/semester of didactic activities)

3.1. Hours per week	<b>3</b>	of which: 3.2 course	<b>2</b>	3.3 seminar/laboratory	<b>1</b>
3.4. Total hours in the curriculum	42	of which: 3.5 course	28	3.6 seminar/laborator	<b>14</b>
<b>Time allotment for individual study (ID) and self-study activities (SA)</b>					<b>hours</b>
Learning using manual, course support, bibliography, course notes (SA)					46
Additional documentation (in libraries, on electronic platforms, field documentation)					36
Preparation for seminars/labs, homework, papers, portfolios and essays					32
Tutorship					24
Evaluations					20
Other activities:					
3.7. Total individual study hours	158				
3.8. Total hours per semester	200				
3.9. Number of ECTS credits	8				

#### 4. Prerequisites (if necessary)

4.1. curriculum	
4.2. competencies	

#### 5. Conditions (if necessary)

5.1. for the course	
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5.2. for the seminar /lab activities	
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### 6.1. Specific competencies acquired <sup>1</sup>

Professional/essential competencies	<p>Knowledge, understanding and use of main concepts and results in Homological Algebra (complexes, homology and cohomology, derived functors)</p> <p>Ability to use fundamental theoretical concepts and in various fields of mathematics fields of mathematics (Algebra, Topology, Banach Spaces, Fixed Point Theory)</p>
Transversal competencies	<p>Ability to inform themselves, to work independently or in a team;</p> <p>Ability to approach complex problems and to use information from various specific fields;</p> <p>Ability to identify and use advanced techniques and methods in order to realize a specific research.</p>

### 6.2. Learning outcomes

Knowledge	The graduate knows fundamental notions related to Homological Algebra, and methods of applying various mathematical structures.
Skills	The graduate is able to ensure the formation of skills specific to the Mathematics-related disciplines needed to complete the assignments.
Responsibility and autonomy:	The graduate is able explore some mathematical content independently, drawing on ideas and tools from previous coursework to extend their understanding.

### 7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	<p>Knowledge, understanding and use of main concepts and results in Homological Algebra</p> <p>Ability to use concepts and fundamental results in some specific fields of mathematics (module theory, topological spaces, Banach spaces)</p>
7.2 Specific objective of the discipline	<p>Understanding the basic concepts about categories, complexes, resolutions, sheaves.</p> <p>Ability to use specific derived functors (Ext, Tor, Pext) in concrete situations.</p>

<sup>1</sup> One can choose either competences or learning outcomes, or both. If only one option is chosen, the row related to the other option will be deleted, and the kept one will be numbered 6.

## 8. Content

8.1 Course	Teaching methods	Remarks
1. Preliminaries	Lectures, didactical demonstration, conversation.	
2. Modules	Lectures, didactical demonstration, conversation.	
3. Categories	Lectures, didactical demonstration, conversation.	
4. Limits and colimits	Lectures, didactical demonstration, conversation.	
5. Functors	Lectures, didactical demonstration, conversation.	
6. Injective and projective modules	Lectures, didactical demonstration, conversation.	
7. Flat modules	Lectures, didactical demonstration, conversation.	
8. Complexes	Lectures, didactical demonstration, conversation.	
9. Homology functors	Lectures, didactical demonstration, conversation.	
10. Derived functors	Lectures, didactical demonstration, conversation.	
11. Ext	Lectures, didactical demonstration, conversation.	
12. Tor	Lectures, didactical demonstration, conversation.	
13. Sheaves	Lectures, didactical demonstration, conversation.	
14. Sheaf cohomology	Lectures, didactical demonstration, conversation.	
<b>Bibliography</b> 1. I. Moerdijk: Notes on Homological Algebra, course notes, <a href="http://www.math.ru.nl/topology/Notes%20on%20Homological%20Algebra.pdf">www.math.ru.nl/topology/Notes%20on%20Homological%20Algebra.pdf</a> 2. J.J. Rotman: An Introduction to Homological Algebra, Springer, 2009		
8.2 Seminar / laboratory	Teaching methods	Remarks
1. The fundamental group	problematization, exercises, problem solving,	
2. Modules	problematization, exercises, problem solving,	
3. Example of Categories	problematization, exercises, problem solving,	
4. Categories of Banach spaces	problematization, exercises, problem solving,	

5. The additive category of Banach spaces	problematization, exercises, problem solving,	
6. The category of Abelian groups	problematization, exercises, problem solving,	
7. Flat modules	problematization, exercises, problem solving,	
8. Directed limits	problematization, exercises, problem solving,	
9. Inverse limits	problematization, exercises, problem solving,	
10. Functors	problematization, exercises, problem solving,	
11. Ext and Tor	problematization, exercises, problem solving,	
12. Ext and Tor for abelian groups	problematization, exercises, problem solving,	
13. Relative homological algebra	problematization, exercises, problem solving,	
14. Projective, injective and flat Banach spaces	problematization, exercises, problem solving,	
Bibliography  1. S. Breaz, G. Calugareanu, G. Miodo, D. Valcan: Exercises in Abelian Group Theory, Kluwer 2003. 2. J. Cigler, V. Losert, P. Michor: Banach Modules and Functors on Categories of Banach Spaces, Marcel Dekker, 1979. 3. A. Hatcher: Algebraic Topology, Cambridge University Press, 2001, <a href="http://www.math.cornell.edu/~hatcher/AT/AT.pdf">http://www.math.cornell.edu/~hatcher/AT/AT.pdf</a> 4. C. Schochet: A Pext primer: Pure extensions and $\lim^1$ for infinite abelian groups, NYJM Monographs, 2003, <a href="http://nyjm.albany.edu/m/2003/1v.pdf">http://nyjm.albany.edu/m/2003/1v.pdf</a>		

### 9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

The content is in accordance with the curricula of many important universities where pure mathematics plays important places in their research.


This discipline is useful since it realizes connections between various mathematical domains, and it is well known that the methods of homological algebra were used during the time to solve important problems in mathematics.

The methods and tools presented here are often used in specific PhD research activities.

### 10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the grade (%)
10.4 Course	Concepts and basic results	Final exam	50%
	Standard examples		
10.5 Seminar/lab activities	Ability to use the concepts in order to solve problems	Final exam and a midterm test.	25%+25%
10.6 Minimum performance standards			
At least grade 5 from 10.			

11. Labels ODD (Sustainable Development Goals)<sup>2</sup>

	General label for Sustainable Development							
								

Date:  
11.04.2025

Signature of course coordinator  
Prof. Simion Breaz

Signature of seminar coordinator  
Prof. Simion Breaz

Date of approval:  
25.04.2025

Signature of the head of department  
Prof. dr. Andrei Mărcuș

Date  
16.04.2024

Signature of course coordinator  
Prof. Simion Breaz

Signature of seminar coordinator  
Prof. Simion Breaz

Date of approval

Signature of the head of department  
Prof. Andrei Marcus

<sup>2</sup> Keep only the labels that, according to the [Procedure for applying ODD labels in the academic process](#), suit the discipline and delete the others, including the general one for *Sustainable Development* – if not applicable. If no label describes the discipline, delete them all and write „*Not applicable.*”.