SYLLABUS

Applied Functional Analysis

University year 2025-2026

1. Information regarding the programme

1.1. Higher education institution	Babeş-Bolyai University
1.2. Faculty	Mathematics and Computer Science
1.3. Department	Mathematics
1.4. Field of study	Mathematics
1.5. Study cycle	Master
1.6. Study programme/Qualification	Advanced Mathematics
1.7. Form of education	Attendance

2. Information regarding the discipline

2.1. Name of the discipline	Applied Functional Analysis					Discipline code	**MME3005**
2.2. Course coordinator Assoc Prof. PHD Brigitte Breckner					Master		
2.3. Seminar coordinator Assoc. Prof. PHD Brigitte Breckner					Advanced Mathematics		
2.4. Year of study	1	2.5. Semester	2	2.6. Type of evaluation	E	2.7. Discipline regime	elective

3. Total estimated time (hours/semester of didactic activities)

3.1. Hours per week	**3**	of which: 3.2 course	**2**	3.3 seminar/laboratory	**1**
3.4. Total hours in the curriculum	42	of which: 3.5 course	28	3.6 seminar/laborator	**14**
Time allotment for individual study (ID) and self-study activities (SA)					**hours**
Learning using manual, course support, bibliography, course notes (SA)					32
Additional documentation (in libraries, on electronic platforms, field documentation)					23
Preparation for seminars/labs, homework, papers, portfolios and essays					32
Tutorship					21
Evaluations					8
Other activities:					17

3.7. Total individual study hours	**133**
3.8. Total hours per semester	**175**
3.9. Number of ECTS credits	**7**

4. Prerequisites (if necessary)

4.1. curriculum	linear algebra; general topology; mathematical analysis; the attendance of the functional analysis course from the bachelor level is NOT necessary
4.2. competencies	abstract and logical thinking

5. Conditions (if necessary)

5.1. for the course	blackboard, chalk, video projector
5.2. for the seminar /lab activities	blackboard, chalk

6.1. Specific competencies acquired [1]

[1] One can choose either competences or learning outcomes, or both. If only one option is chosen, the row related to the other option will be deleted, and the kept one will be numbered 6.

Professional/essential competencies	C1.1 To identify the appropriate notions, to describe the speficic topic and to use an appropriate language. C1.3 To apply correctly basic methods and principles in order to solve mathematical problems.
Transversal competencies	CT1 To apply efficient and rigorous working rules, to manifest responsible attitudes towards the scientific and didactic fields, respecting the professional and ethical principles.

6.2. Learning outcomes

Knowledge	The student - has acquired the mathematics discipline-specific skills needed to complete homework, - knows fundamental concepts related to functional analysis as well as methods of applying them in areas of science related to mathematics and computer science.
Skills	The student is able to: - construct clear and well-supported mathematical arguments to explain mathematical problems, topics and ideas in writing, - prove theorems using mathematical language in theoretical courses and will be able to present these results both orally and in writing.
Responsibility and autonomy:	The student has the ability to work independently to obtain: - independently explore some mathematical content, building on ideas and tools already learned to extend their knowledge, - independently extend mathematical ideas and arguments already learned to a mathematical topic not previously studied.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	• Presentation of the spectral theory of operators on Banach spaces, resp., on Hilbert spaces • Presentation of various applications of the spectral theory of operators
7.2 Specific objective of the discipline	• Acquirement of knowledge specific to higher functional analysis • To become familiar with the abstract thinking and the problematization specific to functional analysis

8. Content

8.1 Course	Teaching methods	Remarks
1. Normed spaces (definition; properties; Banach spaces; inner product spaces; Hilbert spaces; examples)	Lecture with mathematical proofs, problematization, discussion	

2. Linear continuous operators between normed spaces (characterizations of the continuity of linear operators between normed spaces; the normed space of linear continuous operators between normed spaces)	Lecture with mathematical proofs, problematization, discussion	
3. Linear continuous operators between normed spaces (the open mapping theorem; the bounded inverse theorem; the closed graph theorem)	Lecture with mathematical proofs, problematization, discussion	
4. Linear continuous functionals on normed spaces (characterizations of the continuity of linear functionals; the dual of a normed space; the dual of a Hilbert space). Reflexive normed spaces	Lecture with mathematical proofs, problematization, discussion	
5. Spectral theory of operators on Banach spaces (closed operators; the resolvent set, the resolvent, the spectrum, the point spectrum, the approximative point spectrum, the continuous spectrum, and the residual spectrum of an operator)	Lecture with mathematical proofs, problematization, discussion	
6. Spectral theory of operators on Banach spaces (the adjoint of a vector subspace of the product of two normed spaces; the adjoint of a linear densely defined operator)	Lecture with mathematical proofs, problematization, discussion	
7. Spectral theory of operators on Banach spaces (relationships between a linear densely defined operator and its adjoint; properties of the resolvent set and of the spectrum of adjoint operators)	Lecture with mathematical proofs, problematization, discussion	
8. Spectral theory of operators on Banach spaces (compact operators; characterizations and properties of compact operators)	Lecture with mathematical proofs, problematization, discussion	
9. Spectral theory of operators on Banach spaces (the Riesz-Schauder theorem for compact operators; the spectral theorem for compact operators)	Lecture with mathematical proofs, problematization, discussion	
10. Spectral theory of operators on Hilbert spaces (the adjoint operator of a linear continuous operator between Hilbert spaces; properties of the adjoint operator; unitary, selfadjoint, normal, and symmetric operators)	Lecture with mathematical proofs, problematization, discussion	
11. Spectral theory of operators on Hilbert spaces (the Hellinger-Toeplitz theorem; spectral properties of normal operators; spectral properties of selfadjoint operators)	Lecture with mathematical proofs, problematization, discussion	
12. Spectral theory of operators on Hilbert spaces (the spectral theorem for compact, selfadjoint operators; the spectral theorem for compact, normal operators)	Lecture with mathematical proofs, problematization, discussion	
13. Banach algebras (definition; the resolvent set, the spectrum, and the resolvent of an element; properties of the resolvent; ideals and maximal ideals in Banach algebras; characters)	Lecture with mathematical proofs, problematization, discussion	
14. Banach algebras (the Gelfand space of a Banach algebra; the theorem of Gelfand-Mazur; the Gelfand transform)	Lecture with mathematical proofs, problematization, discussion	

Bibliography

1. BRECKNER W. W.: Analiză funcţională, Presa Universitară Clujeană, Cluj-Napoca, 2009.

2. BREZIS H.: Analiză funcţională. Teorie şi aplicaţii, Ed. Academiei Române, Bucureşti, 2002.

3. DUNFORD N. and SCHWARTZ J. T.: Linear Operators. Part 1: General theory, Interscience Publishers, New York, 1958.

4. DUNFORD N. and SCHWARTZ J. T.: Linear Operators. Part 2: Spectral theory, Interscience Publishers, New York, 1963.

5. HEUSER H.: Funktionalanalysis. Theorie und Anwendung, 3. Auflage, B. G. Teubner, Stuttgart, 1992.

6. WERNER D.: Funktionalanalysis, Vierte, überarbeitete Auflage., Springer-Verlag, Berlin - Heidelberg - New York, 2002.

8.2 Seminar / laboratory	**Teaching methods**	**Remarks**
1. From the oscillating swing to Fourier series	Problematization, discussion, team work	
2. Examples of operators (integral, differentiation, interpolation, approximation, from quantum mechanics)	Problematization, discussion, team work	
3. Applications of the results presented in the third lecture (initial value problems for linear differential equations; approximate solutions of operator equations)	Problematization, discussion, team work	
4. Examples/Counterexamples for reflexive normed spaces	Problematization, discussion, team work	
5. The spectral radius of an operator. Determination of the resolvent set, the spectrum, the point spectrum, the approximative point spectrum, the continuous spectrum, and the residual spectrum of concrete operators	Problematization, discussion, team work	
6. Determination of the resolvent set, the spectrum, the point spectrum, the approximative point spectrum, the continuous spectrum, and the residual spectrum of concrete operators	Problematization, discussion, team work	
7. A characterization of adjoint operators. Examples	Problematization, discussion, team work	
8. Examples of compact operators	Problematization, discussion, team work	
9. Applications of the spectral theorem for compact operators (the Sturm-Liouville eigenvalue problem)	Problematization, discussion, team work	
10. Unitary, selfadjoint, normal, and symmetric operators on Hilbert spaces (examples; properties)	Problematization, discussion, team work	
11. Unitary, selfadjoint, normal, and symmetric operators on Hilbert spaces (examples; properties)	Problematization, discussion, team work	
12. Applications of the spectral theorems presented in the lecture (the square root of a positive operator)	Problematization, discussion, team work	
13. Banach algebras (examples)	Problematization, discussion, team work	
14. Banach algebras (the Gelfand space of concrete Banach algebras)	Problematization, discussion, team work	

Bibliography

1. BREZIS H.: Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, 2011.

2. HEUSER H.: Funktionalanalysis. Theorie und Anwendung, 3. Auflage. B. G. Teubner, Stuttgart, 1992.

3. WERNER D.: Funktionalanalysis. Vierte, überarbeitete Auflage, Springer-Verlag, Berlin - Heidelberg - New York, 2002 .

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

- Functional analysis is one of the most important branches of mathematics, having applications in various domains (numerical analysis, approximation theory, optimization, PDEs, probability theory, mathematical and theoretical physics). This discipline both provides the theoretical background for such applications and gives samples of them.

10. Evaluation

Activity type	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Percentage of final grade
10.4 Course	Knowledge of concepts and basic results	Midterm written test	45%
	Ability to perform proofs	Final written test	45%
10.5 Seminar/laboratory	Ability to apply concepts and results acquired in the lecture	Own contributions to the exercise classes	10%
	There are valid the official rules of the faculty concerning the attendance of students to teaching activities.		
10.6 Minimum standard of performance			
• Basic knowledge on the topics from the lectures and seminars			

11. Labels ODD (Sustainable Development Goals)[2]

	General label for Sustainable Development							
								9 INDUSTRY, INNOVATION AND INFRASTRUCTURE

Date:
11.04.2025

Signature of course coordinator

Assoc. Prof. PHD Brigitte Breckner

Signature of seminar coordinator

Assoc. Prof. PHD Brigitte Breckner

---

[2] Keep only the labels that, according to the *Procedure for applying ODD labels in the academic process*, suit the discipline and delete the others, including the general one for *Sustainable Development* – if not applicable. If no label describes the discipline, delete them all and write „*Not applicable*.".

Date of approval:
25.04.2025

Signature of the head of department

Prof. dr. Andrei Mărcuș